GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C1

Paper 5

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**

1. Express $\frac{5}{\sqrt{7}}$ in the form $k\sqrt{7}$, where k is a rational number. [2]

2. The point S has coordinates $(-3, 6)$ and the point T has coordinates $(3, -2)$. Calculate

i) the coordinates of the mid-point of ST, [1]
ii) the distance ST. [2]

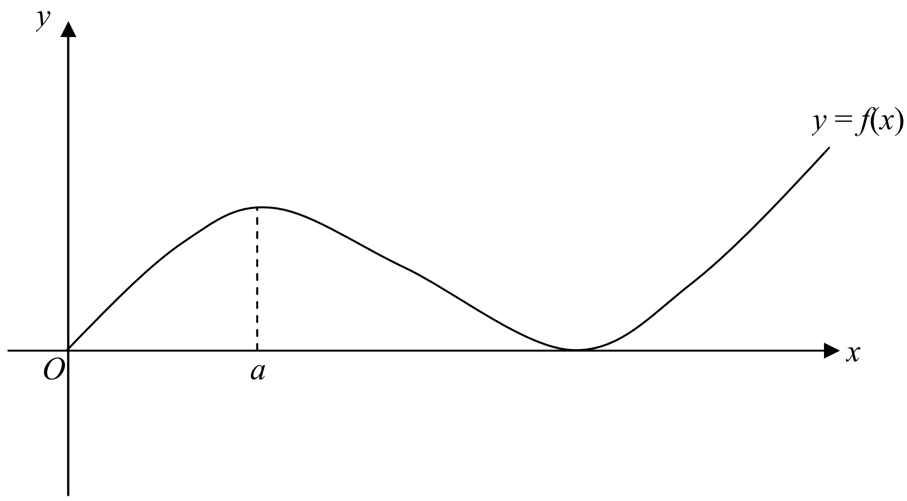
3. i) Solve the simultaneous equations $x^2 + 2y^2 = 9, \quad x + 4y = 9.$ [5]
ii) Give a geometrical interpretation of your solutions to i). [1]

4. Differentiate the following with respect to x.
a) $y = \sqrt{x} + 5x^3 + 7$ [2]
b) $y = (3x - 1)^2$ [2]
c) $y = \frac{9}{\sqrt[3]{x}}$. [2]

5. The coordinates of the points P and Q are $(-2, 8)$ and $(4, 7)$ respectively. Find the equation of the straight line which is perpendicular to PQ and which passes through the mid-point of PQ.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

6. The diagram shows the graph of $y = f(x)$ which has a maximum point as shown.



a) Sketch, on separate diagrams, the graphs of:
i) $y = 2f(x)$ [2]
ii) $y = f(x + a)$ [2]

b) Given that $f(x)$ is given by $f(x) = x^3 - 6x^2 + 9x$, use differentiation to find the value of a. [4]

7. Sketch the graph of $y = (x + 1)(x - 2)(x + 3)$. [3]

Hence solve the inequality $(x + 1)(x - 2)(x + 3) > 0$. [1]

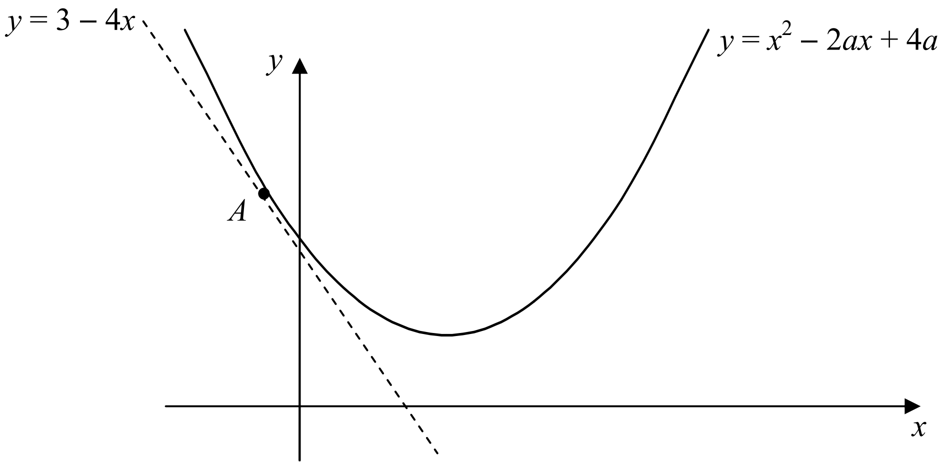
8. Let $f(x) = x^2 - 2kx - 8k^2$, where k is a positive constant.

