GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C1

Paper 6

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**

1. Find the gradient of the curve $y = \sqrt[3]{x^2} - 3x^2$ at the point where $x = -8$. [3]

2. i) Simplify $\left(\sqrt{2} + 2\sqrt{8}\right)^2$. [3]

ii) Find the value of x such that $8^x = \frac{1}{4}$. Show your working clearly. [2]

3. A circle, C, has centre (0, 4) and radius $\sqrt{8}$.

i) Find an equation for C. [2]

ii) Write down the shortest distance from the origin to the circle C. [1]

iii) The line $y = x$ intersects the circle at a single point A. Find the co-ordinates of A. [3]

iv) State the equation of the normal to the circle at the point A. [2]

4. The point A has coordinates $(-1, -13)$. The straight lines with equations $2x + y - 5 = 0$ and $x + 3y = 0$ intersect at the point B.

Show that one of these two lines is perpendicular to AB. [6]

5. It is given that $a^p = 5$ and $a^q = 9$. In each of the following cases, determine the numerical value of the given expression.

i) a^{p+q} [1]

ii) $2a^{-p}$ [1]

iii) $a^{2p - \frac{1}{2}q}$. [3]

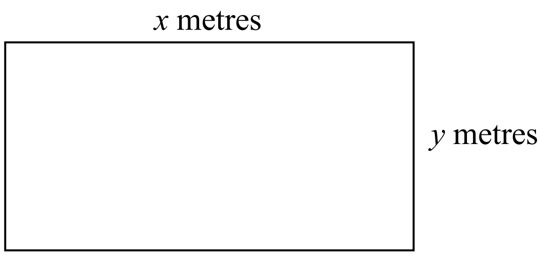
6. i) Solve the inequality $3x^2 - 4x \geq 15$. [3]

ii) The equation $x^2 + 5x + p = 0$ has two distinct real roots. Find the greatest possible **integer** value for p. [3]

7. Solve the equation $x^2 - (7\sqrt{2})x + 20 = 0$, giving your answers in terms of surds, simplified as far as possible. [4]

Hence solve the inequality $x^2 - (7\sqrt{2})x + 20 < 0$. [2]

8.



The diagram shows a rectangular playpen which is to be made out of plastic rods totalling 24 metres in length.

a) Find an expression for y in terms of x. [2]

b) Show that the area, A, of the playpen is given by $A = 12x - x^2$. [1]

It is desired to make the area of the playpen as large as possible. By completing the square, or otherwise, find

c) the value of x for which A is as large as possible, [3]

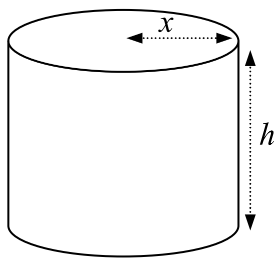
d) the corresponding value of A. [1]

9. The straight line $y = 10 - 3x$ is denoted by L. Show, by means of an appropriate sketch, that the curve $y = x^{\frac{1}{2}}$ and the line L have one and only one point of intersection. [2]

This point of intersection is denoted by P. Calculate the exact coordinates of P. [4]

10. A cylindrical tin, closed at both ends, is made of thin sheet metal and has a volume of $54\pi \text{ cm}^3$.

a) Show that the surface area of the tin, S, is given by
$S = 2\pi x^2 + \dfrac{108\pi}{x}$. [3]



b) Find the dimensions of the tin which has the minimum surface area and verify that for this tin, the height is twice the radius. [6]

ANSWERS / SOLUTIONS.

1. $y = x^{\frac{2}{3}} - 3x^2$.

Differentiating, $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - 6x$.

