

# Pure Core 2 Past Paper Questions

Taken from MAME, MAP1, MAP2, MAP3

## Methods November 2003

5 In this question, no credit will be given for using approximate decimal values.

It is given that  $p = 8^{\frac{1}{2}}$  and  $q = 4^{\frac{3}{4}}$ .

- (a) Show that  $p = 2^{\frac{3}{2}}$ . (1 mark)
- (b) Similarly, express  $q$  as a power of 2. (1 mark)
- (c) Hence express  $pq$  as a power of 2. (2 marks)

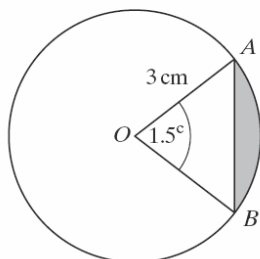
## Pure 1 January 2001

1 Given that

$$y = x^2 - x^{-2},$$

- (a) find the value of  $\frac{dy}{dx}$  at the point where  $x = 2$ , (3 marks)
- (b) find  $\int y \, dx$ . (2 marks)

3



The diagram shows a circle with centre  $O$  and radius  $3 \text{ cm}$ . The points  $A$  and  $B$  on the circle are such that the angle  $AOB$  is  $1.5$  radians.

- (a) Find the length of the minor arc  $AB$ . (1 mark)
- (b) Find the area of the minor sector  $OAB$ . (2 marks)
- (c) Show that the area of the shaded segment is approximately  $2.3 \text{ cm}^2$ . (4 marks)

5 An electrician cuts a piece of wire of length 27 metres from a long roll of wire. He then cuts off more pieces, each piece being two-thirds as long as the preceding piece.

(a) Calculate the length of the fourth piece cut off. (2 marks)

(b) Show that, no matter how many pieces he cuts off, their total length cannot exceed 81 metres. (2 marks)

(c) Show that the total length of the first eleven pieces cut off is more than 80 metres. (3 marks)

(d) The length of the eleventh piece of wire is  $u$  metres. Write down an expression for  $u$ . Hence show that

$$\ln u = a \ln 2 - b \ln 3,$$

where  $a$  and  $b$  are positive integers to be determined. (4 marks)

## Pure 1 June 2001

1 (a) Find the sum of the three hundred integers from 101 to 400 inclusive. (3 marks)

(b) Find the sum of the geometric series

$$2 + 6 + 18 + \dots + 2 \times 3^{n-1},$$

giving your answer in the form

$$p^n - q,$$

where  $p$  and  $q$  are integers. (3 marks)

4 (a) Given that

$$2\cos^2 \theta - \sin \theta = 1,$$

show that

$$2\sin^2 \theta + \sin \theta - 1 = 0. \quad (2 \text{ marks})$$

(b) *In this part of the question, no credit will be given for an approximate numerical method.*

Hence find all the values of  $\theta$  in the interval  $0 < \theta < 2\pi$  for which

$$2\cos^2 \theta - \sin \theta = 1,$$

giving each answer in terms of  $\pi$ . (4 marks)

(c) **Write down** all the values of  $x$  in the interval  $0 < x < \pi$  for which

$$2\cos^2 2x - \sin 2x = 1. \quad (2 \text{ marks})$$

5 (a) Express  $x^2 \sqrt{x}$  in the form  $x^p$ . (1 mark)

(b) Given that

$$y = x^2 \sqrt{x},$$

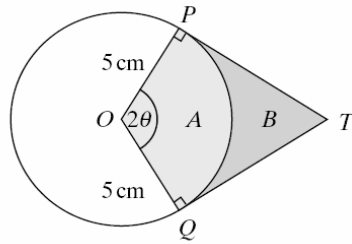
find the value of  $\frac{dy}{dx}$  at the point where  $x = 9$ . (3 marks)

