

Mark scheme January 2004

GCE

Mathematics A

Unit MAP1

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Key to mark scheme

| M | mark is for | method |
|--------------------------|---|--------------------------------------|
| m | mark is dependent on one or more M marks and is for | method |
| A | mark is dependent on M or m mark and is for | accuracy |
| В | mark is independent of M or m marks and is for | method and accuracy |
| E | mark is for | explanation |
| $$ or ft or \mathbf{F} | | follow through from previous |
| | | incorrect result |
| CAO | | correct answer only |
| AWFW | | anything which falls within |
| AWRT | | anything which rounds to |
| AG | | answer given |
| SC | | special case |
| OE | | or equivalent |
| A2,1 | | 2 or 1 (or 0) accuracy marks |
| -x EE | | Deduct <i>x</i> marks for each error |
| NMS | | No method shown |
| PI | | Perhaps implied |
| c | | Candidate |
| | | |

Abbreviations used in marking

| MC-x | deducted x marks for miscopy |
|------|------------------------------|
| MR-x | deducted x marks for misread |
| ISW | ignored subsequent working |
| BOD | gave benefit of doubt |
| WR | work replaced by candidate |

Application of mark scheme

| Correct answer without working | mark as in scheme |
|----------------------------------|---------------------------------------|
| Incorrect answer without working | zero marks unless specified otherwise |
| incorrect answer without working | zero marks umess specified omerwise |

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

| Q | Solution | Marks | Total | Comments |
|---------|--|----------|-------|---|
| 1 (a) | $\int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} (+c)$ | M1A1 | 2 | M1 for the correct power of x |
| (b) | Substitution of $x = 2$ | ml | | |
| | $\int_{0}^{2} x^{\frac{1}{2}} dx = \frac{2}{3} (2^{\frac{3}{2}})$ =\frac{4}{3}\sqrt{2} | A1F | | ft wrong coeff of $x^{\frac{3}{2}}$; decimals not allowed |
| | $\cdots - \frac{1}{3}\sqrt{2}$ | A1F | 3 | ditto |
| | Total | | 5 | |
| 2 (a) | $u_1 = 6, u_2 = 18$ | B1B1 | 2 | Allow 1/2 for answers 2, 6 |
| (b) | Common ratio is 3 | B1 | 1 | Condone 1:3 |
| (c) | Formula for sum of GP stated | M1 | | or used |
| | $S_{10} = \frac{6(3^{10} - 1)}{3 - 1}$ | m1 | | Allow with one numerical error |
| | $=3(3^{10}-1)$ | A1 | 3 | Convincingly shown (AG) |
| | Total | | 6 | |
| 3 (a) | Sector area formula stated Sector area = 12.5 θ (cm ²) | M1 A1 | 2 | or used Condone omission of units throughout |
| (b)(i) | Equating sector area to 6.25 $\theta = 0.5$ | M1 A1 | 2 | |
| (ii) | Arc length formula stated | M1 | | or used |
| | Perimeter = 22.5 (cm) | A1F | 2 | ft wrong value of θ |
| | Total | | 6 | |
| 4(a)(i) | Terms 102, 104 | B1B1 | 2 | |
| (ii) | Formula for <i>n</i> th term stated $100 + 2(n-1) = 200$ | M1 m1 | | or used OE; allow with one numerical error |
| | No of terms = 51 | A1 | 3 | Allow NMS; allow 2/3 for answer 50 |
| (b) | Formula for sum of AP stated Total length = $\frac{51}{2}$ (100+200) | M1 M1 | | or used OE; allow with one numerical error |
| | = 7650 (mm) | A1 | 3 | SC allow 3/3 for correct answer obtained by adding all 51 numbers but NMS 1/3 |
| | Total | | 8 | |

