GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C1

Paper 4

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**

1. The coordinates of the points P and Q are (2, 11) and (–4, 3) respectively. Calculate the length of PQ and write down the coordinates of the mid-point of PQ. [4]

2. Given that a is positive, express $\dfrac{(36a^2)^{\frac{1}{2}}}{2a^{\frac{1}{3}}}$ in the form ka^m, stating the exact values of the constants k and m. [3]

3. Two straight lines have equations $x + 3y = 13a$ and $3x - y = 9a + 10$, where a is a constant. Their point of intersection is denoted by A.

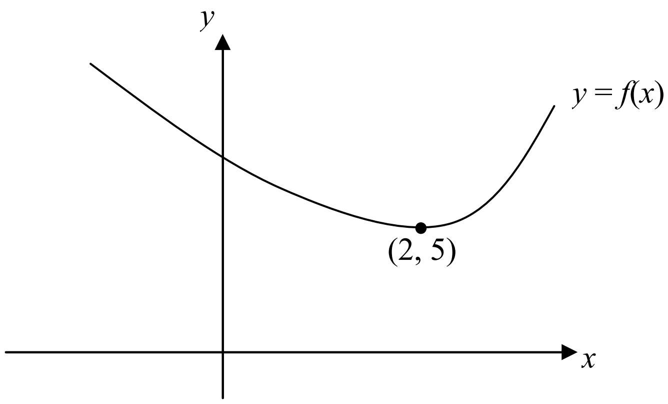
   i) Find, in terms of a, the coordinates of A. [5]

   ii) Show that, for all values of a, the point A lies on the straight line with equation $3x - 4y = 13$. [2]

4. Find the equation of the normal to the curve $y = 2x^{\frac{1}{3}}$ at the point on the curve where $x = 8$. Give your answer in the form $y = mx + c$. [5]

5. Solve the inequality $3x^2 + x - 2 > 0$. [3]

6.



The only stationary point of the curve $y = f(x)$ has coordinates (2, 5), as shown in the diagram. State the coordinates of the stationary point of each of the following curves.

   i) $y = f(x + 1)$. [1]

   ii) $y = f(-x)$. [1]

   iii) $y = 3f(x) - 4$. [1]

7. Factorise $49x^2 - 21x + 2$. [1]

Hence, or otherwise, solve the equation $49y - 21\sqrt{y} + 2 = 0$.
Give your answers as fractions. [3]

8. Find the x coordinate of the stationary point on the curve

$$y = x^{\frac{3}{2}} - 5x.$$ [4]

Show that this stationary point is a minimum point. [2]

9. The circle C has centre $(8, -1)$ and passes through the point $(4, 1)$.

a) Find an equation for C. [3]

b) Show that the line with equation $x + 2y + 4 = 0$ is a tangent to C. [5]

ANSWERS / SOLUTIONS.

1. Use Pythagoras to get $PQ = 10$ units.
Mid-point = (–1, 7).

2. $$\frac{\left(36a^2\right)^{\frac{1}{2}}}{2a^{\frac{1}{3}}} = \frac{\sqrt{36a^2}}{2a^{\frac{1}{3}}} = \frac{6a}{2a^{\frac{1}{3}}} = 3\,a^{\frac{2}{3}}.$$
$k = 3$, $m = \dfrac{2}{3}$.

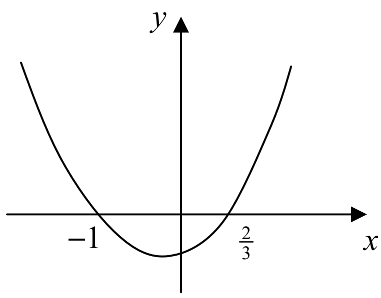
3. i) Solve simultaneously to get (for example):
$$\begin{aligned} 3x + 9y &= 39a \\ 3x - y &= 9a + 10 \\ \text{subtract} \quad 10y &= 30a - 10 \\ \text{and thus} \quad y &= 3a - 1. \end{aligned}$$
Use $x = 13a - 3y$ to get that $x = 13a - 3(3a - 1)$
$= 13a - 9a + 3$
$= 4a + 3$.
Thus $A = (4a + 3, 3a - 1)$.

ii) Put $x = 4a + 3$ and $y = 3a - 1$ to get
$$\begin{aligned} 3x - 4y &= 3(4a + 3) - 4(3a - 1) \\ &= 12a + 9 - 12a + 4 \\ &= 13 \end{aligned}$$
and thus A lies on the line $3x - 4y = 13$.