## Pure 1 January 2002

- 1 It is given that  $y = x^{\frac{1}{3}}$ .
- (a) Find  $\frac{dy}{dx}$ . *(2 marks)*
- (b) (i) Find  $\int y \, dx$ . *(2 marks)*
- (ii) Hence evaluate  $\int_0^8 y \, dx$ . *(2 marks)*
- 2 (a) Show that  $\log_2 8 = 3$ . *(1 mark)*
- (b) Find the value of
- (i)  $\log_2 (8^4)$ , *(1 mark)*
- (ii)  $\log_2 \left( \frac{1}{\sqrt{8}} \right)$ . *(2 marks)*
- 3 A pipeline is to be constructed under a lake. It is calculated that the first mile will take 15 days to construct. Each further mile will take 3 days longer than the one before, so the 1st, 2nd and 3rd miles will take 15, 18 and 21 days, respectively, and so on.
- (a) Find the  $n$ th term of the arithmetic sequence 15, 18, 21, ... . *(2 marks)*
- (b) Show that the total time taken to construct the first  $n$  miles of the pipeline is  $\frac{3}{2}n(n+9)$  days. *(3 marks)*
- (c) Calculate the total length of pipeline that can be constructed in 600 days. *(3 marks)*

- (b) The diagram shows a sector  $A$  of a circle with centre  $O$  and radius 5 cm.  $OP$  and  $OQ$  are the radii forming part of the boundary of  $A$ , and the angle  $POQ$  is  $2\theta$  radians.

The tangents to the circle at  $P$  and  $Q$  intersect at  $T$ , and the shaded region outside the circle is  $B$ .



- (i) Write down the area of the sector  $A$  in terms of  $\theta$ . (2 marks)
- (ii) Find the area of the triangle  $OPT$  in terms of  $\theta$ . (2 marks)
- (iii) Deduce that the area of the region  $B$  is  $25(\tan \theta - \theta) \text{ cm}^2$ . (2 marks)
- (iv) Given that the regions  $A$  and  $B$  have equal areas, show that
- $$\tan \theta - 2\theta = 0. \quad (2 \text{ marks})$$
- (v) Use your result from part (a)(ii) to give the value of  $\theta$  correct to one decimal place. (1 mark)

## Pure 1 June 2002

- 2 (a) Find the sum of the 16 terms of the arithmetic series

$$2 + 5 + 8 + \dots + 47. \quad (3 \text{ marks})$$

- (b) An arithmetic sequence  $u_1, u_2, u_3, \dots$  has  $r$ th term  $u_r$ , where

$$u_r = 50 - 3r.$$

- (i) Write down the values of  $u_1, u_2, u_3$  and  $u_4$ . (2 marks)
- (ii) Show that the sequence has exactly 16 positive terms. (2 marks)

5 (a) Write down the exact values of:

(i)  $\sin \frac{\pi}{4}$ ;

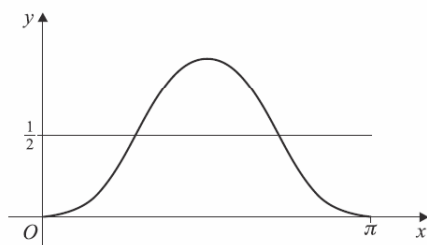
(ii)  $\cos \frac{\pi}{6}$ ;

(iii)  $\tan \frac{\pi}{3}$ .

(3 marks)

The diagram shows the graphs of

$$y = \sin^2 x \text{ and } y = \frac{1}{2} \text{ for } 0 \leq x \leq \pi.$$



(b) Solve  $\sin^2 x = \frac{1}{2}$  for  $0 \leq x \leq \pi$ .

(3 marks)

(c) Hence solve  $\sin^2 x > \frac{1}{2}$  for  $0 \leq x \leq \pi$ .

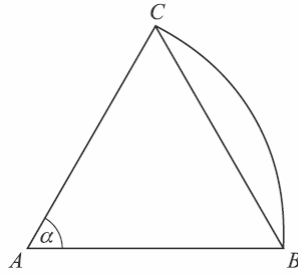
(2 marks)

(d) Prove that

$$\sin^2 x > \frac{1}{2} \Rightarrow \cos^2 x < \frac{1}{2}.$$

(2 marks)

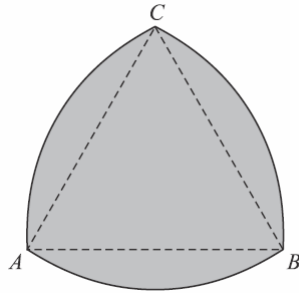
6 (a)



The diagram shows an equilateral triangle  $ABC$  with sides of length 6 cm and an arc  $BC$  of a circle with centre  $A$ .