| 5 (a) $y' = 2e^{2x} \dots$ M1A1 M1 for ke^{2x} 2 x^{-2} B1 3 (b) At SP $2e^{2x} = 2x^{-2}$ m1 OE Multiplication by x^2 m1 Dep on m1 $x^2 e^{2x} = 1$ A1 3 convincingly shown (AG) (c) Take square roots, $xe^x = 1$ B1 AG (square roots must be mentioned); condone no mention of ± Then take logs, $\ln x + x = 0$ M1A1 3 AG; M1 for use of a log law or lin $e^x = x$ or $\ln 1 = 0$ (d) $f(0.5) \approx -0.19, f(0.6) \approx 0.09$ B1B1 Where $f(x) = \ln x + x$ Change of sign, so root between E1 3 AG (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ M1A1 M1 for ke^{2x} + 2 ln x (+ c) B1 3 Modulus not needed here Total 15 6(a)(i) $fg(x) = \sqrt{x-1}$ B1 2 (ii) $fg(x) = \sqrt{x-1}$ B1 1 (iii) $fg(x) = x + x = 0$ B1 1 Accept 'transformation' if clarified 'Positive' may be implied (iii) Range of h is $0 \le h(x) \le 2$ B1 <t< th=""><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></t<> | Q | Solution | Marks | Total | Comments |
|---|---------|--|-------|-------|--|
| | 5 (a) | $y'=2e^{2x}\dots$ | M1A1 | | M1 for ke^{2x} |
| (b) At SP $2e^{2x} = 2x^{-2}$ Multiplication by x^2 $x^2e^{2x} = 1$ Al 3 convincingly shown (AG) (c) Take square roots, $xe^x = 1$ B1 AG (Square roots must be mentioned); condone no mention of \pm Then take logs, $\ln x + x = 0$ M1A1 3 AG; M1 for use of a log law or $\ln e^x = x$ or $\ln 1 = 0$ (d) $f(0.5) \approx -0.19, f(0.6) \approx 0.09$ Change of sign, so root between E1 3 AG (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ M1A1 M1 for ke^{2x} M1A1 M2 M2 for ke^{2x} M1A1 M2 Accept 'transformation' if clarified 'Positive' may be implied (b) (i) Fig(1) = gf(1) = 0 (b) (ii) Range of h is $0 \le h(x) \le 2$ B1 Allow any symbol for h(x); condone < for \le ; allow '0 to 2' (iii) Domain of h^{-1} is $0 \le x \le 2$ Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 2 Allow any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5' (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ | | $\dots -2x^{-2}$ | B1 | 3 | |
| Multiplication by x^2 x^2 e 2x = 1 | | 2 | | 3 | |
| $x^2 e^{2x} = 1$ (c) Take square roots, $xe^x = 1$ $x^2 e^{2x} = 1$ (d) Take square roots, $xe^x = 1$ $x^2 e^{2x} = 1$ $x^2 e^{2x} = 1$ $x^2 e^{2x} = 1$ B1 $x^2 e^{2x} = 1$ A2 $x^3 e^{2x} = 1$ A3 $x^3 e^{2x} = 1$ A3 $x^3 e^{2x} = 1$ A4 $x^3 e^{2x} = 1$ A5 $x^3 e^{2x} = 1$ A6 $x^3 e^{2x} = 1$ A7 $x^3 e^{2x} = 1$ A8 $x^3 e^{2x} = 1$ A1 $x^3 e^{2x} = 1$ A2 $x^3 e^{2x} = 1$ A3 $x^3 e^{2x} = 1$ A6 $x^3 e^{2x} = 1$ A7 $x^3 e^{2x} = 1$ A8 $x^3 e^{2x} = 1$ A9 $x^3 e^{2x} = 1$ A1 $x^3 e^{2x} = 1$ A2 $x^3 e^{2x} = 1$ A1 $x^3 e^{2x} = 1$ A1 $x^3 e^{2x} = 1$ A2 $x^3 e^{2x} = 1$ A1 | (b) | | | | |
| (e) Take square roots, $xe^x = 1$ Then take logs, $\ln x + x = 0$ M1A1 3 AG (square roots must be mentioned); condone no mention of \pm Then take logs, $\ln x + x = 0$ M1A1 3 AG; M1 for use of a log law or $\ln e^x = x$ or $\ln 1 = 0$ Where $f(x) = \ln x + x$ AG (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ M1A1 B1 3 M1 for ke^{2x} Modulus not needed here Total 15 6(a)(i) $fg(x) = \sqrt{x-1}$ B1 B1 g1(x) = $\sqrt{x-1}$ B1 g1(x) = $gf(x) = \sqrt{x-1}$ B1 (ii) $fg(1) = gf(1) = 0$ B1 1 Accept 'transformation' if clarified 'Positive' may be implied (ii) Range of h is $0 \le h(x) \le 2$ B1 AG (square roots must be mentioned); condone no mention of \pm AG; M1 for use of a log law or $\ln e^x = x$ or $\ln 1 = 0$ Where $f(x) = \ln x + x$ AG M1 for ke^{2x} Modulus not needed here | | | | | |
| Then take logs, $\ln x + x = 0$ Then take logs, $\ln x + x = 0$ M1A1 3 AG; M1 for use of a log law or $\ln e^x = x$ or $\ln 1 = 0$ Where $f(x) = \ln x + x$ AG (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ M1A1 B1 3 M3 for e^{2x} M1A1 M1 for e^{2x} M2 M1 for e^{2x} M2 M3 for e^{2x} M4 for e^{2x} M5 for e^{2x} M6 for e^{2x} M6 for e^{2x} M6 for e^{2x} M7 for e^{2x} M8 for e^{2x} M8 for e^{2x} M8 for e^{2x} M9 for e^{2x} P9 for e^{2x} M9 for e^{2x} | | $x^2 e^{2x} = 1$ | A1 | 3 | convincingly shown (AG) |
| $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | (c) | Take square roots, $xe^x = 1$ | B1 | | ` * |
| (d) $f(0.5) \approx -0.19, f(0.6) \approx 0.09$ Change of sign, so root between E1 3 Where $f(x) = \ln x + x$ (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ M1A1 M1 for ke^{2x} Modulus not needed here Total 15 6(a)(i) $fg(x) = \sqrt{x-1}$ B1 2 (ii) $fg(1) = gf(1) = 0$ B1 1 (b)(i) Translation 1 unit in (positive) x direction M1 A1 2 "Positive' may be implied (ii) Range of h is $0 \le h(x) \le 2$ B1 1 Allow any symbol for $h(x)$; condone $<$ for \le ; allow '0 to 2' fit wrong answer in (ii); any symbol for $x = x = x = x = x = x = x = x = x = x $ | | Then take logs, $1n x + x = 0$ | M1A1 | 3 | AG; M1 for use of a log law or |
| Change of sign, so root between E1 3 AG M1A1 M1 for ke^{2x} M1A1 B1 3 Modulus not needed here Total 15 6(a)(i) $fg(x) = \sqrt{x-1}$ $gf(x) = \sqrt{x-1}$ (ii) $fg(1) = gf(1) = 0$ B1 (iii) Range of h is $0 \le h(x) \le 2$ Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 2 ACcept 'transformation' if clarified 'Positive' may be implied A1 Accept 'transformation' if clarified 'Positive' may be implied 1 Allow any symbol for $h(x)$; condone $<$ for \le ; allow '0 to 2 ' ft wrong answer in (ii); any symbol for $h^{-1}(x)$; condone $<$ for \le ; allow '1 to 5 ' (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1 M1 AG M1 ACCEPT 'transformation' if clarified 'Positive' may be implied Allow any symbol for $h(x)$; condone $<$ for \le ; allow '0 to 2 ' ft wrong answer in (ii); any symbol for $x = 1$. | | | | | $\ln e^x = x \text{ or } \ln 1 = 0$ |
