Further Pure 1 Past Paper Questions Pack B

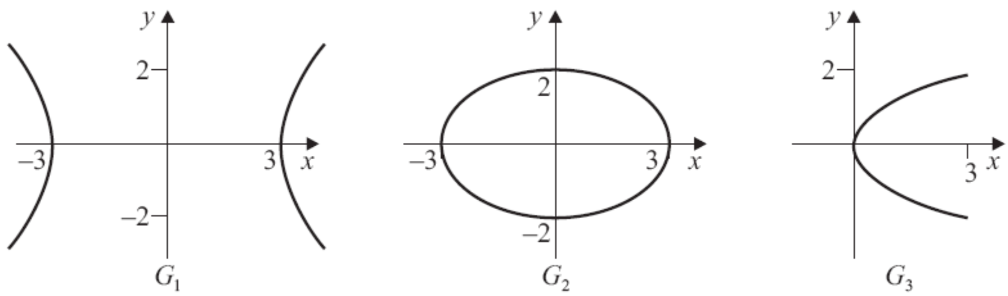
Taken from MBP1, MBP3, MBP4, MBP5

Parabolas, Ellipses and Hyperbolas

Pure 3 January 2002

1 (a) The graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is one of those sketched below.

Identify whether it is graph G_1, G_2 or G_3.



(1 mark)

(b) On separate sets of axes, sketch the graphs of:

(i) $\frac{(x+3)^2}{9} - \frac{y^2}{4} = 1$; *(2 marks)*

(ii) $\frac{y^2}{9} - \frac{x^2}{4} = 1$. *(2 marks)*

Pure 3 January 2003

3 (a) Sketch the graph of $\frac{x^2}{16} + \frac{y^2}{49} = 1$, marking the values of the intercepts with the coordinate axes. *(3 marks)*

(b) Describe a sequence of geometrical transformations that maps the graph of $\frac{x^2}{16} + \frac{y^2}{49} = 1$ onto the graph of $\frac{x^2}{4} + \frac{(y-3)^2}{49} = 1$. *(4 marks)*

Pure 3 January 2004

2 (a) Sketch the curve with equation $y^2 = 8x$. *(2 marks)*

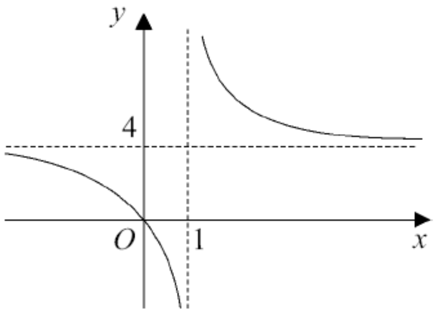
(b) Write down the equation of the curve obtained when the curve $y^2 = 8x$ is reflected in the line $y = x$. *(2 marks)*

(c) Describe a geometrical transformation that maps the curve $y^2 = 8x$ onto the curve with equation $y^2 = 8x - 16$. *(2 marks)*

Rational Functions and Asymptotes

Pure 3 January 2004

3 The graph of $y = f(x)$ is sketched below.
The asymptotes have equations $x = 1$ and $y = 4$.



(a) Given that $f(x) = \dfrac{ax}{x-b}$, use the sketch to find the values of a and b. *(2 marks)*

(b) Sketch the graph of $y^2 = f(x)$ and state the equations of its asymptotes. *(5 marks)*

Pure 3 January 2002

3 (a) Sketch the graph of $y = \dfrac{5x-7}{x-3}$.

State the coordinates of the points where the curve crosses the coordinate axes and write down the equations of its asymptotes. *(5 marks)*

(b) Using the graph from part (a), or otherwise, solve the inequality

$$\frac{5x-7}{x-3} > 0.$$ *(2 marks)*

Pure 3 June 2002

2 (a) Sketch the graph of $y = \dfrac{3x+4}{x-2}$.

State the coordinates of the points where the curve crosses the coordinate axes and write down the equations of its asymptotes. *(6 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{3x+4}{x-2} > 1.$$ *(3 marks)*

Pure 3 January 2003

1 (a) Sketch the graph of $y = \dfrac{3-4x}{2x-5}$.