- (i) Write down, in radians, the value of the angle  $\alpha$ . (1 mark)
- (ii) Find the length of the arc  $BC$ . (2 marks)
- (iii) Show that the area of the triangle  $ABC$  is  $9\sqrt{3}$  cm<sup>2</sup>. (3 marks)
- (iv) Show that the area of the sector  $ABC$  is  $6\pi$  cm<sup>2</sup>. (3 marks)

(b)



The diagram shows an ornament made from a flat sheet of metal. Its boundary consists of three arcs of circles. The straight lines  $AB$ ,  $AC$  and  $BC$  are each of length 6 cm. The arcs  $BC$ ,  $AC$  and  $AB$  have centres  $A$ ,  $B$  and  $C$  respectively.

- (i) The boundary of the ornament is decorated with gilt edging. Find the total length of the boundary, giving your answer to the nearest centimetre. (2 marks)
- (ii) Find the area of one side of the ornament, giving your answer to the nearest square centimetre. (3 marks)

## Pure 1 November 2002

- 2 The  $n$ th term of an arithmetic sequence is  $u_n$ , where

$$u_n = 10 + 0.5n.$$

- (a) Find the values of  $u_1$  and  $u_2$ . (2 marks)
- (b) Write down the common difference of the arithmetic sequence. (1 mark)
- (c) Find the value of  $n$  for which  $u_n = 25$ . (2 marks)
- (d) Evaluate  $\sum_{n=1}^{30} u_n$ . (3 marks)

3 (a) Show that  $\int_1^4 x^{\frac{3}{2}} dx = \frac{62}{5}$ . (4 marks)

- 4 (a) Write down the value of  $\log_2 8$ . (1 mark)
- (b) Express  $\log_2 9$  in the form  $n \log_2 3$ . (1 mark)
- (c) Hence show that

$$\log_2 72 = m + n \log_2 3,$$

where  $m$  and  $n$  are integers. (1 mark)

- 5 The angle  $\theta$  radians, where  $0 \leq \theta \leq 2\pi$ , satisfies the equation

$$3 \tan \theta = 2 \cos \theta.$$

- (a) Show that

$$3 \sin \theta = 2 \cos^2 \theta. \quad (1 \text{ mark})$$

- (b) Hence use an appropriate identity to show that

$$2 \sin^2 \theta + 3 \sin \theta - 2 = 0. \quad (3 \text{ marks})$$

- (c) (i) Solve the quadratic equation in part (b). Hence explain why the only possible value of  $\sin \theta$  which will satisfy it is  $\frac{1}{2}$ . (3 marks)
- (ii) Write down the values of  $\theta$  for which  $\sin \theta = \frac{1}{2}$  and  $0 \leq \theta \leq 2\pi$ . (2 marks)
- (iii) For the smaller of these values of  $\theta$ , write down the exact values, in surd form, of  $\tan \theta$  and  $\cos \theta$ . (2 marks)
- (iv) Verify that these exact values satisfy the original equation. (1 mark)

## Pure 1 January 2003

- 1 The first four terms of a geometric sequence are

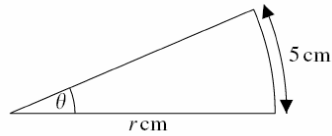
$$10, 9, 8.1, 7.29.$$

- (a) Show that the common ratio of the sequence is 0.9. (1 mark)
- (b) Find the  $n$ th term. (2 marks)
- (c) Show that the sum of the first 25 terms is approximately 92.8. (2 marks)
- (d) Find the sum to infinity. (2 marks)

4 The acute angle  $\theta$  radians is such that

$$\sin \theta = \frac{5}{13}.$$

- (a) (i) Show that  $\cos \theta = \frac{12}{13}$ . (2 marks)
- (ii) Find the value of  $\tan \theta$ , giving your answer as a fraction. (2 marks)
- (b) Use your calculator to find the value of  $\theta$ , giving your answer to three decimal places. (1 mark)
- (c) The diagram shows a sector of a circle of radius  $r$  cm and angle  $\theta$  radians. The length of the arc which forms part of the boundary of the sector is 5 cm.



