A-LEVEL

Further Mathematics

Statistics

Mark scheme

Specimen

Version 1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.2	B1	8
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	$\frac{5}{9}$
	Total		**1**	
3	States value for k	AO1.1b	B1	$k = \frac{1}{b+a}$
	Uses correct integral for E(R)	AO1.1a	M1	E(R) = $\int_{-a}^{b} kr\,\mathrm{d}r = \left[k\frac{r^2}{2}\right]_{-a}^{b}$
	Uses limits correctly with 'their' integral	AO1.1a	M1	$= \frac{k}{2}\left(b^2 - a^2\right)$ $= \frac{1}{2(b+a)}(b-a)(b+a)$
	Completes clear correct rigorous workings to show required result **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$= \frac{(b-a)}{2}$ **AG**
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	States 1.83 AWRT	AO1.1b	B1	1.83
(b)	Comments that David's statement is not or may be not true	AO2.3	R1	David's statement may not be true because:
	Mentions that interval is based on mean from sample or Mentions that interval is for population mean length, not for population of lizard lengths	AO2.4	R1	Interval is centred on sample mean, not population mean or interval is for population mean length, not lizard lengths
(c)	Uses correct expression in b, s and n in equation relating to width value	AO1.1a	M1	$\frac{b \times s}{\sqrt{n}} = 4.27$ or $2 \times \frac{b \times s}{\sqrt{n}} = 8.54$
	Solves for b FT incorrect s	AO1.1b	A1F	$\frac{b \times 4.15(45691)}{\sqrt{10}} = 4.27$ $b = 3.25$
	Deduces that $\beta = 99$ (condone 99 %)	AO2.2a	R1	From t tables, $\beta = 99$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	States both hypotheses using correct notation	AO2.5	B1	H_0: Language choice is independent of sex H_1: Language choice is not independent of sex
	Constructs 'correct' contingency table with frequencies shown (PI)	AO3.1b	M1	<table><tr><th></th><th>German</th><th>Mandarin</th><th></th></tr><tr><td>**M**</td><td>7</td><td>12</td><td>19</td></tr><tr><td>**F**</td><td>23</td><td>8</td><td>31</td></tr><tr><td></td><td>30</td><td>20</td><td>50</td></tr></table>
	Finds expected frequencies (PI)	AO3.4	B1	<table><tr><th></th><th>German</th><th>Mandarin</th></tr><tr><td>**M**</td><td>11.4</td><td>7.6</td></tr><tr><td>**F**</td><td>18.6</td><td>12.4</td></tr></table>
	Evaluates $\chi^2 = \sum \frac{(O-E)^2}{E}$ Allow no Yates correction used FT their observed and expected frequencies	AO1.1a	M1F	$\chi^2 = \sum \frac{(\lvert O-E \rvert - 0.5)^2}{E}$ $= \frac{(3.9)^2}{11.4} + \frac{(3.9)^2}{7.6} + \frac{(3.9)^2}{18.6} + \frac{(3.9)^2}{12.4}$
	Obtains χ^2 test statistic or quotes p-value	AO1.1b	A1	= 1.33 + 2.00 +0.82 + 1.23 ts $\chi^2 = 5.38$ $p = 0.0204$ 1 tail 5%
	Evaluates χ^2 model by comparing test statistic and correct critical value Or by comparing p-value with 0.05	AO3.5a	M1	df = 1 cv = 3.84 ts > 3.84 or $p < 0.05$
	Infers that H_0 should be rejected	AO2.2b	E1	Reject H_0
	Completes test correctly and gives conclusion in context. (conclusion should not be definite)	AO3.2a	E1	Significant evidence to suggest that language choice is not independent of sex
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)(i)	Identifies that total probability must sum to 1	AO3.1a	M1	$c+\left[(1-k(15-t)^2)\right]_0^{12}=1$ or $c+\int_0^{12} f(t)\,dt=1$
	Uses limits to obtain expression in terms of k for $P(0 \le T \le t)$	AO1.1a	M1	$c+\left[(1-k(15-t)^2)\right]_0^{12}=1$ $c+\left[(1-9k)-(1-225k)\right]=1$ or $f(t)=\frac{d}{dt}(1-k(15-t)^2)=2k(15-t)$ $2k\int_0^{12} 15-t\,dt=2k\left[15t-\frac{t^2}{2}\right]_0^{12}$ $=2k(180-72)$
	correct rigorous workings to show required result **AG**	AO2.1	R1	$c+216k=1$ $\Rightarrow 1-c=216k$ or $c+\int_0^{12} 2k(15-t)\,dt=1$ $c+216k=1$ $1-c=216k$