| (e) $\int (e^{2x} + 2x^{-1}) dx = \frac{1}{2}e^{2x}$ $\dots + 2 \ln x (+c)$ B1 $M1 \text{ for } ke^{2x}$ $Modulus \text{ not needed here}$ | (d) | | | | |
| | | Change of sign, so root between | E1 | 3 | AG |
| | (0) | $\int (e^{2x} + 2x^{-1}) dx - \frac{1}{2}e^{2x}$ | MIAI | | NG C 1 2x |
| Total 15 6(a)(i) $fg(x) = \sqrt{x-1}$ B1 $gf(x) = \sqrt{x-1}$ B12(ii) $fg(1) = gf(1) = 0$ B11(b)(i)Translation 1 unit in (positive) x directionM1 Allow any symbol for $h(x)$; condone $h(x)$ for $h(x)$; condone $h(x)$ for $h(x)$ | (6) | _ | | 2 | |
| 6(a)(i) $fg(x) = \sqrt{x-1}$ B1 $gf(x) = \sqrt{x-1}$ B12(ii) $fg(1) = gf(1) = 0$ B11(b)(i)Translation 1 unit in (positive) x directionM1 2 'Positive' may be implied(ii)Range of h is $0 \le h(x) \le 2$ B11 Allow any symbol for $h(x)$; condone < for \le ; allow '0 to 2'(iii)Domain of h^{-1} is $0 \le x \le 2$ B1Fft wrong answer in (ii); any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5'(iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1OE | | + 2 III x (+c) | DI | 3 | Wodulus not needed here |
| (ii) $gf(x) = \sqrt{x-1}$ $fg(1) = gf(1) = 0$ B12 B1(b)(i)Translation 1 unit in (positive) x directionM1 A1Accept 'transformation' if clarified 'Positive' may be implied(ii)Range of h is $0 \le h(x) \le 2$ B11 Allow any symbol for h(x); condone < for \le ; allow '0 to 2' ft wrong answer in (ii); any symbol for h(iii)Domain of h^{-1} is $1 \le h^{-1}(x) \le 5$ B12 B1FAllow any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5'(iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1OE | | | Total | 15 | |
| (ii) $fg(1) = gf(1) = 0$ B1 1 (b)(i) Translation 1 unit in (positive) x direction A1 B1 Accept 'transformation' if clarified 'Positive' may be implied (ii) Range of h is $0 \le h(x) \le 2$ B1 Allow any symbol for $h(x)$; condone $<$ for \le ; allow '0 to 2' Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 Allow any symbol for h(x); condone $<$ for \le ; allow '0 to 2' Allow any symbol for $h^{-1}(x)$; condone $<$ for \le ; allow '1 to 5' (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1 OE | 6(a)(i) | $fg(x) = \sqrt{x - 1}$ | B1 | | |
| (ii) $fg(1) = gf(1) = 0$ B1 1 (b)(i) Translation 1 unit in (positive) x direction A1 B1 Accept 'transformation' if clarified 'Positive' may be implied (ii) Range of h is $0 \le h(x) \le 2$ B1 Allow any symbol for $h(x)$; condone $<$ for \le ; allow '0 to 2' Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 Allow any symbol for h(x); condone $<$ for \le ; allow '0 to 2' Allow any symbol for $h^{-1}(x)$; condone $<$ for \le ; allow '1 to 5' (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1 OE | | $gf(x) = \sqrt{x-1}$ | B1 | 2 | |
| (ii)Range of h is $0 \le h(x) \le 2$ B11Allow any symbol for h(x); condone < for \le ; allow '0 to 2'(iii)Domain of h^{-1} is $0 \le x \le 2$ B1F2Allow any symbol for h(x); condone < for \le ; allow '0 to 2'Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B12Allow any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5'(iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1OE | (ii) | , · | B1 | 1 | |
| (ii)Range of h is $0 \le h(x) \le 2$ B11Allow any symbol for h(x); condone < for \le ; allow '0 to 2'(iii)Domain of h^{-1} is $0 \le x \le 2$ B1F2Allow any symbol for h(x); condone < for \le ; allow '0 to 2'Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B12Allow any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5'(iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1OE | (b)(i) | Translation | M1 | | Accept 'transformation' if clarified |
| (iii) Domain of h^{-1} is $0 \le x \le 2$ Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 B1 Allow any symbol for $h^{-1}(x)$; condone | (1)(1) | | | 2 | |
| (iii) Domain of h^{-1} is $0 \le x \le 2$ Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 B1 Allow any symbol for $h^{-1}(x)$; condone | (**) | D (1: 0 <1/ >2 | D1 | 1 | Allers and comball Comba |
| (iii) Domain of h^{-1} is $0 \le x \le 2$ Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ B1F B1F Allow any symbol for $h^{-1}(x)$; condone $<$ for \le ; allow '1 to 5' OE | (11) | Range of n is $0 \le h(x) \le 2$ | BI | 1 | * |
| Range of h^{-1} is $1 \le h^{-1}(x) \le 5$ B1 2 Allow any symbol for $h^{-1}(x)$; condone < for \le ; allow '1 to 5' OE | (iii) | Domain of h^{-1} is $0 \le x \le 2$ | B1F | | ft wrong answer in (ii); any symbol for x |
| (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1 condone < for \leq ; allow '1 to 5' OE | (111) | | R1 | 2 | Allow any symbol for $h^{-1}(r)$: |
| (iv) $y = \sqrt{x-1} \Rightarrow y^2 = x-1$ M1 | | 131 = 11 (x) = 5 | D1 | _ | |
| | (iv) | $v = \sqrt{x-1} \Rightarrow v^2 = x-1$ | M1 | | , and the second |
| | | • | | | |
| | | | | 2 | |
| So $h^{-1}(x) = x^2 + 1$ Allow NMS 3/3 | | So n $\bar{x} = x^{-} + 1$ | AI | 3 | Allow NIVIS 3/3 |
| Total 11 | | Total | | 11 | |

