General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAS2/W**
Unit Statistics 2

Wednesday 23 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS2/W.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The number of mobile phone calls, X, that Pat makes each day to her boyfriend James has a Poisson distribution with a mean of 4.0. The number of text messages, Y, that James sends each day to Pat has a Poisson distribution with a mean of 3.5. Assume that X and Y are independent random variables.

(a) Calculate the probability that, on a randomly chosen day:

(i) Pat phones James on more than 8 occasions; *(2 marks)*

(ii) James sends fewer than 2 text messages to Pat. *(2 marks)*

(b) The total number of phone calls made and text messages sent, T, is a Poisson random variable with mean λ.

(i) Write down the value of λ. *(1 mark)*

(ii) Calculate the probability that, on a randomly selected day, T is at least eleven. *(2 marks)*

2 In recent practice sessions, a netball player counts the number of attempts at goal that she makes in order for her to score a goal, each attempt being independent of every other attempt.

The results are recorded in the table below.

Number of attempts	1	2	3	4	5	6	7
Frequency	20	14	8	3	3	1	1

(a) Write down:

(i) her total number of attempts;

(ii) the total number of goals that she scores. *(2 marks)*

(b) Hence show that an estimate for the probability of her scoring a goal at each attempt is 0.446, correct to three decimal places. *(1 mark)*

(c) (i) Define a probability distribution that may be an appropriate model for the above data. *(1 mark)*

(ii) Use a χ^2 goodness-of-fit test and the 5% level of significance to determine whether your probability distribution is an appropriate model for the given data. (Expected frequencies should be given to two decimal places.) *(8 marks)*

3 The Upgrade School has 900 students. Assume that the probability that any particular student is excluded from school during a year is 0.01, and that each exclusion is independent of every other.

(a) Write down the probability distribution for the discrete random variable X, the number of students excluded from Upgrade School during a year. *(1 mark)*

(b) Calculate values for the mean and variance of X. *(2 marks)*

(c) (i) State a distributional approximation for X, giving a reason for your choice. *(2 marks)*

(ii) Use your distributional approximation to calculate the probability that more than 15 students are excluded from Upgrade School during a year. *(2 marks)*

(d) Comment on the assumption that:

(i) the probability that any particular student is excluded from school during a year is 0.01; *(1 mark)*

(ii) each exclusion is independent of every other. *(1 mark)*

4 A continuous random variable T has the probability density function $\mathrm{f}(t)$ given by:

$$\mathrm{f}(t) = \begin{cases} \frac{1}{18}t^2 & 0 \leqslant t \leqslant 3 \\ \frac{1}{4}(5 - t) & 3 \leqslant t \leqslant 5 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of f. *(2 marks)*

(b) Determine the distribution function, $\mathrm{F}(t)$, for $0 \leqslant t \leqslant 5$. *(4 marks)*

(c) Hence, or otherwise, calculate $\mathrm{P}(T < 4)$. *(2 marks)*

5 A golf club sells bottles of water in its professional's shop. The amount of water, Y millilitres, in a bottle has a normal distribution with a mean of μ and a standard deviation of 4.

The supplier claims that the mean amount of water per bottle is 300 ml. The golf club members suspected that, over a period of time, the mean amount of water per bottle had decreased. In order to investigate their suspicion, they decided to take a random sample of 20 bottles. From this sample, their calculated value for $\overline{Y}$, the mean amount of water per bottle, is 298.1 ml.

(a) Stating null and alternative hypotheses, test, at the 1% level of significance, their suspicion that the mean amount of water per bottle is now less than 300 ml. *(6 marks)*

(b) For the 1% level of significance, calculate the critical region for $\overline{Y}$, giving your answer to one decimal place. *(2 marks)*

(c) Hence calculate the probability of a Type II error given that in fact $\mu = 296.5$. *(3 marks)*

6 (a) Anchal travels to work each day by initially walking from home to the bus stop, waiting for the bus to arrive and then travelling by bus to her office.

The time, A_1 minutes, that she spends walking from home to the bus stop is a normal random variable with a mean of 12 and a variance of 3.

The time, A_2 minutes, that she spends waiting for the bus to arrive is a normal random variable with a mean of 6 and a variance of 2.

The time, A_3 minutes, spent travelling by bus to her office is a normal random variable with a mean of 32 and a variance of 20.

(i) Assuming that A_1, A_2 and A_3 are independent, write down the distribution of T_A, the total time, in minutes, for Anchal's journey to work. *(2 marks)*

(ii) Calculate the probability that, on a randomly selected day, Anchal will take less than one hour to travel to work. *(2 marks)*

(b) Baldeep drives to work each day. The total time, T_B minutes, that he spends driving to work is a normal random variable with a mean of 53 and a variance of 16. Calculate the probability that, on a randomly selected day, he will take less than one hour to travel to work. *(1 mark)*

(c) Calculate the probability that, on a randomly selected day:

(i) at least one of Anchal and Baldeep will take longer than one hour to travel to work; *(3 marks)*

(ii) Baldeep will take longer than Anchal to travel to work. *(5 marks)*

END OF QUESTIONS