Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Use of Mathematics (Pilot) USE3

Mathematical Comprehension

Thursday 22 May 2014 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 45.

Advice
- You are advised to spend 1 hour on Section A and 30 minutes on Section B.
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Give all answers in this section to the nearest penny where appropriate.

Use **Every penny counts!** *on the Data Sheet.*

1 (a) (i) You invest £1000 at the start of 2010 and leave it untouched in a bank account paying interest at a fixed rate of 4% compounded annually for 10 years.

Calculate how much you will have at the end of the 10 years.

[2 marks]

(ii) Find how much more interest is paid in the second 5 years than in the first 5 years.

[2 marks]

(iii) Explain briefly why this is the case.

[1 mark]

(b) A different bank pays interest of 2% every six months rather than 4% once a year. In this case, how much would you have for your investment of £1000 at the end of 10 years?

[2 marks]

(c) Yet another bank pays interest at a fixed annual rate of 4% compounded continuously throughout the year. In this case, how much would you have for your investment of £1000 at the end of 10 years?

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

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2 In 10 years' time, you want an investment of $£P$, deposited in an account paying a fixed interest rate of 5% compounded annually, to have become $£1500$.

Find P.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 An amount of £5000 is invested at a fixed annual interest rate of 2% compounded continuously. After t years, this will have become an amount, £A.

(a) Write down an expression for A in terms of t.

[1 mark]

(b) Sketch a graph of A against t, showing all important features.

[2 marks]

(c) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2A}{\mathrm{d}t^2}$.

[2 marks]

(ii) Explain what these values tell you about how A varies with t.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 A student has a part-time job and saves £500 per year which she deposits in an account at the start of each year. Assume that this account pays interest at an annual fixed rate of 2.5% compounded continuously.

How much would be in this account at the end of the third year?

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 4

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5 (a) The article, printed on the Data Sheet, gives an illustration of someone who has a continuous stream of money of £1000 each year which arrives in their account at a constant rate. If the interest rate remains fixed at 4%, after five years this will have become £5535.07.

Show calculations that confirm this result.

[4 marks]

(b) Someone else has a continuous stream of money, £$S(t)$, which increases linearly over the same five-year period. The interest rate remains fixed at 4% per year.

The function, S, is given by $S = 1000 + 50t$.

Use integration to find the total amount, £A, in this person's account after five years.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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Section B

Read the article below carefully.

Answer **all** questions.

Answer each question in the space provided for that question.

Cars on our roads

The rise in the number of cars on our roads over decades has been relentless. Although more roads have been built, this has not kept pace with the increasing numbers of cars. Hence there is ever more traffic congestion on our roads. Each year, every car being driven in Great Britain should be licensed. The graph in **Figure 3** shows the data from 1950 to 2010 for the number of car licences issued each year in Great Britain together with a linear model which approximates to the data.

The linear model was found using the data in the year 1950, when almost $2\ 000\ 000$ licences were issued, and in the year 2000 when almost $23\ 000\ 000$ licences were issued. This leads to the equation for the number of car licences issued, c (thousands)

$$c = 2000 + 420n$$

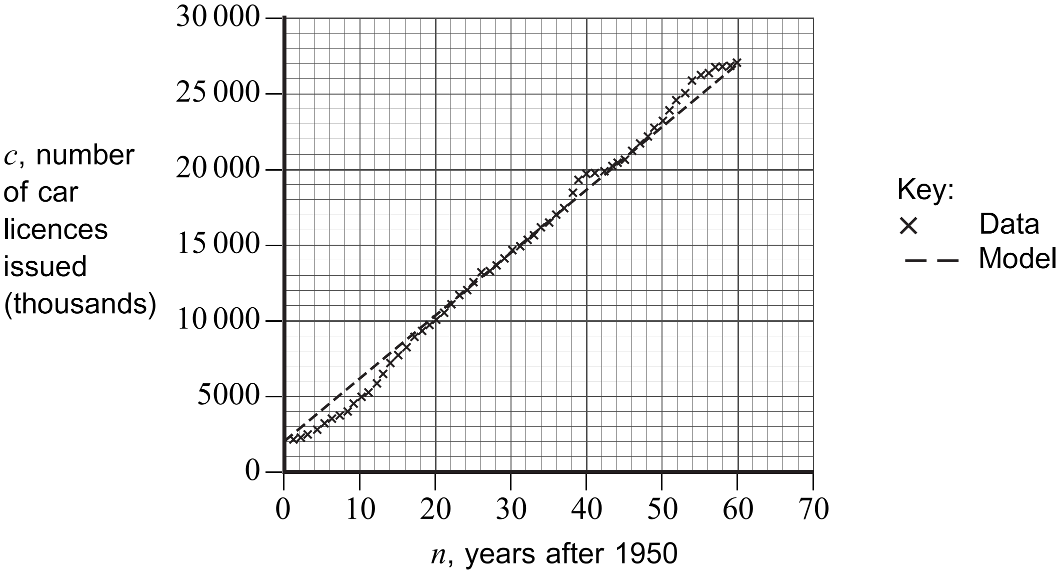
where n is the number of years after 1950.