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)(ii)	Correct expression for $\mathrm{E}(T)$ (PI)	AO1.1a	M1	$\mathrm{E}(T) = (-2 \times c) + \int_0^{12} t \times \mathrm{f}(t)\,\mathrm{d}t$ $\mathrm{f}(t) = \frac{\mathrm{d}}{\mathrm{d}t}(1 - k(15-t)^2) = 2k(15-t)$
	Uses $k = \frac{1}{240}$ and integrates with correct limits	AO1.1a	M1	$\mathrm{E}(T) = -0.2 + 2k\int_0^{12} 15t - t^2\,\mathrm{d}t$ $0.9 = 216k$ so $k = \frac{9}{2160} = \frac{1}{240}$ $\mathrm{E}(T) = -0.2 + \frac{2}{240}\left[\frac{15}{2}t^2 - \frac{1}{3}t^3\right]_0^{12}$
	Obtains correct value for $\mathrm{E}(T)$	AO1.1b	A1	$= -0.2 + \frac{1}{120} \times 504 = -0.2 + 4.2$ $\mathrm{E}(T) = 4$
(b)	Correct expression for $\mathrm{E}(\sqrt{\lvert T \rvert})$	AO1.1a	M1	$\mathrm{E}(\sqrt{\lvert T \rvert}) = 0.1 \times \sqrt{2} + \int_0^{12} 2k \times \sqrt{t} \times (15-t)\,\mathrm{d}t$ =
	Integrates and uses limits	AO1.1a	M1	$\frac{1}{120}\left(10 \times 12^{\frac{3}{2}} - \frac{2}{5} \times 12^{\frac{5}{2}}\right) = \frac{12^{\frac{3}{2}}}{120}(10 - \frac{24}{5})$
	Completes clear correct rigorous workings to show required result **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$= \frac{12 \times \sqrt{12}}{120} \times \frac{26}{5} = \frac{13\sqrt{12}}{25} = \frac{26\sqrt{3}}{25}$ $\mathrm{E}(\sqrt{\lvert T \rvert}) = 0.1 \times \sqrt{2} + \frac{26\sqrt{3}}{25}$ $= \frac{\sqrt{2}}{10} + \frac{26\sqrt{3}}{25}$ $= \frac{5\sqrt{2} + 52\sqrt{3}}{50}$
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)(i)	States both hypotheses correctly	AO2.5	B1	H_0: $\mu = 3$ H_1: $\mu > 3$
	Finds both $\overline{x}$ and s^2 or s (condone n divisor if retrieved in formula)	AO1.1b	B1	$\overline{x} = \frac{28.8}{9} = 3.2$ $s^2 = \frac{\sum(x-\overline{x})^2}{n-1} = 0.075$ $(s = 0.27386)$
	Uses formula to find test statistic PI	AO1.1a	M1	Test statistic $t = \frac{3.2-3}{\sqrt{\frac{0.075}{9}}}$
	States test statistic correctly (AWFW 2.15 – 2.23) or finds p	AO1.1b	A1	Test statistic $t =$ 2.19 $p = 0.020(4)$
	Evaluates t model by comparing test statistic and correct critical value Or by comparing p-value with 0.05	AO3.5a	M1	Critical value for 1-tail test at the 5% level of significance for a t_8 distribution is 1.8595, 2.19 > 1.8595 Or 0.020(4) < 0.05
	Infers that H_0 should be rejected	AO2.2b	E1	Reject H_0
	Completes test correctly and gives conclusion in context. (conclusion should not be definite)	AO3.2a	E1	As 2.19 > 1.8595 The evidence suggests that the impurity, per cent, level in the chemical is too high

Q	Marking Instructions	AO	Marks	Typical Solution
7a(ii)	Identifies the limitation of the t-distribution model if data not normally distributed	AO3.5b	E1	Assumption: The **level of impurity** is normally distributed
7(b)	Makes statement (could be implied by quoting the critical z-value) that z-test would be required	AO3.5c	E1	It would be a z-test rather than a t- test [critical value change to z = 1.645]
	Mentions change to σ or change to test statistic	AO2.4	E1	Would use σ = 0.25 not σ = 0.27386 or Value of ts changes to 2.4
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	States mean = 40 hours	AO1.2	B1	Mean = $\frac{1}{\lambda} = 40$ hours
(b)	Obtains correct probability	AO1.1b	B1	P(time $<$ 12) = 1 – $e^{-12\times0.025}$ = 1 – $e^{-0.3}$ = 0.259
(c)	Uses 'no memory' property PI	AO3.4	M1	Exponential distribution has no memory
	Obtains probability	AO1.1b	A1	P(time $>$ 30) = $e^{-30\times0.025}$ = $e^{-0.75}$ = 0.472
(d)	States or uses new mean (or uses $e^{-0.3}$)	AO3.4	B1	4 consecutive shifts gives 4×12 = 48 hours
	Finds probability P (time $>$ 48) (or uses $(e^{-0.3})^4$)	AO1.1a	M1	P(time $>$ 48) = $e^{-48\times0.025}$ = $e^{-1.2}$
	Obtains correct probability	AO1.1b	A1	= 0.301
(e)(i)	States Poisson for model of situation given	AO3.3	B1	Poisson identified
	States value for λ (= 0.025 per hour) for model	AO3.3	B1	Po (0.025) per hour

Q	Marking Instructions	AO	Marks	Typical Solution
8e (ii)	Uses 60 x 'their' λ for 'their' Po model Or Uses exponential model to find P(Time > 60)	AO3.4	M1F	For **60** hours of process $\lambda = 60 \times 0.025 = 1.5$
	Obtains correct probability using Poisson or exponential model	AO1.1b	A1	$P(X = 0) = \frac{1.5^0 \times e^{-1.5}}{0!} = e^{-1.5} =$ 0.223
	Total		**11**	
	TOTAL		**50**	