Put $x = -8$ to get $\frac{dy}{dx} = \frac{2}{3} \times 8^{-\frac{1}{3}} - 6 \times (-8)$

$$= \frac{2}{3} \times \frac{1}{\sqrt[3]{-8}} + 48$$

$$= \frac{2}{3} \times \frac{1}{-2} + 48$$

$$= 47\tfrac{2}{3}.$$

2. i) $\left(\sqrt{2} + 2\sqrt{8}\right)^2 = \left(\sqrt{2} + 4\sqrt{2}\right)^2 = \left(5\sqrt{2}\right)^2 = \left(5\sqrt{2}\right) \times \left(5\sqrt{2}\right) = 25 \times 2 = 50.$

Alternatively, $\left(\sqrt{2} + 2\sqrt{8}\right)^2 = \left(\sqrt{2} + 2\sqrt{8}\right) \times \left(\sqrt{2} + 2\sqrt{8}\right)$ etc.

ii) $8^x = \left(2^3\right)^x = 2^{3x}$.

$\frac{1}{4} = 2^{-2}$.

Therefore we have that $2^{3x} = 2^{-2}$ and thus $3x = -2$

$\Rightarrow x = -\frac{2}{3}$.

3. i) $x^2 + (y - 4)^2 = 8$ (or $x^2 + y^2 - 8y + 8 = 0$).

ii) Distance of (0, 0) to the centre pf the circle = 4.

Shortest distance from (0, 0) to the circle = $4 - \sqrt{8}$. (Draw a diagram!)

iii) Solve simultaneously: $x^2 + y^2 - 8y + 8 = 0$

$y = x$

$\Rightarrow$ $x^2 + x^2 - 8x + 8 = 0$

$2x^2 - 8x + 8 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$\Rightarrow$ $x = 2, y = 2$.

iv) Tangent is $y = x$ with gradient 1 and so the normal has gradient -1.

Equation is given by $y = 4 - x$ (note that this normal must pass through the centre of the circle).

4. Solve simultaneously to find the co-ordinates of B:

$y = 5 - 2x$

$x + 3y = 0$ $\Rightarrow$ $x + 3(5 - 2x) = 0$

$x + 15 - 6x = 0$

$15 = 5x$

and thus $x = 3, y = -1$.

$B = (3, -1)$.

Gradient of $AB = \frac{-1 - (-13)}{3 - (-1)} = \frac{12}{4} = 3$.

The line $2x + y - 5 = 0$ can be written as $y = 5 - 2x$ and thus has gradient -2.

The line $x + 3y = 0$ can be written as $y = -\frac{1}{3}x$ and thus has gradient $-\frac{1}{3}$.

Thus AB is perpendicular to the second line as the product of the respective gradients $= -1$.

5. i) $a^{p+q} = a^p \times a^q = 5 \times 9 = 45$.

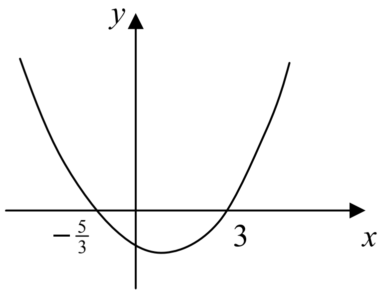
ii) $2a^{-p} = \frac{2}{a^p} = \frac{2}{5}$.

iii) $a^{2p - \frac{1}{2}q} = \frac{a^{2p}}{a^{\frac{1}{2}q}} = \frac{a^p \times a^p}{\sqrt{a^q}} = \frac{5 \times 5}{\sqrt{9}} = \frac{25}{3}$.

6. i) $3x^2 - 4x - 15 \geq 0$

$(3x + 5)(x - 3) \geq 0$

$x \leq -\frac{5}{3}$ or $x \geq 3$.



ii) For distinct roots, the discriminant > 0

$\Rightarrow$ $5^2 - 4 \times 1 \times p > 0$

i.e. $25 > 4p$

which means that $p < 6.25$.

Thus the greatest integer value is 6.

