

General Certificate of Education
June 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 4

MAP4

Wednesday 16 June 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Sheets of graph paper are available on request.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Show that $(3 - i)^2 = 8 - 6i$. (1 mark)

(b) The quadratic equation

$$az^2 + bz + 10i = 0,$$

where a and b are real, has a root $3 - i$.

(i) Show that $a = 3$ and find the value of b . (6 marks)

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + iq$. (3 marks)

2 (a) Show that

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)}. \quad (2 \text{ marks})$$

(b) Hence find the sum of the series

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \cdots + \frac{1}{30 \times 31 \times 32},$$

giving your answer as a rational number. (5 marks)

3 (a) Sketch on one Argand diagram:

(i) the locus of points satisfying $|z - i| = |z - 2|$; (2 marks)

(ii) the locus of points satisfying $\arg(z - i) = \frac{1}{4}\pi$. (2 marks)

(b) Shade on your diagram the region in which

$$|z - i| \leq |z - 2| \quad \text{and} \quad -\frac{1}{2}\pi \leq \arg(z - i) \leq \frac{1}{4}\pi. \quad (3 \text{ marks})$$

4 The roots of the cubic equation

$$x^3 + 9x^2 + 27x + 35 = 0$$

are α , β and γ .

- (a) Use the substitution $x = y - 3$ to show that the cubic equation which has roots $\alpha + 3$, $\beta + 3$ and $\gamma + 3$ is

$$y^3 + 8 = 0. \quad (6 \text{ marks})$$

- (b) (i) Find the **three** roots of $y^3 + 8 = 0$, giving each root in the form $a + ib$. (3 marks)
- (ii) Hence, or otherwise, find the roots of the equation

$$x^3 + 9x^2 + 27x + 35 = 0. \quad (2 \text{ marks})$$

5 (a) Find the constants A and B in the identity

$$\left(z^2 - \frac{1}{z^2}\right)^3 \equiv A\left(z^2 - \frac{1}{z^2}\right) + B\left(z^6 - \frac{1}{z^6}\right). \quad (3 \text{ marks})$$

- (b) (i) Use the result

$$z^n - \frac{1}{z^n} = 2i \sin n\theta,$$

where $z = \cos \theta + i \sin \theta,$

to show that

$$\sin^3 2\theta = \frac{3}{4} \sin 2\theta - \frac{1}{4} \sin 6\theta. \quad (4 \text{ marks})$$

- (ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{4}\pi} \sin^3 2\theta \, d\theta = \frac{1}{3}. \quad (3 \text{ marks})$$

6 (a) Show that

$$\frac{d}{dt} \left(2 \tan^{-1} e^t \right) = \operatorname{sech} t. \quad (5 \text{ marks})$$

(b) A curve C is given parametrically by

$$x = 2 + \tanh t, \quad y = 2 - \operatorname{sech} t.$$

(i) Show that

$$\frac{dy}{dt} = \operatorname{sech} t \tanh t. \quad (3 \text{ marks})$$

(ii) Express $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$ in terms of $\operatorname{sech} t$. (3 marks)

(c) The arc of the curve C between $t = 0$ and $t = 1$ is rotated through 2π radians about the x -axis.

(i) Show that S , the surface area generated, is given by

$$S = 2\pi \int_0^1 (2 - \operatorname{sech} t) \operatorname{sech} t \, dt. \quad (1 \text{ mark})$$

(ii) Hence show that

$$S = 2\pi(4 \tan^{-1} e - \tanh 1 - \pi). \quad (3 \text{ marks})$$

END OF QUESTIONS