SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
FURTHER MATHEMATICS
Paper 1

Exam Date | Morning | Time allowed: 2 hours

Materials
For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A vector is given by **a** = $\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

which vector is **not** perpendicular to **a**?

Circle your answer.

[1 mark]

$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

2 Use the definitions of $\cosh x$ and $\sinh x$ in terms of e^{x} and e^{-x} to show that $\cosh^2 x - \sinh^2 x \equiv 1$

[2 marks]

3 (a) Given that

$$\frac{2}{(r+1)(r+2)(r+3)} \equiv \frac{A}{(r+1)(r+2)} + \frac{B}{(r+2)(r+3)}$$

find the values of the integers A and B

[2 marks]

3 (b) Use the method of differences to show clearly that

$$\sum_{r=9}^{97} \frac{1}{(r+1)(r+2)(r+3)} = \frac{89}{19800}$$

[4 marks]

4 A student states that $\int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x - \sin x} \, dx$ is not an improper integral because $\frac{\cos x + \sin x}{\cos x - \sin x}$ is defined at both $x = 0$ and $x = \frac{\pi}{2}$

Assess the validity of the student's argument.

[2 marks]

5 $\mathrm{p}(z) = z^4 + 3z^2 + az + b$, $a \in \mathbb{R}$, $b \in \mathbb{R}$

$2 - 3\mathrm{i}$ is a root of the equation $\mathrm{p}(z) = 0$

5 (a) Express $\mathrm{p}(z)$ as a product of quadratic factors with real coefficients.

[5 marks]

5 (b) Solve the equation $\mathrm{p}(z) = 0$.

[1 mark]

6 (a) Obtain the general solution of the differential equation

$$\tan x \frac{\mathrm{d}y}{\mathrm{d}x} + y = \sin x \tan x$$

where $0 < x < \frac{\pi}{2}$

[5 marks]

6 (b) Hence find the particular solution of this differential equation, given that $y = \frac{1}{2\sqrt{2}}$

when $x = \frac{\pi}{4}$

[2 marks]

7 Three planes have equations,

$$x - y + kz = 3$$

$$kx - 3y + 5z = -1$$

$$x - 2y + 3z = -4$$

Where k is a real constant. The planes do not meet at a unique point.

7 (a) Find the possible values of k

[3 marks]

7 (b) There are two possible geometric configurations for the given planes.

Identify each possible configurations, stating the corresponding value of k

Fully justify your answer.

[5 marks]

7 (c) Given further that the equations of the planes form a consistent system, find the solution of the system of equations.

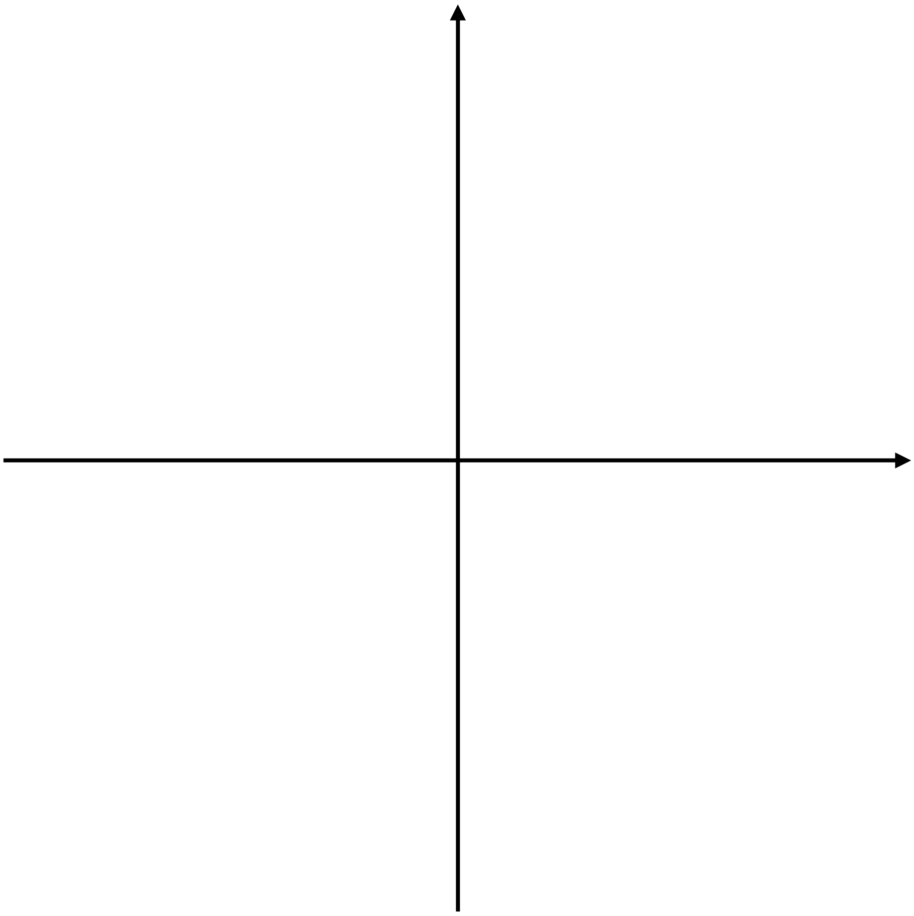
[3 marks]

