ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics – Further Pure

SPECIMEN UNITS AND MARK SCHEMES

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)
ADVANCED PURE MATHEMATICS (6366)
ADVANCED FURTHER MATHEMATICS (6371)

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The equation $x^3 - 5x + 7 = 0$ has a single real root α.

Use the Newton Raphson method with first approximation $x_1 = -3$ to find the value of x_2, giving your answer to 3 significant figures. *(3 marks)*

2 The roots of the quadratic equation $x^2 + 4x - 3 = 0$ are α and β.

(a) Without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$;

(ii) $\left(\alpha^2 + \dfrac{2}{\beta}\right)\left(\beta^2 + \dfrac{2}{\alpha}\right)$. *(6 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots $\left(\alpha^2 + \dfrac{2}{\beta}\right)$ and $\left(\beta^2 + \dfrac{2}{\alpha}\right)$ *(4 marks)*

No credit will be given for simply using a calculator to find α and β in order to find the values in part (a).

3 (a) Sketch the graph of $y = \dfrac{3x+4}{x-2}$.

State the coordinates of the points where the curve crosses the coordinate axes and write down the equations of its asymptotes. *(6 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{3x+4}{x-2} > 1$$ *(3 marks)*

4 The complex number z satisfies the equation

$$\mathrm{i}z + 4 = (2 - \mathrm{i})z^*$$

where z^* is the complex conjugate of z.

Find z in the form $a + \mathrm{i}b$, where a and b are real. *(7 marks)*

5 Find the general solution in radians of the equation

$$\tan\left(2x + \frac{\pi}{5}\right) = \sqrt{3}$$

giving your exact answer in terms of π . *(6 marks)*

6 The matrix $\boldsymbol{A}$ is $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ and the matrix $\boldsymbol{B}$ is $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$.

(a) Find the matrix product $\boldsymbol{AB}$. *(2 marks)*

(b) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{M}\begin{bmatrix} x \\ y \end{bmatrix}.$$

Describe the geometrical transformation represented by **T** for each of the following cases:

(i) $\boldsymbol{M} = \boldsymbol{A}$; *(2 marks)*

(ii) $\boldsymbol{M} = \boldsymbol{B}$; *(3 marks)*

(iii) $\boldsymbol{M} = \boldsymbol{AB}$. *(1 mark)*

7 (a) Find $\int_a^b x^{-\frac{3}{2}}\,\mathrm{d}x$, where $b > a > 0$. *3 marks)*

(b) Hence determine, where possible, the value of the following integrals, giving a reason if the value cannot be found.

(i) $\int_4^\infty x^{-\frac{3}{2}}\,\mathrm{d}x$

(ii) $\int_0^1 x^{-\frac{3}{2}}\,\mathrm{d}x$ *(4 marks)*

8 (a) Express $\sum_{r=1}^{n}(r-1)(3r-2)$ in the form $a\sum_{r=1}^{n}r^2 + b\sum_{r=1}^{n}r + cn$, stating the values of the constants a, b and c. *(3 marks)*

(b) Hence prove that $\sum_{r=1}^{n}(r-1)(3r-2) = n^2(n-1)$. *(4 marks)*

Turn over ►

9 A curve has equation $y = \dfrac{2x^2 - x - 7}{x - 3}$.

(a) (i) Prove that the curve crosses the line $y = k$ when

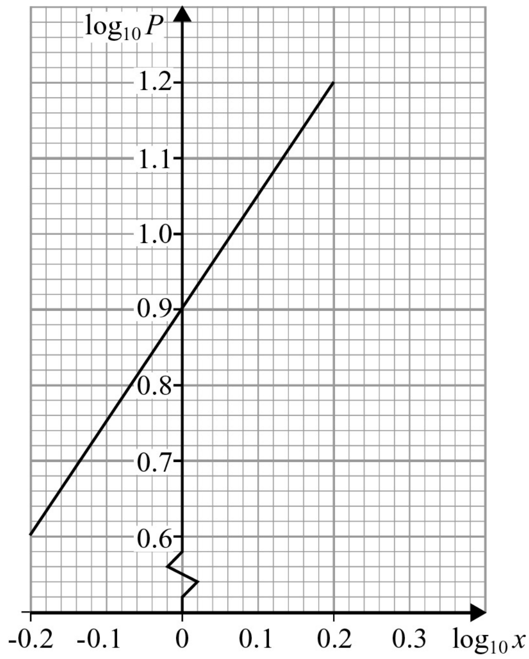
$$2x^2 - (k+1)x + (3k - 7) = 0 \ .$$ *(1 mark)*

(ii) Hence show that if x is real then either $k \leqslant 3$ or $k \geqslant 19$. *(5 marks)*

(b) Use the results from part (a) to find the coordinates of the turning points of the curve. *(4 marks)*

10 A mathematical model is used by an astronomer to investigate features of the moons of a particular planet. The mean distance of a moon from the planet, measured in millions of kilometres, is denoted by x, and the corresponding period of its orbit is P days.

The model assumes that the graph of $\log_{10} P$ against $\log_{10} x$ is the straight line drawn below.



(a) Use the graph to estimate the period of the orbit of a moon for which $x = 1.45$. *(3 marks)*

(b) The graph would suggest that P and x are related by an equation of the form

$$P = k\,x^{\alpha}$$

where k and α are constants.

