General Certificate of Education
January 2004
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS (SPECIFICATION A) **MAM4/W**
Unit Mechanics 4

Thursday 29 January 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM4/W.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 John drops a rock of mass m kg from the top of a vertical cliff. At time t seconds after the rock is dropped, it has speed v m s^{-1} and the air resistance to its motion is of magnitude $0.1mv$ newtons.

(a) Show that the differential equation connecting v and t for this motion is $\frac{dv}{dt} = g - 0.1v$. *(2 marks)*

(b) Solve this differential equation to find v in terms of g and t. *(5 marks)*

(c) Hence show that the speed of the rock cannot exceed 98 m s^{-1}. *(1 mark)*

2 [In this question take Newton's gravitational constant to be 6.7×10^{-11} kg^{-1} m^3 s^{-2}.]

A meteorite of mass 5×10^4 kg is attracted towards the earth. The earth is modelled as a uniform sphere of mass 6×10^{24} kg and radius 6.4×10^6 m.

Initially, the meteorite is at a distance of 8×10^6 m from the earth's surface and is moving directly towards the centre of the earth with a speed of 50 m s^{-1}.

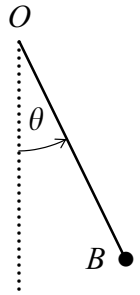
(a) When the distance of the meteorite from the centre of the earth is x metres, show that the magnitude of the acceleration of the meteorite is approximately $\frac{4 \times 10^{14}}{x^2}$ m s^{-2}. *(4 marks)*

(b) Find the speed of the meteorite as it crashes into the earth. *(6 marks)*

3 A simple pendulum consists of a small particle B suspended from a fixed point O by a light inextensible string of length 0.98 m.

The pendulum makes small oscillations under gravity in a medium which offers a resistance to the motion of B. The magnitude of the resistance is $2mv$, where m is the mass of B and v is the speed of B.

The angular displacement of the pendulum at time t is θ, where θ is measured positively in the direction shown by the arrow in the diagram below.



(a) Show that when θ is increasing, the tangential equation of motion of B is

$$\ddot{\theta} + 2\dot{\theta} + 10\theta = 0.$$

State the approximation that you have used. *(5 marks)*

(b) Find the general solution of this differential equation. *(4 marks)*

(c) Given that $\theta = \frac{\pi}{20}$ radians and $\dot{\theta} = 0$ when $t = 0$, find the particular solution of the equation. *(5 marks)*

(d) Find the first time, after the motion starts, at which B is momentarily at rest. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

4 A sledge of mass 40 kg is sliding across a frozen lake at a speed of $8\,\mathrm{m\,s^{-1}}$ when snow starts to fall vertically on to the sledge. The snow falls on to the sledge at a constant rate of $0.05\,\mathrm{kg\,s^{-1}}$. The total resistance to the motion of the sledge is assumed to be 5 N.

(a) By using the impulse/momentum principle, show that the equation of motion of the sledge is

$$(40 + 0.05t)\frac{\mathrm{d}v}{\mathrm{d}t} + 0.05v = -5,$$

where v is the speed of the sledge t seconds after the snow starts to fall. *(5 marks)*

(b) By solving the differential equation in part (a), find v in terms of t. *(7 marks)*

5 A particle P, of mass m, moves in a plane curve. At time t, the polar coordinates of P are (r, θ) relative to a fixed pole O. The path of P is the curve with equation

$$r = \frac{1}{1 + a\cos\theta},$$

where a is a constant.

(a) Given that P moves under a force directed towards O, show that $r^2\dot{\theta} = h$, where h is a constant. *(2 marks)*

(b) Show that the radial component of velocity of P is $ah\sin\theta$. *(4 marks)*

(c) Show that the radial force acting on P is $m\dfrac{h^2}{r^2}$. *(6 marks)*

END OF QUESTIONS