

Q U A L I F I C A T I O N S A L L I A N C E Mark scheme January 2004

GCE

Mathematics A

Unit MAP3

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Key to mark scheme

Μ	mark is for	method
m	mark is dependent on one or more M marks and is for	method
Α	mark is dependent on M or m mark and is for	accuracy
В	mark is independent of M or m marks and is for	method and accuracy
Ε	mark is for	explanation
or ft or F		follow through from previous
		incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
-x EE		Deduct <i>x</i> marks for each error
NMS		No method shown
PI		Perhaps implied
C		Candidate

Abbreviations used in marking

MC - x	deducted x marks for miscopy
MR - x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1(a)(i)		M1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = 6.\frac{1}{6t}$	A1	2	
(ii)	$t = \frac{1}{2}$ gradient = 2	B 1√	1	ft only on $\frac{dy}{dx} = f(t)$
	—			
(b)(i)	$t = \frac{y}{6} \qquad x = 3\left(\frac{y}{6}\right)^2 = \left[\frac{y^2}{12}\right]$	M1A1	2	Accept $\frac{3y^2}{36}$
	$6 \qquad (6) \lfloor 12 \rfloor$			
				Use of tangent $y = 2x + \frac{3}{2}$ or $x = \frac{y}{2} - \frac{3}{4}$
				instead of curve: no marks
(ii)	dx 2y	M1		Alternative:
	$\frac{dx}{dy} = \frac{2y}{12}$ $t = \frac{1}{2} \qquad y = 3$			$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2y}{12} \qquad \qquad M1$
	1	D1		$\frac{dy}{dy} = \frac{12}{12}$
	$t = \frac{1}{2} \qquad y = 3$	B1		$\frac{y}{t} = t$ B1
				6
				$t = \frac{1}{2}; \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{2} \qquad \text{M1}$
	dx = 6 $dy = 12$			$\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ A1
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{6}{12} \qquad \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12}{6} = 2$	M1A1	4	dx = 111
	Total		9	
			9	
2	x y step x $\frac{dy}{dx}$ step y			$\frac{\mathrm{d}y}{\mathrm{d}x} = 0.5$ M1
		M1A1		$\frac{dx}{dt} = 0.125$
	10.50.250.50.1251.250.6250.250.33860.0846	MIAI MIAI		Step dy = 0.125 A1
	1.5 0.7096			1.25; step $y + 0.5$; step $y = 0.25 \frac{dy}{dx}$ M1
	x = 0.71	A1	5	0.08 (46) AWRT A1
	Total		5	
2 (2)(2)	t = 0 $D = 50$	D1	1	
	$t = 0 \qquad P = 50$	B1	1	
(ii)	$e^{\frac{-t}{4}} \rightarrow 0 P \rightarrow 100$	B1	1	
				Allow $25 \text{ for } 1$
(b)	$75 = 100 - 50e^{\frac{-t}{4}} \frac{1}{2} = e^{\frac{-t}{4}}$	M1A1		Allow $\frac{25}{50}$ for $\frac{1}{2}$
	-t			SC trial and improvement
	$\ln \frac{1}{2} = \frac{-t}{4} \qquad t = 2.8$	M1A1	4	2.8 4/4, 2.77 3/4 else 0
	Total		6	

	Q	Solution	Marks	Total	Comments
		8 + 3x = A(2 - x) + B(1 + 3x)	M1		Any equivalent method
		x = 2 $14 = 7B$ $B = 2$	M1		
		$x = \frac{-1}{3}$ $7 = \frac{7}{3}A$ $A = 3$	A1	3	
	(b)	$\frac{1}{1+3x} = (1+3x)^{-1}$			Alternative by Maclaurin
					$f' = \frac{\pm 3}{(1+3x)^2}; f'' = \frac{\pm 18 \text{ or } 6}{(1+3x)^3} \qquad M1$
					and $f(0) = f'(0) = f''(0)$ seen
		$=1 + -1(3x) + \frac{-1 - 2}{2}(3x)^2$	M1		Allow $3x^2$
		$=1-3x+9x^{2}$ 1 1	A1	2	
	(c)	$\frac{1}{2-x} = \frac{1}{2(1-\frac{x}{2})}$	B1		Alternative:
					$(2-x)^{-1} = 2^{-1} + (-1)2^{-2}(-x) + \frac{(-1,-2)}{2!}2^{-3}(-x)^2$
		$= \left(1 + -1\left(\frac{-x}{2}\right) + \frac{-1 \cdot -2}{2}\left(\frac{-x^{2}}{2}\right)\right)$	M1		M1 – use negative powers of 2 A1 – coefficients correct A1 – all correct, with use of $-x$ seen
		$=\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}$	A1	3	Answer given, convincingly obtained Alternative:
		2 7 0			$(1-\frac{x}{2})^{-1}$ by Maclaurin
					$f' = \frac{\pm 1}{(2-x)^2} \qquad f'' = \frac{\pm 2}{(2-x)^3} \qquad M1$
					f(0) f'(0) f''(0) seen M1
		8 + 2 ~			AG convincingly obtained A1
	(d)	$\frac{8+3x}{(1+3x)(2-x)}$			
		$= 3(1 - 3x + 9x^{2}) + 2\left(\frac{1}{2} + \frac{x}{4} + \frac{x^{2}}{8}\right)$	M1M1		M1 – use series M1 – use PFs and multiply out
		$4 - \frac{17}{2}x + \frac{109}{4}x^2$	A1	3	Alternative: $(x_{1}, x_{2})(x_{2}, x_{2}) = (x_{1}, x_{2}, x_{2})$
					$(8+3x)(1-3x+9x^2)\left(\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}\right)$ M1
	(e)	Valid for $ x < \frac{1}{3}$	B2	2	B1 for $x < \frac{1}{3}$ Multiply out M1
		3			B1 for $ x < \frac{1}{3}$ and $ x < 2$ or $ x < 1$
		Total		13	