| Q | Solution | Marks | Total | Comments |
|-------|---|------------|-------|--|
| 7 (a) | $\sin \frac{\pi}{6} = \frac{1}{2}$ | B1 | | Allow 0.5 |
| | $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ | B1 | | OE surd, eg $\sqrt{0.75}$ |
| | $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ | B1 | 3 | OE surd, eg $\sqrt{\frac{1}{3}}$ or $\frac{\sqrt{3}}{3}$ |
| (b) | Either $\sin^2 x + \cos^2 x \equiv 1$ stated | M1 | | or used |
| | Elimination of $\sin x$ or of $\cos x$ | ml | | |
| | $4\cos^2 x = 3 \text{ or } 4\sin^2 x = 1$ | A1 | | OE |
| | Or $\tan x \equiv \sin x / \cos x$ stated | M1 | | or used |
| | Equation in terms of tan x only | m1 | | |
| | $3 \tan^2 x = 1$ | A 1 | | OE |
| | Then one value is $\frac{\pi}{6}$ | B1 | | Condone 0.52; condone degrees or decimals throughout |
| | At least one other value found | M1 | | NMS 2/2 if completely correct list given |
| | Values are $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$ only | A1 | 6 | Ignore values outside domain |
| | To | otal | 9 | |
| | To | otal | 60 | |