In 2010, $n = 60$ and this gives $c = 27\ 200$, predicting that $27\ 200\ 000$ car licences were issued.

An approximation to the total number of car licences issued, T, over the sixty-year period between 1950 and 2010 can be found using the area under the graph of the model. Considering this area as a trapezium, and using the values the model gives for c found above, it can be shown that $T = 876\ 000\ 000$.

Many may think that this gives the total number of cars that have been driven on the roads of Great Britain, but this is not the case, as most individual cars will be licensed every year during their lifetime, which in 2010 was about 14 years on average.

Figure 3 *Graph showing the number of car licences issued each year in Great Britain between 1950 and 2010*



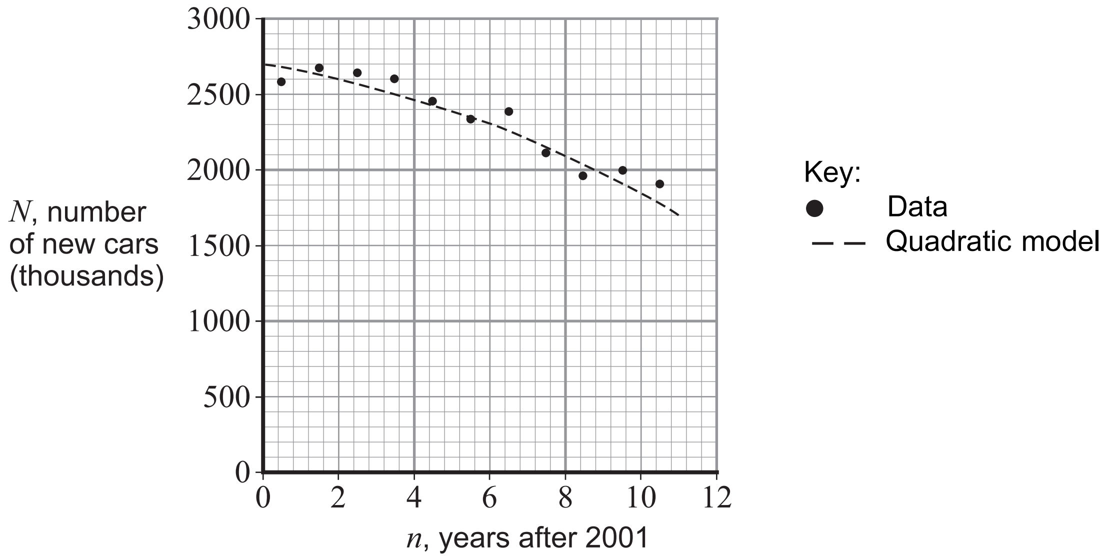
The graph in **Figure 4** shows the number, N (thousands), of new cars each year in Great Britain plotted against n years after 1 January 2001, with N plotted at the midpoint of the year. As you can see, the value of N fell over this period.

The quadratic function

$$N = -5n^2 - 35n + 2700$$

models these data.

Figure 4 *Graph showing the number of new cars each year in Great Britain between 2001 and 2011*



This model suggests that when $n = 5$, that is at the start of 2006, the rate of decrease in the number of new cars in Great Britain was $85\ 000$ per year.

For the model, the area under the curve gives an approximation to the total number of new cars in Great Britain.

This approximation can be found by evaluating the integral

$$S = \int_0^{11} (-5n^2 - 35n + 2700)\, \mathrm{d}n$$

which gives a value of $25\ 364$, suggesting that there were approximately 25 million new cars in Great Britain during the 11 years from 1 January 2001 to 31 December 2011.

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6 The linear model for the number of car licences issued, c (thousands), n years after 1950 is $c = 2000 + 420n$.

Explain fully how the values of 2000 and 420 in the formula relate to the data.

[2 marks]

QUESTION PART REFERENCE

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7 In the article, printed on pages 12 and 13, the total number of car licences issued, T, for the sixty-year period between 1950 and 2010 is found to be approximately $876\ 000\ 000$.

(a) Show calculations that confirm this result.

[2 marks]

(b) (i) If the average lifetime of a car in 2010 was 14 years, use your answer to part **(a)** to find an approximation to the number of individual cars for which licences were issued in Great Britain for the sixty-year period between 1950 and 2010.

[2 marks]

(ii) Give a reason why the approximation found in part **(b)(i)** is unlikely to be valid.

[2 marks]

QUESTION PART REFERENCE

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8 By finding $\frac{\mathrm{d}N}{\mathrm{d}n}$, show how the rate of increase in new cars per year in Great Britain in 2006 is $-85\ 000$.

[2 marks]

QUESTION PART REFERENCE

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9 If the model $N = -5n^2 - 35n + 2700$ holds for $n < 0$, find the year that it suggests there would have been a peak in the number of new cars in Great Britain.

[2 marks]

QUESTION PART REFERENCE

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10 Confirm that there were approximately 25 million new cars in Great Britain during the 11 years from 1 January 2001 to 31 December 2011 by evaluating the integral

$$S = \int_0^{11} (-5n^2 - 35n + 2700)\,\mathrm{d}n\,.$$

[3 marks]

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QUESTION PART REFERENCE

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END OF QUESTIONS

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