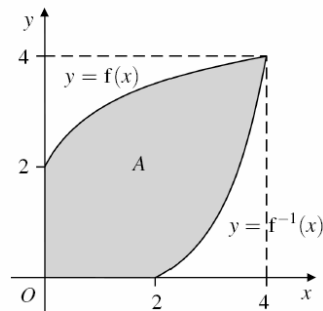
- (i) Show that  $r \approx 12.7$ . (2 marks)
- (ii) Find the area of the sector, giving your answer to the nearest square centimetre. (3 marks)

6 The function  $f$  is defined for  $x \geq 0$  by

$$f(x) = x^{\frac{1}{2}} + 2.$$

- (a) (i) Find  $f'(x)$ . (2 marks)
- (ii) Hence find the gradient of the curve  $y = f(x)$  at the point for which  $x = 4$ . (1 mark)
- (b) (i) Find  $\int f(x) dx$ . (3 marks)
- (ii) Hence show that  $\int_0^4 f(x) dx = \frac{40}{3}$ . (2 marks)
- (c) Show that  $f^{-1}(x) = (x - 2)^2$ . (2 marks)
- (d) The diagram shows a symmetrical shaded region  $A$  bounded by:

parts of the coordinate axes;  
the curve  $y = f(x)$  for  $0 \leq x \leq 4$ ; and  
the curve  $y = f^{-1}(x)$  for  $2 \leq x \leq 4$ .



- (i) Write down the equation of the line of symmetry of  $A$ . (1 mark)
- (ii) Calculate the area of  $A$ . (4 marks)

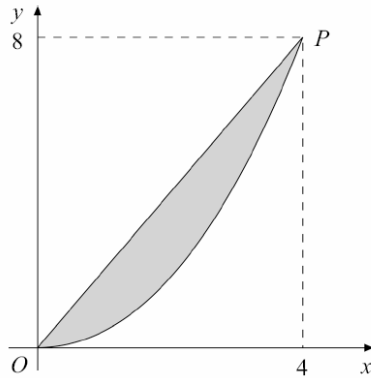


## Pure 1 June 2003

1 The diagram shows the graph of

$$y = x^{\frac{3}{2}}, \quad 0 \leq x \leq 4,$$

and a straight line joining the origin to the point  $P$  which has coordinates  $(4, 8)$ .



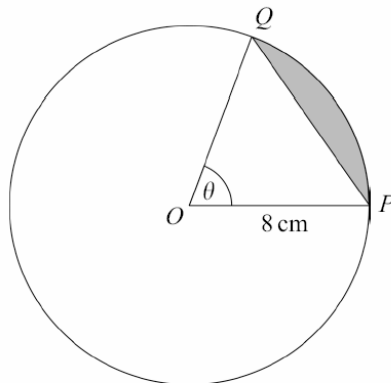
- (a) (i) Find  $\int x^{\frac{3}{2}} dx$ . (2 marks)
- (ii) Hence find the value of  $\int_0^4 x^{\frac{3}{2}} dx$ . (2 marks)
- (b) Calculate the area of the shaded region. (2 marks)

2 The graph of

$$y = x + 4x^{-2}$$

has one stationary point.

- (a) Find  $\frac{dy}{dx}$ . (2 marks)
- (b) Find the coordinates of the stationary point. (3 marks)
- (c) Find the value of  $\frac{d^2y}{dx^2}$  at the stationary point, and hence determine whether the stationary point is a maximum or a minimum. (4 marks)
- 3 The diagram shows a circle with centre  $O$  and radius 8 cm. The angle between the radii  $OP$  and  $OQ$  is  $\theta$  radians.