4. Require $\dfrac{dy}{dx}$ for the gradient of the curve.
$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{2}{3}}.$$
Put $x = 8$ to get that the gradient of the **tangent** is $\dfrac{2}{3}\times 8^{-\frac{2}{3}} = \dfrac{2}{3}\times\dfrac{1}{8^{\frac{2}{3}}} = \dfrac{2}{3}\times\dfrac{1}{4} = \dfrac{1}{6}$.
Now, since the **tangent** and **normal** are perpendicular, the product of their gradients = –1.
$\Rightarrow m \times \dfrac{1}{6} = -1$ and thus $m = -6$.
Therefore the **normal** has equation $y = -6x + c$ for some constant c.
Since the **normal** passes through the point (8, 4), we quickly see that $c = 52$.

Hence the required equation is: $y = 52 - 6x$.

5. Factorise to get $(3x - 2)(x + 1) > 0$.
Now sketch the graph of $y = (3x - 2)(x + 1)$,
from which we see that either $x < -1$ or $x > \dfrac{2}{3}$.



6. i) (1, 5). {Translation of 1 unit along the negative x-axis}
ii) (–2, 5). {Reflection in the y-axis}
iii) (2, 11). {Stretch along y-axis, scale-factor × 3 followed by a translation of 4 units along the negative y-axis}

7. $(7x-1)(7x-2)$.

$49y - 21\sqrt{y} + 2 = 0$

$\left(7\sqrt{y} - 1\right)\left(7\sqrt{y} - 2\right) = 0$

$\Rightarrow \sqrt{y} = \frac{1}{7}$ or $\sqrt{y} = \frac{2}{7}$.

Squaring gives $y = \frac{1}{49}$ or $y = \frac{4}{49}$.

8. Differentiate to get $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 5$.

For stationary points, $\frac{dy}{dx} = 0$ which means $\frac{3}{2}x^{\frac{1}{2}} - 5 = 0$

and thus $\frac{3}{2}x^{\frac{1}{2}} = 5$

or $x^{\frac{1}{2}} = \frac{10}{3}$.

Square to get $x = \frac{100}{9}$.

To show that this point is a minimum, we need to examine the gradient, $\frac{dy}{dx}$, to the left of $x = \frac{100}{9}$ and then to the right.

{To make things easier, choose values of x which are square numbers because of the square-root in the formula for $\frac{dy}{dx}$!}

When $x = 9$, $\frac{dy}{dx} = \frac{3}{2} \times 3 - 5 = -0.5$

When $x = 16$, $\frac{dy}{dx} = \frac{3}{2} \times 4 - 5 = 1$.

Therefore, when $x = \frac{100}{9}$, we have a minimum point.

9. a) The equation of the circle centre (a, b), radius r is given by $(x-a)^2 + (y-b)^2 = r^2$.
Thus the given circle has equation $(x-8)^2 + (y+1)^2 = r^2$.

Now the radius is the distance between the points $(8, -1)$ and $(4, 1)$ which equals $\sqrt{20}$ by Pythagoras.

This the circle has equation $(x-8)^2 + (y+1)^2 = 20$.

b) The best way to show that the line is a tangent is to show that the two graphs **<u>intersect in a single point only</u>**. This automatically means that the line is a tangent to the curve etc.

First expand the brackets: $(x - 8)^2 + (y + 1)^2 = 20$

$x^2 - 16x + 64 + y^2 + 2y + 1 = 20$

$\Rightarrow \quad x^2 - 16x + y^2 + 2y + 45 = 0.$

Now put $x = (-4 - 2y)$ to get $(-4 - 2y)^2 - 16(-4 - 2y) + y^2 + 2y + 45 = 0.$

Simplify carefully $16 + 16y + 4y^2 + 64 + 32y + y^2 + 2y + 45 = 0$

$5y^2 + 50y + 125 = 0$

$y^2 + 10y + 25 = 0$

and thus $(y + 5)(y + 5) = 0.$

This means that $y = -5$ and, more importantly, there is only one point of intersection.

Hence the line is a tangent to the curve.