General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) MAM2/W
Unit Mechanics 2

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM2/W.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Two particles, of masses 2 kilograms and m kilograms, are placed at the points (1, 8) and (7, 11), respectively, in the x-y plane. The unit of distance is the metre.

The centre of mass of this system of particles lies on the line $x = 5$.

(a) Show that $m = 4$. *(3 marks)*

(b) Determine the y-coordinate of the centre of mass of this system. *(3 marks)*

2 Two cars, A and B, are travelling in the same direction on a straight horizontal road. Car A is travelling at $12\,\text{m s}^{-1}$ when it collides with car B which is travelling at $8\,\text{m s}^{-1}$, as shown in the diagram.

Car A has mass 800 kg and car B has mass 1000 kg. To model this collision, the cars can be considered as particles. The coefficient of restitution between these particles is $\frac{1}{8}$.

Show that the speed of car B immediately after the collision is $10\,\text{m s}^{-1}$ and find the speed of car A. *(7 marks)*

3 A car of mass 760 kg is travelling down a straight road inclined at an angle of $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal. At the point A on the road, the car is travelling with speed $10\,\text{m s}^{-1}$, as shown in **Figure 1**. The point B is 200 m along the road from A. When the car reaches B, its speed is $25\,\text{m s}^{-1}$, as shown in **Figure 2**.

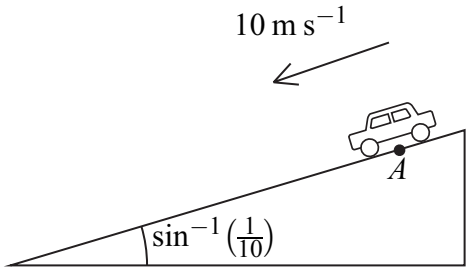


Figure 1

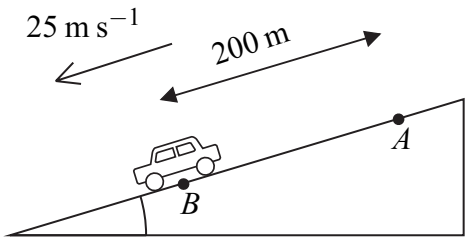


Figure 2

(a) Using a simple model, resistance forces are neglected and the driving force of the car is assumed to be constant.

(i) Find the gain in mechanical energy of the car when it travels from A to B. *(4 marks)*

(ii) Deduce that the driving force of the car is approximately 253 N. *(2 marks)*

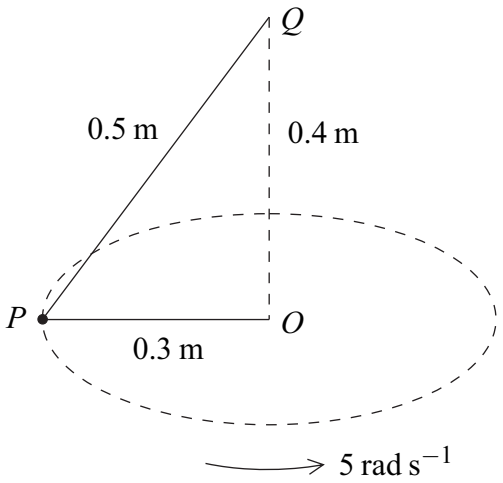
(b) Using a different model, the resistance force on the car is assumed to be 1000 N and the driving force is **not** assumed to be constant. The car reaches its maximum speed of $25\,\text{m s}^{-1}$ at B.

(i) Draw a diagram to show all the forces acting on the car at B. *(1 mark)*

(ii) Determine the driving force of the car at B using this model. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

4 A particle of mass 0.4 kg is attached at the point P to two light strings, QP and OP. The points O and Q are fixed with Q at a distance of 0.4 m vertically above O. The string QP is inextensible and of length 0.5 m. The string OP is elastic and of natural length 0.2 m and stiffness k N m^{-1}. The particle moves in a horizontal circle, centre O and radius 0.3 m, at a constant angular speed of 5 rad s^{-1}.



(a) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(b) Show that the tension in the string QP is 4.9 N. *(3 marks)*

(c) Write down, in terms of k, the tension in the string OP. *(1 mark)*

(d) Show that $k = 0.6$. *(5 marks)*

(e) Find the elastic potential energy stored in the string OP. *(2 marks)*

5 A body of mass 1.5 kg is moving under the action of a single force, **F** newtons. At time t seconds, the velocity of the body is **v** metres per second, where

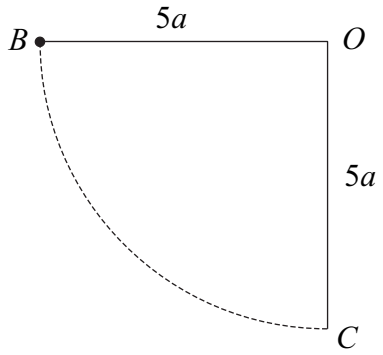
$$\mathbf{v} = \begin{bmatrix} 1 + 4\sin 2t \\ 4\cos 2t \end{bmatrix}.$$

(a) (i) Find an expression for **F** in terms of t. *(3 marks)*

(ii) Show that, for all values of t, $|\mathbf{F}| = 12$. *(2 marks)*

(b) Determine the work done by **F** over the interval $0 \leqslant t \leqslant \frac{\pi}{4}$ seconds. *(6 marks)*

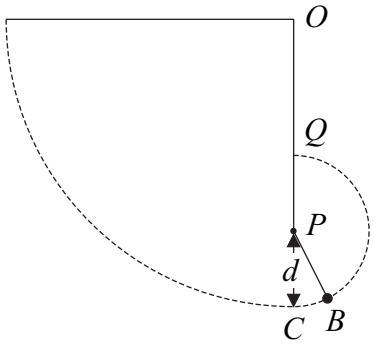
6 Adam has set up an experiment for his Mechanics coursework. He has attached a small ball, B, of mass m, to one end of a light inextensible string of length $5a$. The other end of the string is attached to a fixed point O. The ball is released from rest with the string taut and horizontal, as shown in the diagram. The ball subsequently passes through the point C, which is a vertical distance $5a$ below O.



(a) Find an expression, in terms of a and g, for the speed of B when it reaches C.

(2 marks)

(b) A small smooth peg, P, is fixed at a distance d vertically above C. When the string reaches the vertical position, B begins to move in a vertical circle with centre P and radius d, as shown in the diagram.



The ball reaches Q, the point at a distance d vertically above P, with speed v. At Q, the string is taut.

(i) Show that $v^2 = 2g(5a - 2d)$. *(4 marks)*

(ii) Find, in terms of a, d, g and m, the tension in the string when the ball is at Q.

(4 marks)

(iii) Hence show that $d < 2a$. *(2 marks)*

(c) State **one** modelling assumption used in this question. *(1 mark)*

END OF QUESTIONS

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