(i) Express $\log_{10} P$ in terms of $\log_{10} k$, $\log_{10} x$ and α. *(1 mark)*

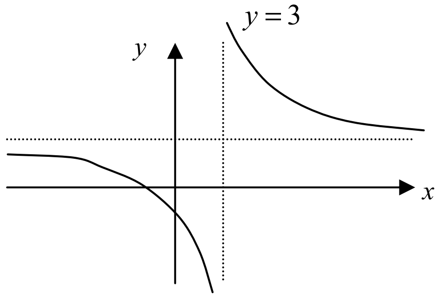
(ii) Use the graph to determine the values of k and α, giving your answers to 2 significant figures. *(4 marks)*

END OF QUESTIONS

MFP1 Specimen

Question	Solution	Marks	Total	Comments
1	$\mathrm{f}(x) = x^3 - 5x + 7 \Rightarrow \mathrm{f}'(x) = 3x^2 - 5$	B1		
	$x_2 = -3 - \dfrac{\mathrm{f}(-3)}{\mathrm{f}'(-3)}$	M1		
	$= -3 - (-5)/22 = -2.77$ (to 3 SF)	A1	3	
	Total		**3**	
2(a)(i)	$\alpha + \beta = -4; \quad \alpha\beta = -3$	B1		Likely to be earned in (ii)
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		
	$= 16 + 6 = 22$	A1		
(ii)	$\alpha^2\beta^2 + 2(\alpha + \beta) + \dfrac{4}{\alpha\beta}$	B1		
	$9 - 8 - \dfrac{4}{3}$	M1		Substitution into similar form as above
	$= -\dfrac{1}{3}$	A1	6	
(b)	Sum of roots			
	$= \alpha^2 + \beta^2 + \dfrac{2}{\alpha} + \dfrac{2}{\beta}$			
	$= \alpha^2 + \beta^2 + \dfrac{2}{\alpha\beta}(\alpha + \beta)$	M1		Essentially this
	$= 22 + \dfrac{2}{-3} \times -4 \quad = \dfrac{74}{3}$	A1		
	New equation			
	$y^2 - (\text{sum of new roots})\,y + \text{product} = 0$	M1		Condone single sign error or missing = 0
	$\Rightarrow y^2 - \dfrac{74}{3}y - \dfrac{1}{3} = 0$			
	$\Rightarrow 3y^2 - 74y - 1 = 0$	A1√	4	(ft any variable fractional values) Must have = 0
	Total		**10**	

MFP1 (cont)

Question	Solution	Marks	Total	Comments
3(a)	$(0,-2)$ accept $x=0$, $y=-2$	B1		
	$\left(-\frac{4}{3},0\right)$ accept $y=0$, $x=-\frac{4}{3}$	B1		
	Asymptotes $x=2$	B1		x asymptote is 2, y asymptote is 3 B1 only
	$y=3$	B1		$x\to 2, y\to 3$ B1 only
		M1 A1√	6	One branch of hyperbola ft asymptotes Condone lack of symmetry to show second branch
(b)	Appropriate method	M1		Multiply both sides by $(x-2)^2$
	Consideration of graph $y=1 \Rightarrow 3x+4=x-2$ $\Rightarrow x=-3$			$\frac{3x+4}{x-2}-1>0$ Considering (x–2) > 0 and (x–2) < 0 3(x+4)>(x–2) ⇒ $x>-3$ only M0
	Solution: $x<-3$	A1		
	$x>2$	A1	3	Solution offered as $2<x<-3$ unless ISW scores A1, A0
	Total		**9**	
4	$\mathrm{i}(a+\mathrm{i}b)+4=(2-\mathrm{i})(a-\mathrm{i}b)$	M1		
	$\mathrm{i}a-b+4$	A1		
	$=2a-\mathrm{i}a-2\mathrm{i}b-b$	A1		Allow i^2b if cancelled
	Equating real parts $2a=4$	M1		
	$a=2$	A1√		
	Equating imaginary parts $a=-a-2b$	M1		And attempt to find b
	$b=-2$	A1√		2–2i is complex number
	Total		**7**	

MFP1 (cont)

Question	Solution	Marks	Total	Comments
5	$\tan^{-1}\sqrt{3}$	M1		Attempt at inverse tangent
	$=\frac{\pi}{3}$	A1		
	General solution of form $n\pi+\alpha$	M1		
	$2x+\frac{\pi}{5}=n\pi+\alpha$	A1√		
		m1		Making x the subject
	$x=\frac{n\pi}{2}+\frac{\pi}{6}-\frac{\pi}{10}$	A1	6	$x=\frac{n\pi}{2}+\frac{\pi}{15}$
	Total		**6**	
6 (a)	$\boldsymbol{AB}=\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	M1		Clear attempt to multiply correctly
		A1	2	Correct
(b) (i)	Rotation about origin	M1		
	through $45°$ clockwise	A1	2	**oe**
(ii)	Reflection	M1		
	in line y=x tan *	m1		and attempt at $\cos 2\theta=\frac{1}{\sqrt{2}}$ etc
	$y=x\tan 22\frac{1}{2}°$	A1	3	
(iii)	Reflection in x-axis	B1	1	
	Total		**8**	
7(a)	$-2x^{-\frac{1}{2}}$	M1		Power -0.5
		A1		correct
	Value of Integral = $\frac{2}{\sqrt{a}}-\frac{2}{\sqrt{b}}$	A1	3	Or equivalent
(b) (i)	$\frac{1}{\sqrt{b}}\to 0$ as $b\to\infty$	M1		
	Hence value of integral is 1	A1	2	
(ii)	Integral does not exist/ cannot find value etc	B1		
	Reason: $\frac{1}{\sqrt{a}}\to\infty$ as $a\to 0^+$	E1	2	
	Total		**7**	

MFP1 (cont)