State the coordinates of the points where the graph crosses the coordinate axes and write down the equations of its asymptotes. *(6 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{3-4x}{2x-5} < 0$$ *(3 marks)*

Pure 5 June 2002

7 A curve has equation $y = \dfrac{3x^2 - 9x + 7}{(2x-3)(x-2)}$.

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) (i) Prove that there are no real values of x for which $-3 < y < 1$. *(7 marks)*

(ii) Hence find the coordinates of the turning points on the curve. *(4 marks)*

Pure 5 January 2003

2 A curve has equation $y = \dfrac{2x^2 + 1}{x^2}$.

(a) Find the equations of the asymptotes to the curve. *(2 marks)*

(b) Sketch the curve. *(2 marks)*

Pure 5 June 2003

3 A curve has equation $y = \dfrac{x^2 + 2}{2x + 1}$.

Prove that there are no real values of x for which $-2 < y < 1$. *(6 marks)*

Pure 3 June 2004

2 (a) A curve has equation $y = \dfrac{3x+4}{1-2x}$.

(i) Find the coordinates of the points where the curve crosses the coordinate axes. *(2 marks)*

(ii) State the equations of its asymptotes. *(2 marks)*

(iii) Sketch the curve. *(2 marks)*

(b) Calculate the x-coordinate of the point where the curve $y = \dfrac{3x+4}{1-2x}$ intersects the line $y = 1$. *(2 marks)*

(c) Hence, or otherwise, solve the inequality $\dfrac{3x+4}{1-2x} \leqslant 1$. *(3 marks)*

Pure 5 January 2004

5 A curve has equation $y = \dfrac{x^2}{x^2+3x+3}$.

(a) Write down the equation of the horizontal asymptote to the curve. *(1 mark)*

(b) (i) Prove that, for all real values of x, y satisfies the inequality $0 \leqslant y \leqslant 4$. *(6 marks)*

(ii) Hence find the coordinates of the turning points on the curve. *(3 marks)*

(c) Given that there are no vertical asymptotes, sketch the curve. *(3 marks)*

Pure 5 June 2004

5 A curve has equation $y = \dfrac{x^2}{x+1}$.

(a) Find the equations of the two asymptotes to the curve. *(3 marks)*

(b) Given that $y \leqslant -4$ or $y \geqslant 0$ for all real values of x, and that there are no values of y for which $-4 < y < 0$, find the coordinates of the two turning points of the curve. *(3 marks)*

(c) Sketch the curve. *(3 marks)*

Complex Numbers

Pure 3 June 2002

4 (a) Given that $z = -2 + 2\sqrt{3}\,i$, show that $z^2 + 4z$ is real. *(3 marks)*

Pure 3 January 2004

6 (a) Find the value of the following, giving each answer in the form $a + b$i, where a and b are integers.

(i) $(2 + 3i)^2$ *(2 marks)*

(ii) $(2 + 3i)^4$ *(2 marks)*

Roots of Quadratic Equations

Pure 3 January 2002

9 (a) The roots of the quadratic equation $x^2 + 4x + 13 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^3 + \beta^3$; *(4 marks)*

(ii) $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2}$. *(2 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots $\dfrac{\alpha}{\beta^2}$ and $\dfrac{\beta}{\alpha^2}$. *(3 marks)*

(c) Find the complex roots of the equation $x^2 + 4x + 13 = 0$. *(3 marks)*

Pure 3 June 2002

1 (a) The roots of the quadratic equation $x^2 + 4x - 3 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$;

(ii) $\left(\alpha^2 + \dfrac{2}{\beta}\right)\left(\beta^2 + \dfrac{2}{\alpha}\right)$. *(6 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots

$$\left(\alpha^2 + \frac{2}{\beta}\right) \text{ and } \left(\beta^2 + \frac{2}{\alpha}\right).$$ *(4 marks)*

Pure 3 January 2003

7 The roots of the quadratic equation $x^2 + 3x - 2 = 0$ are α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(1 mark)*

(b) Without solving the equation, find the value of:

(i) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ *(3 marks)*

(ii) $\left(\alpha - \dfrac{3}{\beta^2}\right)\left(\beta - \dfrac{3}{\alpha^2}\right)$ *(3 marks)*

(c) Determine a quadratic equation with integer coefficients which has roots

$$\alpha - \frac{3}{\beta^2} \quad \text{and} \quad \beta - \frac{3}{\alpha^2}$$ *(4 marks)*

Pure 3 June 2003

9 The roots of the quadratic equation $x^2 - 3x + 1 = 0$ are α and β.

(a) Without solving the equation:

(i) show that $\alpha^2 + \beta^2 = 7$; *(3 marks)*

(ii) find the value of $\alpha^3 + \beta^3$. *(3 marks)*

(b) (i) Show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$. *(1 mark)*

(ii) Hence find the value of $\alpha^4 + \beta^4$. *(2 marks)*

(c) Determine a quadratic equation with integer coefficients which has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$. *(5 marks)*

Pure 3 January 2004

1 The roots of the quadratic equation $x^2 + 2x + 3 = 0$ are α and β.

(a) Without solving the equation:

(i) write down the value of $\alpha + \beta$ and the value of $\alpha\beta$; *(2 marks)*

(ii) show that $\alpha^3 + \beta^3 = 10$; *(3 marks)*

(iii) find the value of $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3}$. *(2 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots

$$\frac{1}{\alpha^3} \quad \text{and} \quad \frac{1}{\beta^3}$$ *(3 marks)*

Pure 3 June 2004

3 The roots of the quadratic equation $x^2 + (7 + p)x + p = 0$ are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$, in terms of p. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$ in terms of p. *(2 marks)*

(c) (i) Show that $(\alpha - \beta)^2 = p^2 + 10p + 49$. *(2 marks)*

(ii) Given that α and β differ by 5, find the possible values of p. *(3 marks)*

Series

Pure 1 June 2003

3 (a) Find the value of:

(i) $\sum_{r=1}^{100} r^3$ *(1 mark)*

(ii) $\sum_{r=51}^{100} r^3$ *(2 marks)*

(b) Find the sum of the fifty integers from 51 to 100 inclusive. *(3 marks)*

(c) Hence find the value of $\sum_{r=51}^{100} (r^3 - 6325r)$. *(2 marks)*

Pure 1 June 2004

6 (a) Find the value of $\sum_{r=1}^{29} r^2$. *(2 marks)*

(b) (i) The first two terms of an arithmetic series are 3 and 7 respectively.

Write down the rth term of the series, giving your answer in its simplest form. *(3 marks)*

(ii) Express the sum of the following arithmetic series in sigma notation

$$3 + 7 + 11 + \ldots + 799$$

(*You are not required to evaluate this sum.*) *(2 marks)*

Calculus

Pure 1 June 2004

(d) The points P and Q lie on the curve with equation $y = x^2 - 6x + 10$.
The x-coordinate of P is 1 and the x-coordinate of Q is $1 + h$.

(i) Show that the gradient of the chord PQ is $h - 4$. *(3 marks)*

(ii) Deduce the value of the gradient of the curve at the point P. *(1 mark)*

Pure 3 January 2002

6 [A sheet of graph paper is supplied for use in this question.]

The energy, E, lost in a cycle of magnetization of a transformer core is thought to relate to the flux density, B, by a law of the form $E = kB^{\alpha}$ where k and α are constants.

(a) Express $\ln E$ in terms of $\ln k$, α and $\ln B$. *(1 mark)*

For a given material, the values of B and E in appropriate units are:

B	3.16	9.56	18.3	29.0	41.4
E	1	2	3	4	5

(b) Plot $\ln E$ against $\ln B$ on graph paper. *(3 marks)*

(c) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(d) Use your line to estimate:

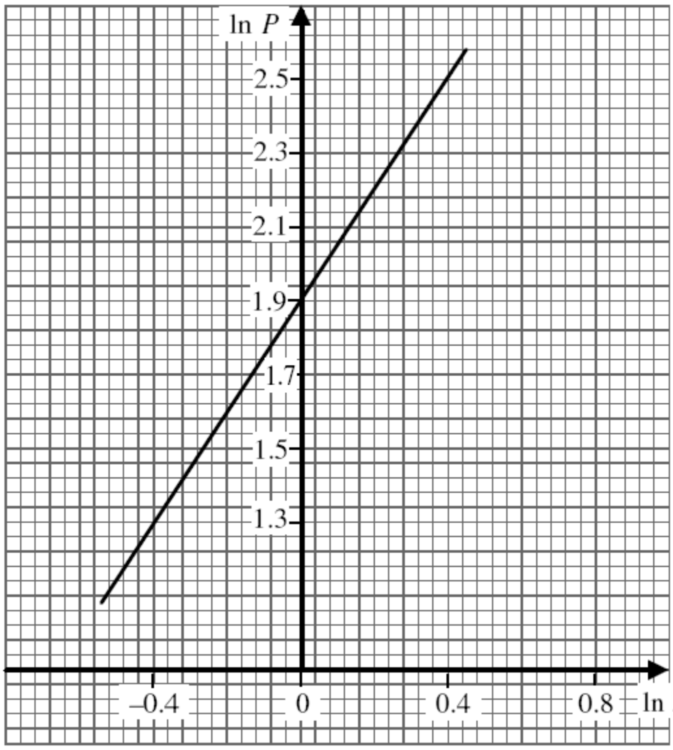
(i) the value of E when $B = 25.5$ giving your answer to 2 significant figures; *(3 marks)*

(ii) the values of k and α, giving your answers to 2 significant figures. *(4 marks)*

Pure 3 January 2003

5 A mathematical model is used by an astronomer to investigate features of the moons of a particular planet. The mean distance of a moon from the planet, measured in millions of kilometres, is denoted by x, and the corresponding period of its orbit is P days.