8 A curve has equation

$$y = \frac{5 - 4x}{1 + x}$$

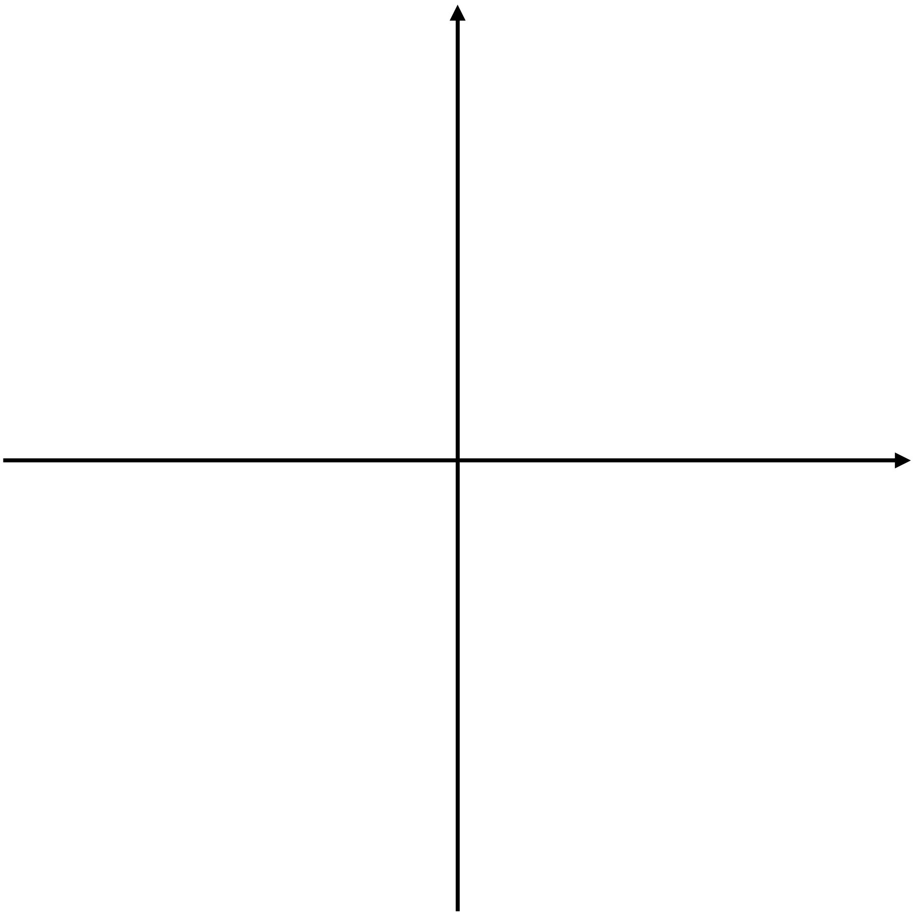
8 (a) Sketch the curve.

[4 marks]



8 (b) Hence sketch the graph of $y = \left|\dfrac{5-4x}{1+x}\right|$.

[1 mark]



9 The line L has Cartesian equations $x - p = \dfrac{y+2}{q} = 3 - z$ and the plane π has equation $\mathbf{r}.\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + 3 = 0$

9 (a) In the case where the plane fully contains the line, find the values of p and q.

[3 marks]

9 (b) In the case where the line intersects the plane at a single point, find the range of values of p and q.

[3 marks]

9 (c) In the case where the angle θ between the line and the plane satisfies $\sin\theta = \frac{1}{\sqrt{6}}$ and the line intersects the plane at $z = 0$

9 (c) (i) Find the value of q.

[4 marks]

9 (c) (ii) Find the value of p.

[3 marks]