Question	Solution	Marks	Total	Comments
8 (a)	$(r-1)(3r-2)=3r^2-5r+2$	M1		
	$\sum 1 = n$	B1		
	Printed answer with a = 3, b = –5, c = 2	A1	3	$3\sum r^2 - 5\sum r + 2n$
(b)	Use of $\sum r^2$ and $\sum r$ formulae	M1		
	$\frac{3n}{6}(n+1)(2n+1)-\frac{5n}{2}(n+1)+2n$	A1		ft
		m1		Factorising or multiplying out
	$=n^2(n-1)$	A1	4	**ag**
	Total		**7**	
9 (a)(i)	$k(x-3)=2x^2-x-7$ leading to $2x^2-(k+1)x+(3k-7)=0$	B1	1	**ag**
(ii)	Use of discriminant b^2-4ac	M1		
	$(k+1)^2-8(3k-7)$	A1		
	Solving quadratic equation or factorising	m1		Or use of formula
	$(k-3)(k-19)$	A1		k = 3, k=19
	$b^2-4ac \geq 0$. Hence $k \leq 3, or\ k \geq 19$	A1	5	**ag** be convinced
(b)	k = 3: $2x^2-4x+2=0$	M1		Either k value and attempt to solve/factorise
	$\Rightarrow x=1$,	A1		
	k = 19: $2x^2-20x+50=0 \Rightarrow x=5$	A1		
	Coordinates of TPs (1, 3) and (5, 19)	A1	4	Both
	Total		**10**	
10 (a)	log 1.45 = 0.161…	M1		
	From graph log P = 1.14	m1		
	P = 14 days (to nearest day)	A1	3	
(b)(i)	$\log_{10} P = \log_{10} k + \alpha \log_{10} x$	B1	1	
(ii)	Intercept on vertical axis is 0.9			
	$\log_{10} k = 0.9$	M1		
	k = 7.9	A1		
	Gradient of graph is given by α	M1		
	α =1.5	A1	4	
	Total		**8**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

MATHEMATICS **MFP2**
Unit Further Pure 2

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The cubic equation

$$x^3 + 2x^2 + 5x + k = 0$$

where k is real, has roots α, β and γ.

(a) Write down the values of:

(i) $\alpha + \beta + \gamma$; *(1 mark)*

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$. *(1 mark)*

(b) (i) Show that $\alpha^2 + \beta^2 + \gamma^2 = -6$. *(3 marks)*

(ii) Hence explain why the cubic equation must have two non-real roots. *(2 marks)*

(c) Given that one root is $-2 + 3\mathrm{i}$, find the value of k. *(5 marks)*

2 (a) Given that

$$3\sinh^2 x = 2\cosh x + 2 \qquad (x > 0)$$

find the value of $\cosh x$. *(4 marks)*

(b) Hence obtain x in the form $\ln p$, where p is an integer to be determined. *(2 marks)*

3 (a) Express $\dfrac{1}{(r-1)(r+1)}$ in partial fractions. *(2 marks)*

(b) Hence find

$$\sum_{r=2}^{n} \frac{1}{(r^2 - 1)}$$

giving your answer in the form

$$A + \frac{B}{n} + \frac{C}{n+1}$$ *(5 marks)*

4 (a) Draw an Argand diagram to show the points A and B which represent the complex numbers $1-3\text{i}$ and $5-\text{i}$ respectively. *(1 mark)*

(b) (i) The circle C has AB as a diameter. Find its radius and the coordinates of its centre. *(4 marks)*

(ii) Write down the equation of C in the form

$$\left|z - z_0\right| = k$$ *(2 marks)*

5 (a) Use de Moivre's theorem to show that if $z = \cos\theta + \text{i}\sin\theta$, then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$ *(3 marks)*

(b) (i) Write down the expansion of $\left(z - \frac{1}{z}\right)^4$ in terms of z. *(2 marks)*

(ii) Hence, or otherwise, show that

$$8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$$ *(5 marks)*

(c) Solve the equation

$$8\sin^4\theta = \cos 4\theta + 1$$

in the interval $-\pi < \theta \le \pi$, giving your answers in terms of π. *(3 marks)*

6 (a) Express $\sqrt{3} + \text{i}$ in the form r $(\cos\theta + \text{i}\sin\theta)$, where $\text{r} > 0$ and $-\pi < \theta < \pi$. *(3 marks)*

(b) Obtain similar expressions for:

(i) $\sqrt{3} - \text{i}$ *(2 marks)*

(ii) $\dfrac{1}{\sqrt{3} + \text{i}}$ *(2 marks)*

7 Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} (2r-1)^2 = \tfrac{1}{3}n(2n-1)(2n+1)$$ *(7 marks)*

Turn over ►

8 (a) Use the definition $\cosh t = \frac{1}{2}\left(e^{t} + e^{-t}\right)$ to show that

$$2\cosh^2 t = 1 + \cosh 2t$$ *(3 marks)*

(b) A curve is given parametrically by the equations

$$x = 2\sinh t, \quad y = \cosh^2 t$$

(i) Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4\cosh^4 t$ *(6 marks)*

(ii) Hence show that the length of arc of the curve from the point where $t = 0$ to the point where $t = \frac{1}{2}$ is

$$\frac{1}{2}(1 + \sinh 1)$$ *(4 marks)*

(c) Find the Cartesian equation of the curve. *(3 marks)*

END OF QUESTIONS

MFP2 Specimen

Question	Solution	Marks	Total	Comments
1(a)(i)	$\alpha+\beta+\gamma=-2$	B1	1	
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=5$	B1	1	
(b)(i)	$\alpha^2+\beta^2+\gamma^2=(\sum\alpha)^2-2\sum\alpha\beta$	M1A1		
	$=4-10=-6$	A1	3	
(ii)	Since sum of squares < 0, some of $\alpha,\ \beta,\ \gamma$ must be non-real	E1		
	As coefficients real, non real roots come in conjugate pairs	E1	2	
(c)	$-2+3\text{i}$ is a root, so is $-2-3\text{i}$	B1		p.i
	$(-2+3\text{i})(-2-3\text{i})=13$	B1		Alternative solution – substituting $-2+3\text{i}$ into cubic $(-2+3\text{i})^2=-5-12\text{i}$
	and third root is +2	B1√		$(-2+3\text{i})^3=46+9\text{i}$
	$\alpha\beta\gamma=26$	B1√		equation involving k $k=-26$
	$k=-\alpha\beta\gamma=-26$	B1√	5	4/5 for 1 slip
	Total		**12**	
2(a)	$3(\cosh^2 x-1)-2\cosh x-2=0$	M1		
	$(3\cosh x-5)(\cosh x+1)=0$	A1		Or use of formula
	$\cosh x\neq-1$	E1		Some indication of rejection
	$\cosh x=\dfrac{5}{3}$	A1√	4	
(b)	$x=\ln\left(\dfrac{5}{3}+\sqrt{\dfrac{16}{9}}\right)=\ln 3$	M1 A1√	2	ft provided p is an integer
	Total		**6**	