7. Either factorise or use the quadratic formula.

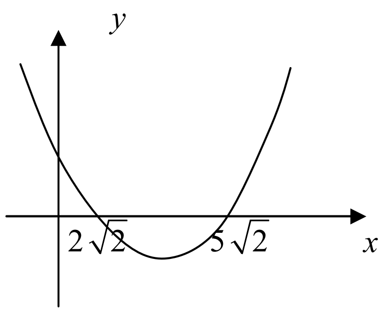
$x^2 - (7\sqrt{2})x + 20 = 0$

$\left(x - 2\sqrt{2}\right)\left(x - 5\sqrt{2}\right) = 0$

$\Rightarrow x = 2\sqrt{2}$ or $x = 5\sqrt{2}$.

(Take care if you choose to use the quadratic formula instead.)

If $x^2 - (7\sqrt{2})x + 20 < 0$ then $2\sqrt{2} < x < 5\sqrt{2}$.



8. a) Perimeter = 24 $\Rightarrow$ $2x + 2y = 24$

$\Rightarrow$ $y = 12 - x$.

b) Area $= xy = x(12 - x) = 12x - x^2$ as required.

c) $A = 12x - x^2 = -\{x^2 - 12x\}$

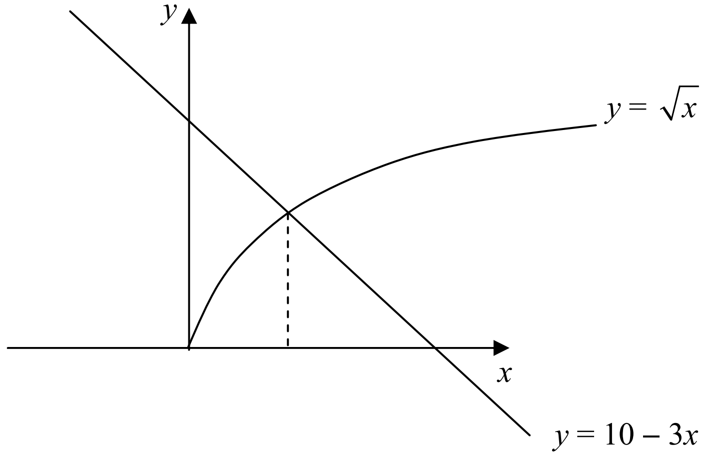
$= -\{(x - 6)^2 - 36\}$

$= 36 - (x - 6)^2$.

Largest A when $x = 6$.

d) Largest area = 36 m^2.

9.



Solve simultaneously to find P: $10 - 3x = \sqrt{x}$

square both sides
$(10 - 3x)^2 = x$
$100 - 60x + 9x^2 = x$
$9x^2 - 61x + 100 = 0$
$(9x - 25)(x - 4) = 0$
$x = \frac{25}{9}$ or $x = 4$.

Now, the problem with our solution is that by originally squaring both sides we have possibly introduced a '*false*' solution! E.g. consider the equation $x = 2$ which has one obvious solution. Square both sides to get $x^2 = 4$ which has 2 solutions!

We therefore have to check our 2 'solutions'; $x = \frac{25}{9}$ or $x = 4$ in the original equations.

We soon find that, in this case, our solution is $x = \frac{25}{9}$, $y = \frac{5}{3}$.

$P = \left(\frac{25}{9}, \frac{5}{3}\right)$.

10. a) Volume $= 54\pi = \pi x^2 h$ which means that $h = \frac{54}{x^2}$.

$S = \pi x^2 + \pi x^2 + 2\pi x h = 2\pi x^2 + 2\pi x \frac{54}{x^2}$ which equals $2\pi x^2 + \frac{108\pi}{x}$ as required.

b) Differentiate to get $\frac{\mathrm{d}S}{\mathrm{d}x} = 4\pi x - \frac{108\pi}{x^2}$.

For a minimum we require this gradient to equal zero.

$\Rightarrow \quad 4\pi x - \frac{108\pi}{x^2} = 0$

$4x^3 = 108$

$\Rightarrow \quad x^3 = 27$ and hence $x = 3$.

Also, $h = \frac{54}{x^2} = \frac{54}{3^2}$ which equals 6 and which confirms that the height is twice the radius for this particular tin.