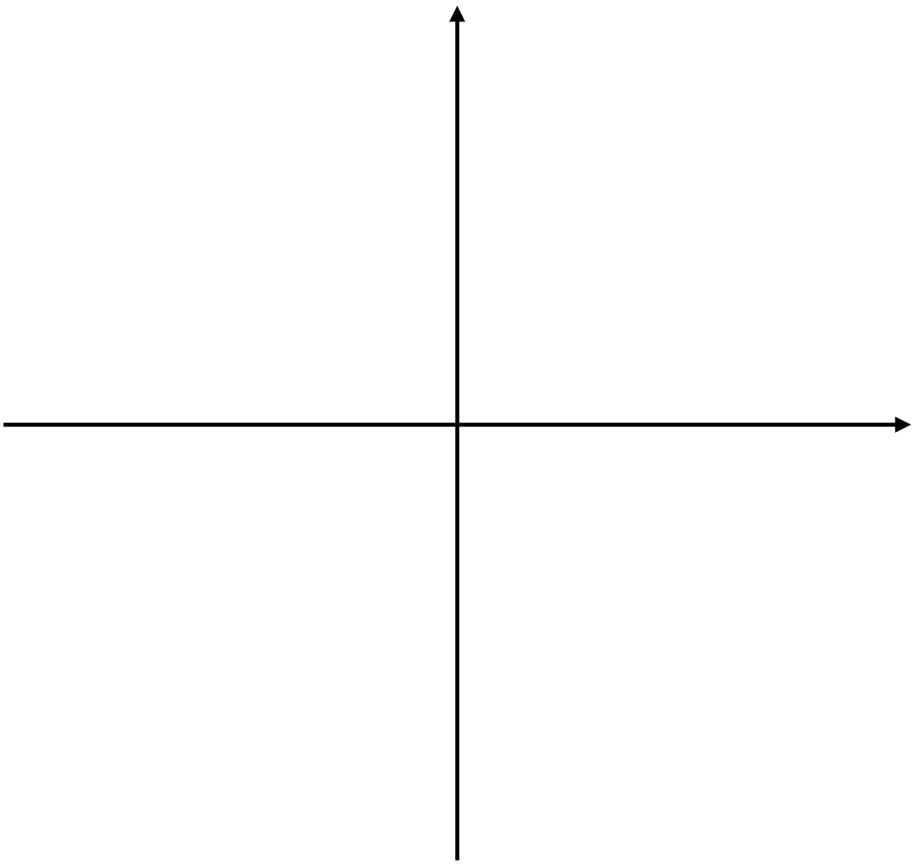
10 The curve, C, has equation $y = \dfrac{x}{\cosh x}$

10 (a) Show that the x-coordinates of any stationary points of C satisfy the equation $\tanh x = \dfrac{1}{x}$

[3 marks]

10 (b) (i) Sketch the graphs of $y = \tanh x$ and $y = \dfrac{1}{x}$ on the axes below.

[2 marks]



10 (b) (ii) Hence determine the number of stationary points of the curve C.

[1 mark]

10 (c) Show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = 0$ at each of the stationary points of the curve C.

[4 marks]

11 (a) Prove that $\dfrac{\sinh\theta}{1+\cosh\theta}+\dfrac{1+\cosh\theta}{\sinh\theta}\equiv 2\coth\theta$

Explicitly state any hyperbolic identities that you use within your proof.

[4 marks]

11 (b) Solve $\dfrac{\sinh\theta}{1+\cosh\theta}+\dfrac{1+\cosh\theta}{\sinh\theta}=4$ giving your answer in an exact form.

[2 marks]

12 The function $\mathrm{f}(x) = \cosh(\mathrm{i}x)$ is defined over the domain $\{x \in \mathbb{R} : -a\pi \le x \le a\pi\}$, where a is a positive integer.

By considering the graph of $y = [\mathrm{f}(x)]^n$, find the mean value of $[\mathrm{f}(x)]^n$, when n is a positive odd number.

Fully justify your answer.

[3 marks]

13 Given that $\mathbf{M}=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, prove that $\mathbf{M}^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$ for all $n\in\mathbb{N}$

[5 marks]

14 A particle, P, of mass M is released from rest and moves along a horizontal straight line through a point O. When P is at a displacement of x metres from O, moving with a speed v ms^{-1}, a force of magnitude $|8Mx|$ acts on the particle directed towards O. A resistive force, of magnitude $4Mv$, also acts on P.

14 (a) Initially P is held at rest at a displacement of 1 metre from O. Describe completely the motion of P after it is released.

Fully justify your answer.

[8 marks]

14 (b) It is decided to alter the resistive force so that the motion of P is critically damped.

Determine the magnitude of the resistive force that will produce critically damped motion.

[4 marks]

15 An isolated island is populated by rabbits and foxes. At time t the number of rabbits is x and the number of foxes is y.

It is assumed that:

- The number of foxes increases at a rate proportional to the number of rabbits. When there are 200 rabbits the number of foxes is increasing at a rate of 20 foxes per unit period of time.
- If there were no foxes present, the number of rabbits would increase by 120% in a unit period of time.
- When both foxes and rabbits are present the foxes kill rabbits at a rate that is equal to 110% of the current number of foxes.
- At time $t = 0$, the number of foxes is 20 and the number of rabbits is 80.

15 (a) (i) Construct a mathematical model for the rate of change of the number of rabbits.

[9 marks]

15 (a) (ii) Use this model to show that the number of rabbits has doubled after approximately 0.7 units of time.

[1 mark]

15 (b) Suggest one way in which the model that you have used for the rate of change of the number of rabbits could be refined.

[1 mark]

END OF QUESTIONS