MFP2 (cont)

Question	Solution	Marks	Total	Comments
3(a)	$\dfrac{1}{r^2-1}=\dfrac{1}{2}\left(\dfrac{1}{r-1}-\dfrac{1}{r+1}\right)$	M1A1	2	
(b)	$\sum_{r=2}^{n}\dfrac{1}{r^2-1}=\dfrac{1}{2}\left(\dfrac{1}{2-1}-\dfrac{1}{2+1}\right)$			
	$+\dfrac{1}{2}\left(\dfrac{1}{3-1}-\dfrac{1}{3+1}\right)$	M1A1		
	$+\dfrac{1}{2}\left(\dfrac{1}{4-1}-\dfrac{1}{4+1}\right)$			
	$+\dfrac{1}{2}\left(\dfrac{1}{n-2}-\dfrac{1}{n}\right)$			
	$+\dfrac{1}{2}\left(\dfrac{1}{n-1}-\dfrac{1}{n+1}\right)$	A1		
	$S_n=\dfrac{1}{2}\left(\dfrac{3}{2}-\dfrac{1}{n}-\dfrac{1}{n+1}\right)$	M1A1	5	
	Total		**7**	
4(a)	Points plotted correctly	B1	1	
(b)(i)	The centre must be $\dfrac{1-3\text{i}+5-\text{i}}{2}=3-2\text{i}$	M1A1		Accept $(3,-2)$, but $(3,-2\text{i})$ gets A0
	The radius must be $\sqrt{\left((3-1)^2+(-2+3)^2\right)}=\sqrt{5}$	M1A1	4	$\sqrt{(3-1)^2+(-2\text{i}+3\text{i})^2}$ M0 If diameter is taken as $\sqrt{20}$ or radius taken as $\sqrt{20}$ allow B1
(ii)	$\therefore$ equation is $\lvert z-3+2\text{i}\rvert=\sqrt{5}$	M1 A1√	2	
	Total		**7**	

MFP2 (cont)

Question	Solution	Marks	Total	Comments
5(a)	$z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta) = \cos n\theta - \mathrm{i}\sin n\theta$	M1A1		Allow B1 only if z^{-n} is quoted as $\cos n\theta - \mathrm{i}\sin n\theta$
	$z^n + \dfrac{1}{z^n} = 2\cos n\theta$	A1	3	
(b)(i)	$\left(z - \frac{1}{z}\right)^4 = z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}$	M1A1	2	M1 for attempt at expansion
(ii)	$z - \frac{1}{z} = 2\mathrm{i}\sin\theta$	M1A1		
	$(2\mathrm{i}\sin\theta)^4 = 2\cos 4\theta - 8\cos 2\theta + 6$	M1		Any form
	$16\sin^4\theta = 2\cos 4\theta - 8\cos 2\theta + 6$	A1√		ft if i missing in $(2\mathrm{i}\sin\theta)^4$
	$8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$	A1	5	**ag** (no error) If M0, allow B1 for $2\cos 4\theta$ and $8\cos 2\theta$
(c)	$4\cos 2\theta = 2$	M1		Allow B1 for any two correct answers
	$2\theta = \pm\dfrac{\pi}{3}, \pm\dfrac{5\pi}{3}$	A1		
	$\theta = \pm\dfrac{\pi}{6}, \pm\dfrac{5\pi}{6}$	A1√	3	
	Total		**13**	
6(a)	$r = 2$	B1		
	$\theta = \dfrac{\pi}{6}$	M1A1	3	
(b)(i)	$r = 2$	B1√		ft wrong answer in (a)
	$\theta = -\dfrac{\pi}{6}$	B1√	2	ditto
(b)(ii)	$r = \dfrac{1}{2}$	B1√		ditto
	$\theta = -\dfrac{\pi}{6}$	B1√	2	ditto
	Total		**7**	

MFP2 (cont)

Question	Solution	Marks	Total	Comments
7	Assume result true for $n=k$			
	Then $\sum_{r=1}^{k+1}(2r-1)^2=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$	M1		
	$=\frac{1}{3}\left(4k^3-k+3\left(4k^2+4k+1\right)\right)$	A1		
	$=\frac{1}{3}(k+1)\left(4k^2+8k+3\right)$	M1A1		Any factor. If $(2k+1)$ taken out at start, all marks up to this point earned.
	$=\frac{1}{3}(k+1)(2k+1)(2k+3)$	A1		
	Shown true for $n=1$	B1		
	$\mathrm{T}_k \Rightarrow \mathrm{T}_{k+1}$ and T_1 true	E1	7	
	Total		**7**	

MFP2 (cont)