Q	Solution	Marks	Total	Comments
5 (a)(i)	$f(x) = e^{-2x}$ $f'(x) = -2e^{-2x}$	B1		
		B1√	2	
(ii)	$f''(x) = 4e^{-2x}$ $f(x) = f(0) + f'(0)x + f''(0) \frac{x^2}{2}$			
	f(0) = 1 $f'(0) = -2$	M1		Use $x = 0$ in Maclaurin series
	f(0) = 1 f'(0) = -2 f''(0) = 4 $f(x) \approx 1 - 2x + 2x^{2}$	A1	2	AG convincingly obtained
(b)(i)	$\cos 3x \approx 1 - \frac{(3x)^2}{2}$	B1	1	
	$1 - 2x + 2x^2 = 1 - \frac{9}{2}x^2$	M1		Set up equation
	$\frac{13}{2}x^2 - 2x = 0$	m1		Rearrange to soluble form
	$x = \frac{4}{13}, 0.308$	A1	3	Accept 0.31 Ignore $x = 0$
	Total		8	
6 (a)	$\int \frac{\mathrm{d}v}{10-5v} = \int \mathrm{d}t$	M1		Attempt to separate and integrate
	$-\frac{1}{5}\ln(10-5v) = t+c$	M1 A1A1		$\pm k \ln (10 - 5v)$ c required
	$t = 0$ $v = 0$ $c = -\frac{1}{5}\ln 10$	B1√		Find c or use limits
	$t = \frac{1}{5} \ln\left(\frac{10}{10 - 5v}\right) = \frac{1}{5} \ln\left(\frac{2}{2 - v}\right)$	A1	6	AG convincingly obtained
(b)	$e^{5t} = \frac{2}{2-v}$	M1		Alternative: $0.5 = \frac{1}{5} (\ln 2 - \ln (2 - v))$ M1
	$t = 0.5 \ 2 - v = 2e^{-2.5}$ $v = 1.8358 \ v = 1.8 \text{ m s}^{-1}$	m1 A1	3	$e^{\ln 2-2.5} = e^{\ln(2-\nu)}$ M1 $\nu = 1.8$ A1
	Total		9	
	Total		,	

Q	Solution	Marks	Total	Comments
7 (a)(i)	$\overrightarrow{AB} = \begin{bmatrix} 2\\4\\-4 \end{bmatrix}$			No marks for \overrightarrow{AB} alone
	$\left \overrightarrow{AB}\right = \sqrt{2^2 + 4^2 + 4^2} = 6$	M1A1	2	
(ii)	<i>M</i> is (4, 1, 0)	B1	1	Accept $\begin{bmatrix} 4\\1\\0 \end{bmatrix}$
(b)	$\overrightarrow{CM} \bullet \overrightarrow{AB} = \begin{bmatrix} -4\\3\\1 \end{bmatrix} \bullet \begin{bmatrix} 2\\4\\-4 \end{bmatrix}$	M1A1	2	M1 – sensible attempt at $\overrightarrow{CM} \bullet \overrightarrow{AB}$ Allow \overrightarrow{MC} for \overrightarrow{CM}
	= -8 + 12 - 4 = 0			$\mp 8 \mp 12 \mp 4 = 0$ must be seen
(c)	$\mathbf{r} \bullet \begin{bmatrix} -4\\3\\1 \end{bmatrix} = \begin{bmatrix} -4\\3\\1 \end{bmatrix} \bullet \begin{bmatrix} 3\\-1\\2 \end{bmatrix} = -13$ $4x - 3y - z = 13$	M1A1	2	$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \text{ or } \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix} \text{ or } \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$ a b m AG convincingly obtained
(d)	4(8+5t) - 3(-2-3t) - (-1+3t) = 13 39 + 26t = 13 t = -1	M1 m1		Solve for <i>t</i>
	<i>P</i> is (3, 1, -4)	A1	3	Accept $\begin{bmatrix} 3\\1\\-4 \end{bmatrix}$
	Total		10	
	Total		60	