AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme
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GCE

Mathematics A

Unit MAM4

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Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – x	deducted x marks for miscopy
MR – x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1 (a)	$mg - 0.1mv = m\dfrac{dv}{dt}$	M1		Newton's 2nd law
	$\dfrac{dv}{dt} = g - 0.1v$	A1	2	CAO
(b)	$\int_0^v \dfrac{dv}{g - 0.1v} = \int_0^t dt$	M1		Attempt at integration with correct separation of variables
	$\left[-10\ln(g - 0.1v)\right]_0^v = \left[t\right]_0^t$	A2, 1		-1 EE
	$\ln\dfrac{g - 0.1v}{g} = -0.1t$	A1F		
	$v = 10g\left(1 - e^{-0.1t}\right)$	A1F	5	Or equivalent
(c)	As $t \to \infty$, $e^{-0.1t} \to 0$	M1	1	
	$\therefore v \to 10g = 98$			
	Total		**8**	

Q	Solution	Marks	Total	Comments
2 (a)	Force of attraction on meteor is			
	$\dfrac{6.7\times10^{-11}\times5\times10^{4}\times10^{24}}{x^2}$	M1		
	Using $'F = ma'$			
	$-\dfrac{6.7\times10^{-11}\times5\times10^{4}\times6\times10^{24}}{x^2}=5\times10^{4}a$	m1		
		A1		Correct sign
	$a=-\dfrac{4\times10^{14}}{x^2}$	A1	4	CAO
(b)	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=-\dfrac{4\times10^{14}}{x^2}$	M1		
	$\int_{50}^{v} v\mathrm{d}v=-4\times10^{14}\int_{1.44\times10^{7}}^{6.4\times10^{6}} x^{-2}\mathrm{d}x$	M1		
		A1		Signs and limits
	$\frac{1}{2}v^2\Big\vert_{50}^{v}=4\times10^{14}x^{-1}\Big\vert_{1.44\times10^{7}}^{6.4\times10^{6}}$	A1F		Integration
	$v=\sqrt{8\times10^{14}\left(\dfrac{1}{6.4\times10^{6}}-\dfrac{1}{1.44\times10^{7}}\right)+50^2}$	A1F		
	$v=8.33\times10^{3}\left(\mathrm{m}\,s^{-1}\right)$	A1F	6	AWRT
	Total		**10**	

Q	Solution	Marks	Total	Comments
3 (a)	$mg\sin\theta + 2mv = -m0.98\ddot{\theta}$	M1		Newton's 2nd law
	$g\sin\theta + 2\times 0.98\dot{\theta} = -0.98\ddot{\theta}$	A2,1		
	For small θ, $\sin\theta = \theta$	M1		For using $v = l\dot{\theta}$
	$\therefore \ddot{\theta} + 2\dot{\theta} + 10\theta = 0$			
	Auxiliary eqn $m^2 + 2m + 10 = 0$	A1	5	CAO
	$m = -1 \pm 3\mathrm{i}$			
(b)	$\theta = a\mathrm{e}^{-t}\sin(3t + \varepsilon)$	M1A1		M1 for attempt to solve
	$\dot{\theta} = -a\mathrm{e}^{-t}\sin(3t+\varepsilon) + 3a\mathrm{e}^{-t}\cos(3t+\varepsilon)$	M1A1	4	For A1: CAO
(c)	$0 = -a\sin\varepsilon + 3a\cos\varepsilon$	M1		
	$\varepsilon = 1.25$	A1F		
	$\frac{\pi}{20} = ae^0\sin 1.25$	m1		Attempt to solve
	$a = 0.166$	A1F		
	$\theta = 0.166\mathrm{e}^{-t}\sin(3t + 1.25)$	A1F	5	
(d)	$\dot{\theta} = -0.166\mathrm{e}^{-t}\sin(3t+1.25)$ $+ 3\times 0.166\mathrm{e}^{-t}\cos(3t+1.25) = 0$	M1		
	$\tan(3t + 1.25) = 3$	A1		For diff and setting to zero
	$3t + 1.25 = 4.39$	m1		
	$t = 1.05$	A1F	4	AWRT
	Total		**18**	

Q	Solution	Marks	Total	Comments
4 (a)	Snow $\boxed{\delta m} \longrightarrow 0$			
	Sledge			
	$\boxed{m} \longrightarrow v \quad \boxed{m+\delta m} \longrightarrow v+\delta v$			
	Time t Time $t+\delta t$			
	Change in momentum = Impulse of ext. force	B1		
	$(m+\delta m)(v+\delta v)-mv=-5\delta t$	M1 A1		Correct sign
	As $\delta t \longrightarrow 0$			
	$mv+m\delta v+v\delta m-mv=-5\delta t$	A1		
	$(40+0.05t)\dfrac{\mathrm{d}v}{\mathrm{d}t}+0.05v=-5$	A1F	5	
(b)	$\displaystyle\int_8^v \frac{\mathrm{d}v}{5+0.05v}=\int_0^t \frac{\mathrm{d}t}{40+0.05t}$	M1 A1		
	$-\dfrac{1}{0.05}[\ln(5+0.05v)]_8^v$			
	$=\dfrac{1}{0.05}[\ln(40+0.05t)]_0^t$	A2,1F		-1 EE
	$-\ln(5+0.05v)+\ln 6.2=\ln(40+0.05t)-\ln 40$	A1F		
	$v=\dfrac{192-t}{8+0.01t}$	m1 A1F	7	OE
	Total		**12**	

Q	Solution	Marks	Total	Comments
5 (a)	No transverse force $\Rightarrow \dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta}) = 0$	M1		FB gives formula for acceleration
	$\therefore r^2\dot{\theta} = h$	A1	2	CAO
(b)	$\dot{r} = -(1 + a\cos\theta)^{-2}(-a\dot{\theta}\sin\theta)$	M1A1		
	$\dot{r} = a\dfrac{\dot{\theta}}{(1 + a\cos\theta)^2}\sin\theta$			
	$\dot{r} = ar^2\dot{\theta}\sin\theta$	m1		Substitution
	$\dot{r} = ah\sin\theta$	A1	4	CAO
(c)	$\ddot{r} = ah\dot{\theta}\cos\theta$	M1A1		
	$F = -m(ah\dot{\theta}\cos\theta - r\dot{\theta}^2)$	M1A1		M1 for radial eqn of motion
	$F = -m\left(ah\dfrac{h}{r^2}\left(\dfrac{1}{r} - 1\right) - r\left(\dfrac{h}{r^2}\right)^2\right)$	m1		Substitutions
	$F = -m\left(\dfrac{h^2}{r^3} - \dfrac{h^2}{r^2} - \dfrac{h^2}{r^3}\right)$			
	$F = m\dfrac{h^2}{r^2}$	A1	6	
	Total		**12**	
	Total		**60**	