Question	Solution	Marks	Total	Comments
8(a)	$2\cosh^2 t = 2\times\frac{1}{4}\left(e^t + e^{-t}\right)^2$	M1		Or $\cosh^2 t = \frac{1}{4}\left(e^t + e^{-t}\right)^2 = \frac{1}{4}\left(e^{2t} + 2 + e^{-2t}\right)$
	$= \frac{1}{2}\left(e^{2t} + 2 + e^{-2t}\right)$	A1		
	$= 1 + \frac{1}{2}\left(e^{2t} + e^{-2t}\right)$			
	$= 1 + \cosh 2t$	A1	3	**ag**
(b)(i)	$\frac{dx}{dt} = 2\cosh t$	B1		
	$\frac{dy}{dt} = 2\cosh t \sinh t$	B1		
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4\cosh^2 t + 4\cosh^2 t \sinh^2 t$	M1 A1√		
	$= 4\cosh^2 t\left(1 + \sinh^2 t\right)$	m1		Used relevantly
	$= 4\cosh^4 t$	A1	6	
(ii)	$s = \int_0^{\frac{1}{2}} 2\cosh^2 t \, dt$	M1		
	$= \int_0^{\frac{1}{2}} (1 + \cosh 2t) dt$	m1		
	$= \left[t + \frac{1}{2}\sinh 2t\right]_0^{\frac{1}{2}}$	A1		
	$= \frac{1}{2} + \frac{1}{2}\sinh 1$	A1	4	
(c)	Use of $\cosh^2 t - \sinh^2 t = 1$ $y = 1 + \sinh^2 t$ $y = 1 + \frac{1}{4}x^2$	M1 A1 A1√	3	
	Total		**16**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

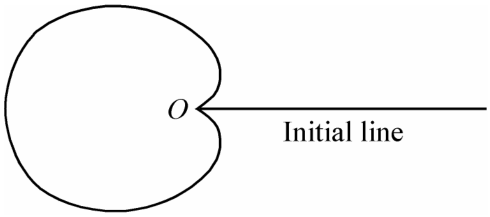
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1



The diagram shows the curve C with polar equation

$$r = a\sin\frac{1}{2}\theta, \quad 0 < \theta \leq 2\pi$$

Find the area bounded by C. *(6 marks)*

2 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 6\cos x$$

(7 marks)

(b) (i) Find the value of the constant λ for which $\lambda x\sin 2x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 6\cos 2x$$

(ii) Hence find the general solution of this differential equation. *(7 marks)*

3 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$$

and $$y(1) = 0.5$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$ to obtain an approximation to $y(1.1)$ giving your answer in four decimal places. *(3 marks)*

(b) (i) Use the formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h\,\mathrm{f}(x_r, y_r)$

and $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$

with $h = 0.1$ to obtain a further approximation to $y(1.1)$. *(5 marks)*

(ii) Use the formula given in part (b)(i), together with your value for $y(1.1)$ obtained in part (b)(i), to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(5 marks)*

Turn over ►

4 (a) A point has Cartesian coordinates (x, y) and polar coordinates (r, θ), referred to the same origin.

Express $\cos\theta$ and $\sin\theta$ in terms of $x, y,$ and r. *(1 mark)*

(b) (i) Hence find the Cartesian equation of the curve with polar equation

$$r = 2\cos\theta - 4\sin\theta$$

(3 marks)

(ii) Deduce that the curve is a circle and find its radius and the Cartesian coordinates of its centre. *(3 marks)*

5 (a) Using the substitution $y = \frac{1}{x}$, or otherwise, evaluate $\lim_{x \to \infty} \frac{\ln x}{x^k}$

where $k > 0$. *(3 marks)*

(b) Hence evaluate

$$\int_1^{\infty} \frac{\ln x}{x^2} \mathrm{d}x$$

(3 marks)

6 (a) The function f (x) is defined by

$$\mathrm{f}(x) = \mathrm{e}^{(\cos x - 1)}$$

Use Maclaurin's theorem to show that when $\mathrm{f}(x)$ is expanded in ascending powers of x:

(i) the first two non-zero terms are

$$1 - \tfrac{1}{2}x^2$$

(6 marks)

(ii) the co-efficient of x^3 is zero. *(3 marks)*

(b) Find

$$\lim_{x \to 0} \frac{1 - \mathrm{e}^{(\cos x - 1)}}{\sin^2 x}$$

(3 marks)

7 (a) Show that the substitution

$$u = \frac{dy}{dx} - y$$

transforms the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 5e^{-4x}$$

into the equation

$$\frac{du}{dx} + 3u = 5e^{-4x}$$

(3 marks)

(b) Show that e^{3x} is an integrating factor of

$$\frac{du}{dx} + 3u = 5e^{-4x}$$

Hence find the general solution of this differential equation, expressing u in terms of x.
(5 marks)

(c) Hence, or otherwise, solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 5e^{-4x}$$

completely, given that $\frac{dy}{dx} = 0$ and $y = 3$ when $x = 0$. *(9 marks)*

END OF QUESTIONS

MFP3 Specimen

Question	Solution	Marks	Totals	Comments
1	$A=\frac{1}{2}\int_0^{2\pi} a^2 \sin^2 \frac{1}{2}\theta \,\mathrm{d}\theta$	M1A1 B1		M1 for $\int \frac{1}{2} r^2 \mathrm{d}\theta$ used A1 if used correctly B1 for limits M0 if $\cos 2\theta$ used
	$=\frac{1}{2}\int_0^{2\pi} a^2\left(\frac{1-\cos\theta}{2}\right)\mathrm{d}\theta$	M1		
	$=\left[\frac{1}{2}a^2\left(\frac{\theta}{2}-\frac{\sin\theta}{2}\right)\right]_0^{2\pi}$	A1		cao
	$=\frac{1}{2}\pi a^2$	A1√	6	
	Total		**6**	
2(a)	$m=\pm 2\mathrm{i}$	B1		
	C.F. is $A\cos 2x + B\sin 2x$ or $A\cos(2x+B)$	M1 A1√		If m is real give M0 A1 ft is for m complex but incorrect
	but **not** $A\mathrm{e}^{2\mathrm{i}x}+B\mathrm{e}^{-2\mathrm{i}x}$			
	P.I. Try $y=p\cos x+q\sin x$	M1		
	$-p\cos x-q\sin x+4(p\cos x+q\sin x)=6\cos x$	A1		
	$p=2$	A1√		
	GS $y=A\cos 2x+B\sin 2x+2\cos x$	B1√	7	For adding their C.F. to their P.I. Must be 2 constants
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=2\lambda x\cos 2x+\lambda\sin 2x$	M1A1		If $y=\lambda x\sin 2x+\mu x\cos 2x$ used, then working at each stage must be correct for equivalent marks
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\lambda\cos 2x-4\lambda x\sin 2x+2\lambda\cos 2x$	A1√		
	$4\lambda\cos 2x-4\lambda x\sin 2x+4\lambda x\sin 2x=6\cos 2x$	M1A1		cao
	$\lambda=\frac{3}{2}$	A1√		
(ii)	GS $y=A\cos 2x+\left(B+\frac{3}{2}x\right)\sin 2x$	B1√	7	
	Total		**14**	