- (a) (i) Find the area of the sector  $OPQ$  in terms of  $\theta$ . (2 marks)
- (ii) Find the area of the triangle  $OPQ$  in terms of  $\sin \theta$ . (2 marks)
- (iii) Hence write down the area of the shaded segment. (1 mark)

4 It is given that  $x$  satisfies the equation

$$2 \cos^2 x = 2 + \sin x.$$

(a) Use an appropriate trigonometrical identity to show that

$$2 \sin^2 x + \sin x = 0. \quad (2 \text{ marks})$$

(b) Solve this quadratic equation and hence find all the possible values of  $x$  in the interval  $0 \leq x < 2\pi$ . (6 marks)

6 (a) The first three terms of a geometric sequence are  $a, b, c$ . Each term represents an increase of  $p$  per cent on the preceding term.

(i) Show that the common ratio is  $\left(1 + \frac{p}{100}\right)$ . (2 marks)

(ii) It is given that  $a = 2000$ . Express  $b$  and  $c$  in terms of  $p$ . (2 marks)

(b) A deposit of £2000 is put into a bank account. After each year, the balance in the account is increased by  $p$  per cent. There are no other deposits or withdrawals. After two years the balance is £2332.80.

(i) Show that  $p = 8$ . (3 marks)

(ii) Given that after  $n$  years the balance is £ $u_n$ , write down an expression for  $u_n$  in terms of  $n$ . (2 marks)

(iii) Use your answer to part (b)(ii) to find the balance after 10 years. (2 marks)

## Pure 1 November 2003

1 The first four terms of a geometric sequence are

$$6, \quad 2, \quad \frac{2}{3}, \quad \frac{2}{9}.$$

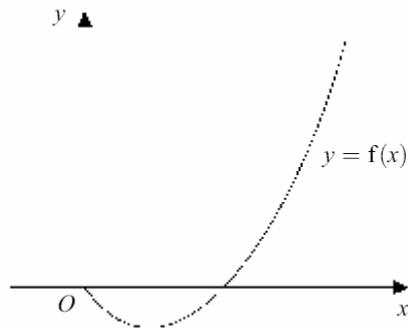
(a) Write down the common ratio. (1 mark)

(b) Find the tenth term, giving your answer to three significant figures. (2 marks)

(c) Find the sum to infinity. (2 marks)

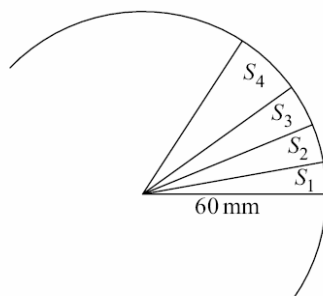
- 2 The diagram shows the graph of  $y = f(x)$ , where

$$f(x) = 5x^{\frac{3}{2}} - 3x.$$



- (a) (i) Differentiate  $f(x)$  to find  $f'(x)$ . (2 marks)
- (ii) Show that, at the stationary point on the graph,  $x = 0.16$ . (3 marks)
- 3 (a) Sketch the graph of
- $$y = \tan x \quad \text{for } 0 \leq x \leq 2\pi,$$
- giving the equations of the asymptotes. (4 marks)
- (b) Solve the equation
- $$\tan x = \sqrt{3} \quad \text{for } 0 \leq x \leq 2\pi. \quad \text{(3 marks)}$$
- 5 (a) Explain briefly why  $\log_5 125 = 3$ . (1 mark)
- (b) Find the value of:
- (i)  $\log_5(125^2)$ ; (1 mark)
- (ii)  $\log_5 \sqrt{125}$ ; (1 mark)
- (iii)  $\log_5 \left( \frac{1}{\sqrt{125}} \right)$ . (1 mark)
- (c) Solve the equation  $\log_5(125x) = 4$ . (2 marks)

- 6 Ashley is cutting a circular disc of radius 60 mm into unequal sectors  $S_1, S_2, S_3, \dots$

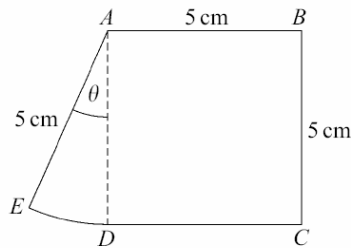


- (a) The first sector,  $S_1$ , has an angle of  $10^\circ$ .
- Write the angle  $10^\circ$  in radians, in terms of  $\pi$ . (2 marks)
  - Show that the area of  $S_1$  is  $100\pi \text{ mm}^2$ . (3 marks)
- (b) The angles of the sectors  $S_2, S_3, S_4, \dots$  are  $12^\circ, 14^\circ, 16^\circ$  and so on in an arithmetic sequence.
- Find the areas of  $S_2, S_3$  and  $S_4$  in terms of  $\pi$ . (3 marks)
  - Show that the sum of the areas of  $S_1, S_2, S_3, \dots, S_n$  is  $10\pi n(n+9) \text{ mm}^2$ . (4 marks)
  - Verify that Ashley will obtain exactly 15 sectors from the circular disc. (3 marks)