The model assumes that the graph of $\ln P$ against $\ln x$ is the straight line drawn below.



(a) Use the graph to estimate the period of the orbit of a moon for which $x = 1.43$. *(3 marks)*

(b) The graph would suggest that P and x are related by an equation of the form

$$P = kx^{\alpha}$$

where k and α are constants.

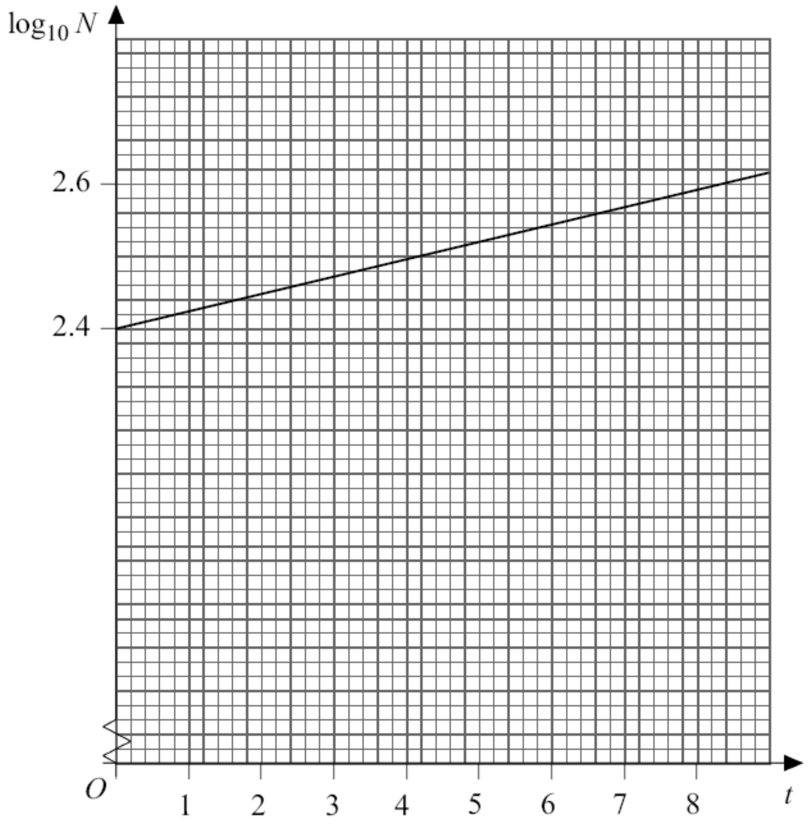
(i) Express $\ln P$ in terms of $\ln k$, $\ln x$ and α. *(1 mark)*

(ii) Use the graph to determine the values of k and α, giving your answers to 2 significant figures. *(4 marks)*

Pure 3 June 2003

7 A mathematical model is required to estimate the number, N, of a certain strain of bacteria in a test tube at time t hours after a certain instant.

After values of $\log_{10} N$ are plotted against t, a straight line graph can be drawn through the points as shown below.



(a) Use the graph to estimate the number of bacteria when $t = 5$. *(3 marks)*

(b) The graph would suggest that N and t are related by an equation of the form

$$N = a \times b^t$$

where a and b are constants.

(i) Express $\log_{10} N$ in terms of $\log_{10} a$, $\log_{10} b$ and t. *(2 marks)*

(ii) Use the graph to determine the values of a and b, giving your answers to 3 significant figures. *(4 marks)*

(c) Suggest why the model $N = a \times b^t$ is likely to give an overestimate of the number of bacteria in the test tube for large values of t. *(1 mark)*

Pure 3 January 2004

5 [An insert is provided for use in answering this question.]

The variables Q and x satisfy a relationship of the form $Q = ax^b$, where a and b are constants.