MFP3 (cont)

Question	Solution	Marks	Totals	Comments
3(a)	$y_1 = 0.5 + 0.1\dfrac{1\times 0.5}{\sqrt{0.5^2+1^2}} = 0.5447(2136)$	M1A1 A1	3	
(b)(i)	$k_1 = 0.1\text{f}(1, 0.5) = 0.04472(136)$	M1 A1√		M1 for candidate's value from part (a) ×0.1
	$k_2 = 0.1\text{f}(1.1, 0.5447) = 0.48813162$	M1 A1√		
	$y_1 = 0.5 + \dfrac{1}{2}(0.04472 + 0.048813)$			
	$= 0.5468$	A1√	5	
(ii)	$k_1 = 0.1\text{f}(1.1, 0.5468) = 0.04896$	M1 A1√		
	$k_2 = 0.1\text{f}(1.2, 0.5468 + 0.04896)$			
	=0.05336	M1 A1√		
	$y_2 = 0.5979... \approx 0.598$	A1√	5	If answer not given to 3dp withhold this mark
	Total		**13**	
4(a)	$\cos\theta = \dfrac{x}{r},\ \sin\theta = \dfrac{y}{r}$	B1	1	
(b)(i)	$r = 2\dfrac{x}{r} - 4\dfrac{y}{r}$	M1		
	use of $x^2 + y^2 = r^2$	M1		
	$x^2 + y^2 = 2x - 4y$	A1	3	
(ii)	$(x-1)^2 + (y+2)^2 = 5$	M1 A1√		
	Centre (1, −2), radius $\sqrt{5}$	A1√	3	
	Total		**7**	
5(a)	$\left(\dfrac{1}{y}\right)^{-k}\ln\left(\dfrac{1}{y}\right) = -\dfrac{\ln y}{y^{-k}} = -y^k \ln y$	M1A1		
	$x \to \infty, y \to 0 \text{ so } \lim y^k \ln y = 0$	A1	3	
(b)	$\int_1^\infty \dfrac{\ln x}{x^2}\,\text{d}x = \left[-\dfrac{\ln x}{x} - \dfrac{1}{x}\right]_1^\infty$	M1A1		
	=1	A1	3	
	Total		**6**	

Question	Solution	Marks	Totals	Comments
6(a)	$\mathrm{f}(x)=\mathrm{e}^{\cos x-1}$ $\mathrm{f}(0)=1$	B1		
	$\mathrm{f}'(x)=-\sin x\mathrm{e}^{\cos x-1}$ $\mathrm{f}'(0)=0$	M1A1		
	$\mathrm{f}''(x)=\left(-\cos x+\sin^2 x\right)\mathrm{e}^{\cos x-1}$	M1A1		
	$\mathrm{f}''(x)=1$	A1√	6	
(b)	$\mathrm{f}'''(x)=(\sin x+2\sin x\cos)\mathrm{e}^{\cos x-1}$	M1A1		ft
	$+\left(-\cos x+\sin^{-2} x\right)(-\sin x)\mathrm{e}^{\cos x-1}$			
	$\mathrm{f}'''(x)=0$	A1√	3	
(c)	$\sin^2 x\approx x^2$	B1		[Ignore higher power of x]
	$\therefore \lim\limits_{x\to 0}\dfrac{1-e^{\cos x-1}}{\sin^2 x}=\dfrac{1}{2}$	M1A1	3	ft
	Total		**12**	
7(a)	$\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}-\dfrac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}-\dfrac{\mathrm{d}y}{\mathrm{d}x}+3\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}-y\right)=5\mathrm{e}^{-4x}$	M1		oe
	$\dfrac{\mathrm{d}u}{\mathrm{d}x}+3u=5\mathrm{e}^{-4x}$	A1	3	**ag**
(b)	Integrating factor is $\mathrm{e}^{\int 3dx}=\mathrm{e}^{3x}$	B1		
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(u\mathrm{e}^{3x}\right)=5\mathrm{e}^{-x}$	M1A1		
	$u\mathrm{e}^{3x}=-5\mathrm{e}^{-x}+A$	A1√		
	$u=-5\mathrm{e}^{-4x}+A\mathrm{e}^{-3x}$	A1√	5	Provided A appears

MFP3 (cont)