i) Solve, for x in terms of k, the inequality $f(x) \geq 0$, [3]

ii) Sketch the graph of $y = f(x)$. [2]

Find, in terms of k, the least value of c for which the equation $f(x) = c$ has a real solution for x. [3]

9. The diagram shows the graph of $y = x^2 - 2ax + 4a$ where a is a positive constant. The line $y = 3 - 4x$, shown below, **touches** the curve at the point A.



Write down the quadratic equation satisfied by the x–coordinate of A, and hence show that $a = 1$ or $a = 7$. [6]

ANSWERS / SOLUTIONS.

1. $\frac{5}{\sqrt{7}} = \frac{5}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$.

This means that $k = \frac{5}{7}$.

(Note that the question does not require us to state the value of k !)

2. i) $M = \left(\frac{-3+3}{2}, \frac{6+-2}{2}\right) = (0, 2)$.

ii) Use Pythagoras to get $ST = \sqrt{6^2 + 8^2} = 10$ units.

3. i) Replace x by $9 - 4y$ to get:

$$x^2 + 2y^2 = 9$$
$$(9 - 4y)^2 + 2y^2 = 9$$
$$\underline{81 - 72y + 16y^2} + 2y^2 = 9$$
$$18y^2 - 72y + 72 = 0$$

(÷18) $$y^2 - 4y + 4 = 0$$
$$(y - 2)(y - 2) = 0$$

and thus $y = 2$ and $x = 9 - 4 \times 2 = 1$.

ii) Since the line and the curve meet in one point only, the line $x + 4y = 9$ is a **tangent** to the curve $x^2 + 2y^2 = 9$.

4. a) $y = x^{\frac{1}{2}} + 5x^3 + 7$.

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 15x^2$.

b) $y = 9x^2 - 6x + 1$.

$\Rightarrow \frac{dy}{dx} = 18x - 6$.

c) $y = 9x^{-\frac{1}{3}}$.

$\Rightarrow \frac{dy}{dx} = 9 \times \left(-\frac{1}{3}x^{-\frac{4}{3}}\right) = -3x^{-\frac{4}{3}}$.

5. Required equation is of the form $y = mx + c$.

Gradient of $PQ = \frac{7 - 8}{4 - (-2)} = -\frac{1}{6}$.

Therefore m (the gradient of the perpendicular line) is such that $m \times -\frac{1}{6} = -1$

$\Rightarrow m = 6$.
Thus $y = 6x + c$.

Now the line passes through the mid-point of P and $Q = \left(\frac{-2+4}{2}, \frac{8+7}{2}\right) = (1, 7.5)$.
Substitute $x = 1$, $y = 7.5$ to get $c = 1.5$.

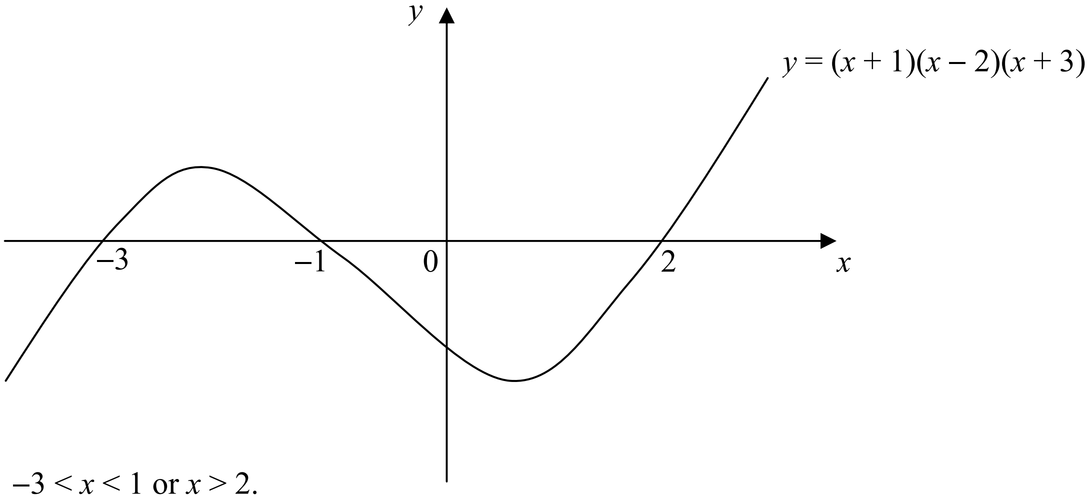
Thus the required equation is given by $y = 6x + 1.5$
which is identical to $2y = 12x + 3$
or $12x - 2y + 3 = 0$.