Measurements of Q for given values of x gave the following results.

x	0.4	0.5	0.6	0.7	0.8
Q	1.72	3.02	4.74	6.98	9.73

(a) Express $\ln Q$ in terms of $\ln a$, b and $\ln x$. *(1 mark)*

(b) (i) Complete the table on the insert and plot $\ln Q$ against $\ln x$ on the axes provided. *(3 marks)*

(ii) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(c) Use your line to estimate:

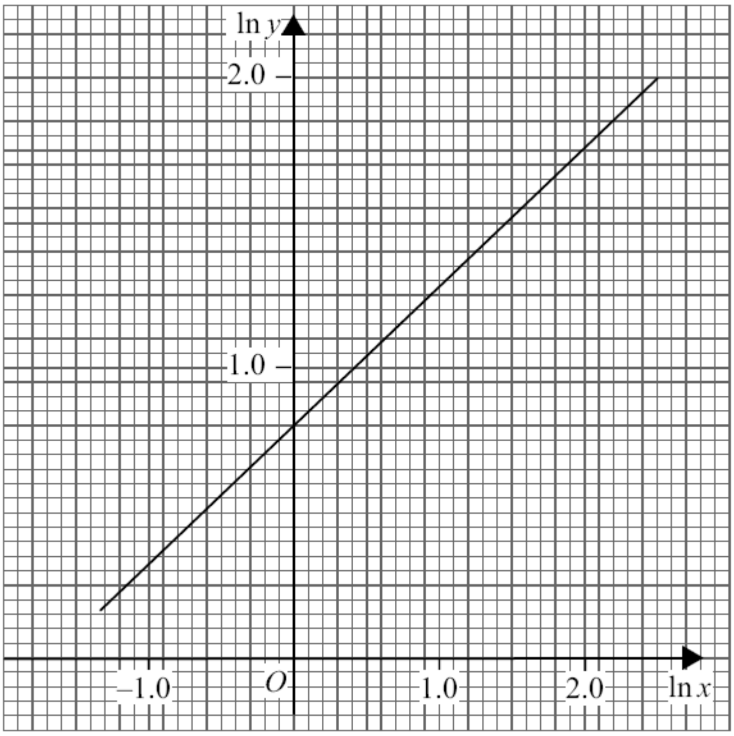
(i) the value of Q when $x = 0.54$, giving your answer to two significant figures; *(2 marks)*

(ii) the values of a and b, giving your answers to two significant figures. *(4 marks)*

Pure 3 June 2004

6 A student performs an experiment and records data for two variables x and y.

Values of $\ln x$ and $\ln y$ are calculated and a line of best fit is drawn as shown below.



(a) Use this graph to find the value of y when $x = 3.0$, giving your answer to two significant figures. *(3 marks)*

(b) The student believes there is a relationship between x and y of the form $y = Ax^n$, where A and n are constants.

(i) Express $\ln y$ in terms of $\ln A$, $\ln x$ and n. *(1 mark)*

(ii) Use the graph to estimate the values of A and n, giving your answers to two significant figures. *(4 marks)*

Numerical Methods

Pure 4 January 2002

2 A polynomial is defined by $p(x) = 4x^3 - 5x^2 + 2$.

(a) Find the remainder when $p(x)$ is divided by $(2x+1)$. *(2 marks)*

(b) The equation $p(x) = 0$ has a single real root, α.

Use the Newton-Raphson method once with first approximation -0.5 to find a second approximation to α, giving your answer to 3 decimal places. *(3 marks)*

Pure 4 January 2003

2 (a) Use logarithms to solve the equation $2^x = 7$, giving your answer to three significant figures. *(2 marks)*

(b) The equation

$$2^x = 7 - x$$

has a single root, α.

(i) Show that α lies between 2.0 and 2.4. *(1 mark)*

(ii) Use the bisection method to find an interval of width 0.1 in which α lies. *(3 marks)*

Pure 4 June 2003

5 A curve is defined for $0 \leqslant x \leqslant \pi$ by the equation

$$y = 2x - 1 + \sin 2x$$

(a) (i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) The curve crosses the x-axis when $x = \alpha$. Use the Newton-Raphson iterative formula with first approximation 0.2 to find a second approximation for α, giving your answer to three significant figures. *(2 marks)*

Pure 4 June 2004

4 The polynomial $p(x)$ is given by

$$p(x) = x^3 - 6x^2 + 12x - 11$$

(a) Find the remainder when $p(x)$ is divided by $(x - 3)$. *(2 marks)*

(b) The equation $p(x) = 0$ has a single real root α.

(i) Show that α lies between 3 and 4. *(1 mark)*

(ii) Use the bisection method to find an interval of width 0.25 in which α lies. *(3 marks)*

Matrix Transformations

Pure 3 January 2002

2 The matrix A is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(a) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}.$$

Describe fully the geometrical transformation represented by **T**. *(2 marks)*

(b) Find the matrix A^3. *(2 marks)*

Pure 3 January 2003

2 The matrix $\boldsymbol{M}$ is $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$.

(a) Find:

(i) $\boldsymbol{M}^2$; *(2 marks)*

(ii) $\boldsymbol{M}^3$. *(1 mark)*

(b) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Describe fully the geometrical transformation represented by **T**. *(2 marks)*