Question	Solution	Marks	Totals	Comments
7 (c)	$\frac{\mathrm{d}y}{\mathrm{d}x} - y = -5\mathrm{e}^{-4x} + A\mathrm{e}^{-3x}$	M1		
	Integrating factor is $\mathrm{e}^{-\int 1\mathrm{d}x} = \mathrm{e}^{-x}$	B1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\mathrm{e}^{-x}\right) = -5\mathrm{e}^{-5x} + A\mathrm{e}^{-4x}$	M1 A1√		
	$y\mathrm{e}^{-x} = \mathrm{e}^{-5x} - \frac{1}{4}A\mathrm{e}^{-4x} + B$	A1√		
	$y = \mathrm{e}^{-4x} - \frac{1}{4}A\mathrm{e}^{-3x} + B\mathrm{e}^{x}$	A1√		
	$A = 2, \quad B = \frac{5}{2}$	B1 B1√		Can be given at any stage
	$y = \mathrm{e}^{-4x} - \frac{1}{2}\mathrm{e}^{-3x} + \frac{5}{2}\mathrm{e}^{x}$	A1√	9	
alt (c)	Auxillary equation is $m^2 + 2m - 3 = 0$	M1		
	$m = -3, 1$	A1		
	Complementary function is $y = A\mathrm{e}^{-3x} + B\mathrm{e}^{x}$	A1		
	Particular integral: try $y = k\mathrm{e}^{-4x}$	M1		
	$16k\mathrm{e}^{-4x} - 8k\mathrm{e}^{-4x} - 3k\mathrm{e}^{-4x} = 5\mathrm{e}^{-4x}$	A1		
	$5k = 5, \quad k = 1$	A1√		
	$\therefore y = \mathrm{e}^{-4x} + A\mathrm{e}^{-3x} + B\mathrm{e}^{x}$			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$	B1		
	$A = -\frac{1}{2},\ B = \frac{5}{2}\ \left[y = \mathrm{e}^{-4x} - \frac{1}{2}\mathrm{e}^{-3x} + \frac{5}{2}\mathrm{e}^{x}\right]$	B2,1,0	9	
	Total		**17**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

MATHEMATICS **MFP4**
Unit Further Pure 4

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

Find the matrices:

(a) **AB**; *(2 marks)*

(b) $\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$. *(2 marks)*

2 The position vectors **a**, **b** and **c** of three points *A, B* and *C* are

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$$

respectively.

(a) Calculate $(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})$, *(4 marks)*

(b) Hence find the exact value of the area of the triangle *ABC*. *(3 marks)*

3 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{bmatrix}$$

(a) Give a geometrical description of each of the transformations represented by the matrices **A** and **B**. *(6 marks)*

(b) For each of these transformations, find the line of invariant points. *(2 marks)*

4 (a) Factorise

$$\begin{vmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

(4 marks)

(b) It is given that

$$\mathbf{A} = \begin{bmatrix} x & 0 & 2 \\ 0 & x & 9 \\ 0 & 1 & x \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} x^2 & x & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

Using your result in part (a), or otherwise, express det(AB) in factorised form. *(4 marks)*

5 The matrix **M** is defined by

$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

(a) Show that M has just two eigenvalues, 1 and 3. *(6 marks)*

(b) Find an eigenvector corresponding to each eigenvalue. *(5 marks)*

The matrix M represents a linear transformation, T, of three dimensional space.

(c) Write down a vector equation of the line of invariant points of T. *(1 mark)*

(d) Write down a vector equation of another line which is invariant under T. *(1 mark)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

6 The planes Π_1 and Π_2 have equations

$$x - 2y + 4z = 0$$

$$\text{and } 2x + 3y + z = 0$$

respectively.

(a) Show that the plane Π_1 is perpendicular to the plane Π_2. *(2 marks)*

(b) Find the Cartesian equation of l, the line of intersection of the planes Π_1 and Π_2 . *(3 marks)*

(c) The line l meets the plane Π_3 whose equation is

$$3x - 4y + z = 18$$

at the point A. *(3 marks)*

Find:

(i) the coordinates of the point A ; *(3 marks)*

(ii) the acute angle between the line l and the plane Π_3 ; *(3 marks)*

(iii) the direction cosines of l . *(3 marks)*

7 Given that $\mathbf{b} \times \mathbf{c} = \mathbf{i}$ and $\mathbf{c} \times \mathbf{a} = 2\mathbf{j}$

express

$$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} + \mathbf{b} + 5\mathbf{c})$$

in terms of i and j. *(6 marks)*

8 A matrix M is defined by

$$\mathbf{M} = \begin{bmatrix} 3 & 1 & 8 \\ 2 & -1 & 5 \\ 1 & 2 & a \end{bmatrix}$$

(a) Find det M in terms of a. *(3 marks)*

(b) Find the value of a for which the matrix M is singular. *(1 mark)*

(c) Find M^{-1}, giving your answer in tems of a. *(6 marks)*

(d) Hence, or otherwise, solve

$$3x + y + 8z = 3$$
$$2x - y + 5z = 0$$
$$x + 2y + 2z = 2$$

(5 marks)

END OF QUESTIONS

MFP4 Specimen

Question	Solution	Marks	Total	Comments
1(a)	$\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$	M1A1	2	
	$= \begin{bmatrix} 1 & 1 & 4 \\ -1 & 2 & -1 \end{bmatrix}$			
(b)	$\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = (\mathbf{AB})^{\mathrm{T}} = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 4 & -1 \end{bmatrix}$	M1 A1$\sqrt{}$	2	
	Total		**4**	
2(a)	$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ or $\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$ seen $\mathbf{c}-\mathbf{a}$ $\quad$ $\mathbf{b}-\mathbf{a}$	B1		Either
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix}$	M1		
	$\mathbf{i}+\mathbf{j}+\mathbf{k}$ or $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	A1A1	4	A1 1 correct (or all −ve) ft A1 all correct
(b)	$\frac{1}{2}\lvert(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\rvert$	M1		$\frac{1}{2}$ prev. result
	$\frac{1}{2}\sqrt{1^2+1^2+1^2} = \frac{1}{2}\sqrt{3}$	M1A1	3	$\sqrt{}$ M1 A1 cao allowing −ves in (a)
	Total		**7**	
3(a)	**A** Shear Parallel to y-axis $(1,0)\to(1,3)$	M1 A1 B1		e.g. (check suggested point) $(1,\ 1)\to(1,\ 4)$ **not** sf
	B Rotation About x-axis of 45°	M1 A1 A1	6	Or "in $y-z$ plane"
(b)	**A** y-axis	B1		Or $x=0$, $\begin{bmatrix} 0 \\ \lambda \end{bmatrix}$
	B x-axis	B1	2	Or $y=z=0$, $\begin{bmatrix} \lambda \\ 0 \\ 0 \end{bmatrix}$
	Total		**8**	