6. a) i) {Stretch along the y-axis, scale factor ×2.}
ii) {Translation of 2 units along the negative x-axis.}

b) Differentiate to get $f'(x) = 3x^2 - 12x + 9$.
For maximum points we require $f'(x) = 0$ and thus $3x^2 - 12x + 9 = 0$.
(÷3) $x^2 - 4x + 3 = 0$
and hence $(x - 1)(x - 3) = 0$.
$\Rightarrow x = 1$ or $x = 3$.

From the graph it is thus clear that $a = 1$.

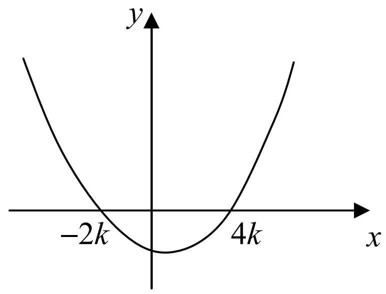
7.



$-3 < x < 1$ or $x > 2$.

8. i) $x^2 - 2kx - 8k^2 \geq 0$
$(x - 4k)(x + 2k) \geq 0$
which means that $x \leq -2k$ or $x \geq 4k$.

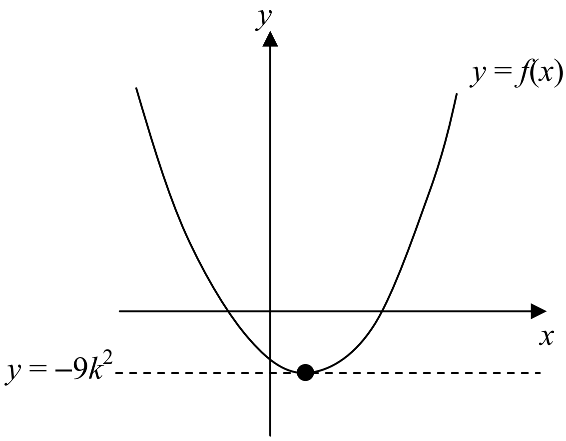
ii)



There are several ways of solving the final part to this question.
One way (avoiding discriminants!) is to complete the square:

$$x^2 - 2kx - 8k^2 = (x - k)^2 - 9k^2.$$

This means that the graph of $y = x^2 - 2kx - 8k^2$ has a minimum point where $y = -9k^2$ as shown below.



It is clear from the graph that providing $c \geq -9k^2$, then the equation $f(x) = c$ has at least one solution.

Thus the least value of c is $-9k^2$.

9. Solve simultaneously in an attempt to find the co-ordinates of A:

$3 - 4x = x^2 - 2ax + 4a$
$0 = x^2 + 4x - 2ax + 4a - 3$
$0 = x^2 + (4 - 2a)x + (4a - 3)$.

Knowing that this equation only has one solution means that it's discriminant is zero.

⇒ $(4 - 2a)^2 - 4\times1\times(4a - 3) = 0$
i.e. $16 - 16a + 4a^2 - 16a + 12 = 0$
⇒ $4a^2 - 32a + 28 = 0$
$a^2 - 8a + 7 = 0$
$(a - 1)(a - 7) = 0$
and thus $a = 1$ or $a = 7$ as required.