## Pure 1 January 2004

- 1 (a) Find  $\int x^{\frac{1}{2}} dx$ . (2 marks)
- (b) Hence find the value of  $\int_0^2 x^{\frac{1}{2}} dx$ ,  
giving your answer in the form  $p\sqrt{2}$ , where  $p$  is a rational number. (3 marks)
- 2 The  $n$ th term of a geometric sequence is  $u_n$ , where
- $$u_n = 2 \times 3^n.$$
- Find the values of  $u_1$  and  $u_2$ . (2 marks)
  - Write down the common ratio of the geometric sequence. (1 mark)
  - Show that the sum of the first 10 terms is  $3(3^{10} - 1)$ . (3 marks)

- 3 The diagram shows a shape  $ABCDE$ . The shape consists of a square  $ABCD$ , with sides of length 5 cm, and a sector  $ADE$  of a circle with centre  $A$  and radius 5 cm. The angle of the sector is  $\theta$  radians.



- (a) Find the area of the sector  $ADE$  in terms of  $\theta$ . (2 marks)
- (b) The area of the sector  $ADE$  is a quarter of the area of the square  $ABCD$ .
- (i) Find the value of  $\theta$ . (2 marks)
- (ii) Find the perimeter of the shape  $ABCDE$ . (2 marks)
- 4 (a) An arithmetic sequence has first term 100 and common difference 2.
- (i) Write down the second and third terms. (2 marks)
- (ii) Given that the last term is 200, find the number of terms. (3 marks)
- (b) A tape dispenser has a length of tape wrapped round a circular cylinder. The length of tape in the first layer (nearest to the cylinder) is 100 mm. Each further layer is 2 mm longer than the one before. The outer layer has 200 mm of tape.
- Calculate the total length of tape. (3 marks)

- 7 (a) Write down the exact values of  $\sin \frac{\pi}{6}$ ,  $\cos \frac{\pi}{6}$  and  $\tan \frac{\pi}{6}$ . (3 marks)
- (b) It is given that  $x$  satisfies the equation

$$3 \sin^2 x = \cos^2 x.$$

By first using an appropriate trigonometrical identity to simplify this equation, find all the solutions of the equation in the interval  $0 \leq x \leq 2\pi$ . (6 marks)

## Pure 1 June 2004

- 1 (a) Find the sum of the 100 terms of the arithmetic series

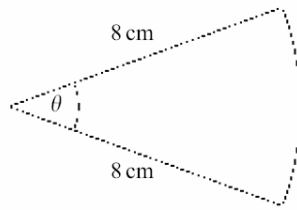
$$3 + 7 + 11 + \dots + 399. \quad (3 \text{ marks})$$

- (b) An arithmetic sequence  $u_1, u_2, u_3, \dots$  has  $r$ th term  $u_r$ , where

$$u_r = 8r - 2.$$

- (i) Write down the values of  $u_1, u_2, u_3$  and  $u_4$ . (2 marks)
- (ii) Using your answer to part (a), or otherwise, find the sum of the first 100 terms of this sequence. (2 marks)

- 2 The diagram shows a sector of a circle of radius 8 cm. The sector has angle  $\theta$  radians. The perimeter of the sector is  $P$  cm and its area is  $A$  cm<sup>2</sup>.