Question	Solution	Marks	Total	Comments
4(a)	$x^2(1-2)-x(1-4)+1(2-4)$	M1A1		
	$-x^2+3x-2$	A1		
	$-(x-1)(x-2)$	A1√	4	
(b)	det **A** det **B** used	M1		
	det **B** = x^3-9x	M1A1		
	det **AB** = $-x(x-3)(x+3)(x-1)(x-2)$	A1√	4	If any other method is used, it must be complete.
	Total		**8**	
5(a)	$\begin{vmatrix} 2-\lambda & -1 & 1 \\ 2 & 3-\lambda & 2 \\ 1 & 1 & 2-\lambda \end{vmatrix}$	M1		
	$(2-\lambda)(6-5\lambda+\lambda^2-2)+1(2-2\lambda)+1(-1+\lambda)$	M1		o.e.
	$-\lambda^3+7\lambda^2-15\lambda+9$	A1		
	$(\lambda-1)(\lambda-3)^2=0$	M1A1		
	$\lambda=1$ or 3	A1	6	**ag**
(b)	$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\mathbf{v}=0, \begin{bmatrix} -1 & -1 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & -1 \end{bmatrix}\mathbf{v}=0$	M1M1		
	e.g. $\mathbf{v}=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}=\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$	M1A1 A1	5	M1A1 for either
(c)	$r=\lambda\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$	B1√	1	
(d)	$r=\mu\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$	B1√	1	
	Total		**13**	

Question	Solution	Marks	Total	Comments
6(a)	$\begin{bmatrix}1\\-2\\4\end{bmatrix}\cdot\begin{bmatrix}2\\3\\1\end{bmatrix}$ $=2-6+4=0$	M1A1	2	
(b)	$\begin{bmatrix}1\\-2\\4\end{bmatrix}\times\begin{bmatrix}2\\3\\1\end{bmatrix}=\begin{bmatrix}-14\\7\\7\end{bmatrix}$	M1A1		
	l is $\dfrac{x}{-2}=\dfrac{y}{1}=\dfrac{z}{1}\ (=\lambda)$	A1√	3	oe
(c)(i)	Substitute $x=-2\lambda,\ y=\lambda, z=\lambda$ into Π_3 $-6\lambda-4\lambda+\lambda=18$ $\lambda=-2$ $\therefore$ A is $(4,-2,-2)$	M1 A1 A1	3	
(ii)	$\cos\theta=\pm\dfrac{\begin{bmatrix}3\\-4\\1\end{bmatrix}\cdot\begin{bmatrix}-2\\1\\1\end{bmatrix}}{\sqrt{26}\sqrt{6}}$	M1		
	$\theta=43.9°$ required angle is 46.1°	A1 A1√	3	
(iii)	direction cosines are $\left(-\dfrac{2}{\sqrt{6}},\dfrac{1}{\sqrt{6}},\dfrac{1}{\sqrt{6}}\right)$	M1 A2,1,0	3	
	Total		**14**	
7	Sensible expansion	M1		If $a^2+ab+5ac...$ used M0 unless some indication of understanding e.g. $a^2=0$. $b=j,\ a=2i,\ c=k$ or similar 0.
	Cancelling out $\mathbf{a}\times\mathbf{a}$ **etc.** Cancelling out $\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}$ $\mathbf{a}\times5\mathbf{c}+\mathbf{b}\times5\mathbf{c}$ $5\mathbf{i}-10\mathbf{j}$	M1 M1 A1 A1A1	6	All A's depend on M3
	Total		**6**	

Question	Solution	Marks	Total	Comments
8(a)	$3(-a-10)-(2a-5)+8(4+1)$	M1A1		M attempt; A correct unsimplified
	$15-5a$	A1	3	cao
(b)	$a=3$	B1√	1	ft $\theta = 0$
(c)	$\begin{bmatrix} -a-10 & 5-2a & 5 \\ 16-a & 3a-8 & -5 \\ 13 & 1 & -5 \end{bmatrix}$	M1 A1		Finding 2×2 determinants (co-factors) Any one correct row/column
	Use of $\dfrac{1}{\det \mathbf{M}}$	B1		ft (a) wrong or correct having stated again
	$\dfrac{1}{15-5a}\begin{bmatrix} -a-10 & 16-a & 13 \\ 5-2a & 3a-8 & 1 \\ 5 & -5 & -5 \end{bmatrix}$	M1 M1 A1	6	Signs Transpose cao
(d)	Realisation that a = 2	B1	1	
	$\dfrac{1}{5}\begin{bmatrix} -12 & 14 & 13 \\ 1 & -2 & 1 \\ 5 & -5 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$	M2		
	$\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$	A1√ A1	4	A1 any one correct (ft) A1 all cao
Alt 1 to (d)	3 equations $\to 2 \to 1 \to$ Answers	M1 M1 A1 A1		3 equations $\to 2$ 2 equations $\to 1$ Any one correct All cao
Alt 2 to (d)	Cramer's Rule $x = \dfrac{\Delta x}{\Delta}$ etc	M1		
	$x=-2,\ y=1,\ z=1$	A1A1 A1		
Alt 3 to (d)	Gaussian Elimination	M1A1 A1A1		
	Total		**15**	
	TOTAL		**75**	