- (a) Show that  $P = 8(\theta + 2)$ . (2 marks)
- (b) Find  $A$  in terms of  $\theta$ . (2 marks)
- (c) Given that  $A = P$ , find the value of  $\theta$ . (3 marks)
- 3 (a) Show that the equation

$$2x^{\frac{3}{2}} - 9x + 6 = 0$$

has a root between 0 and 1. (3 marks)

- (b) A curve has equation

$$y = 2x^{\frac{3}{2}} - 9x.$$

- (i) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . (5 marks)
- (ii) Calculate the coordinates of the stationary point on the curve. (3 marks)
- (iii) Find the value of  $\frac{d^2y}{dx^2}$  at the stationary point and hence determine whether this point is a maximum or a minimum. (2 marks)

4 Write each of the following in the form  $a \ln p + b \ln q$ :

(a)  $\ln(pq)$ ; *(1 mark)*

(b)  $\ln(p^2q^3)$ ; *(1 mark)*

(c)  $\ln\left(\frac{p}{q}\right)$ ; *(1 mark)*

(d)  $\ln\sqrt{\frac{p}{q}}$ . *(1 mark)*

5 (a) A geometric sequence has first term 230 and second term 345.

(i) Show that the common ratio is 1.5. *(1 mark)*

(ii) Calculate the third and fourth terms. *(2 marks)*

(b) In 1501 the population of a country was 2 300 000.

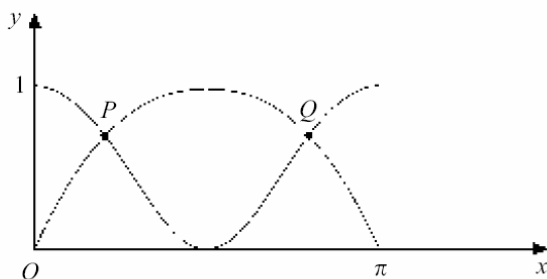
In 1601 the population was 3 450 000.

Assuming that the population in the years 1501, 1601, 1701 and 1801 can be modelled as a geometric sequence, write down an estimate for the population in 1801. Give your answer to three significant figures. *(2 marks)*

6 The diagram shows the graphs of

$$y = \cos^2 x \text{ and } y = \sin x \text{ for } 0 \leq x \leq \pi.$$

The graphs intersect each other at two points  $P$  and  $Q$ .



(a) Use a trigonometric identity to show that the  $x$ -coordinates of  $P$  and  $Q$  satisfy the equation

$$\sin^2 x + \sin x - 1 = 0. \quad (2 \text{ marks})$$

(b) (i) Solve this quadratic equation. *(2 marks)*

(ii) Show that the only possible value for  $\sin x$  is approximately 0.618. *(2 marks)*

(c) Find the  $x$ -coordinates of  $P$  and  $Q$ , giving each answer to two decimal places. *(3 marks)*

## Pure 2 June 2002

- 2 In a clinical trial, the concentration of a drug in the blood at time  $t$  hours from the start of the trial is denoted by  $p_t$ .

It is given that

$$p_{t+1} = a + bp_t,$$

where  $a$  and  $b$  are constants.

Measurements give  $p_0 = 5.0$ ,  $p_1 = 13.0$  and  $p_2 = 14.6$ .

(a) Find:

(i)  $a$  and  $b$ , *(3 marks)*

(ii) the concentration of the drug in the blood after 3 hours. *(1 mark)*

(b) The concentration converges to a limiting value  $w$ . Write down and solve an equation for  $w$ . *(2 marks)*

## Pure 2 June 2004

- 6 (a) The circle  $(x - 4)^2 + (y - 3)^2 = 4$  has centre  $C$  and radius  $r$ .

Write down:

(i) the coordinates of  $C$ ;

(ii) the value of  $r$ . *(2 marks)*

(b) The line  $y = x + 1$  intersects this circle at two points  $A$  and  $B$ .

(i) Find the coordinates of  $A$  and  $B$ . *(5 marks)*

(ii) Show that the area of the minor segment bounded by the circle and the chord  $AB$  is  $\pi - 2$ . *(3 marks)*

## Pure 3 January 2002

- 1 Find the coefficient of the term in  $x^4$  in the binomial expansion of

$$(3 + 2x)^7. \quad \text{span style="float: right;">*(3 marks)*$$

## Pure 3 June 2003

- 1 Find the coefficient of  $x^3$  in the binomial expansion of  $(2 + 3x)^9$ . Give your answer as an integer. *(3 marks)*