A-LEVEL
MATHEMATICS

Mechanics 4 – MM04
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	$\mathbf{r}_1\mathrm{x}\mathbf{F}_1=\begin{vmatrix} \boldsymbol{i} & 1 & 3 \\ \boldsymbol{j} & -2 & 1 \\ \boldsymbol{k} & a & 4 \end{vmatrix} = \begin{pmatrix} -8-a \\ 3a-4 \\ 7 \end{pmatrix}$	**M1**		Use of **r x F** or **F x r** – at least two components correct in any vector
	$\mathbf{r}_2\mathrm{x}\mathbf{F}_2=\begin{vmatrix} \boldsymbol{i} & 6 & \frac{1}{2} \\ \boldsymbol{j} & 2 & -1 \\ \boldsymbol{k} & 8 & \frac{1}{2}a \end{vmatrix} = \begin{pmatrix} a+8 \\ 4-3a \\ -7 \end{pmatrix}$			
	$\mathbf{r}_3\mathrm{x}\mathbf{F}_3=\begin{vmatrix} \boldsymbol{i} & 1 & a \\ \boldsymbol{j} & 0 & 2 \\ \boldsymbol{k} & -1 & -a \end{vmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$	**A(2,1,0)**		**A1** – Two vector products fully correct **A2** – All three correct
	$\mathbf{r}_1\mathrm{x}\mathbf{F}_1 + \mathbf{r}_2\mathrm{x}\mathbf{F}_2 + \mathbf{r}_3\mathrm{x}\mathbf{F}_3 = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$	**m1A1F**		**m1** – Dependent on summing three vector products and the first **M1** **A1F** – their correct sum – following through one slip only
	a cancels - hence independent of a	**E1**		Comment indicating independent of a - **CSO**
	Total		**6**	

Q	Solution	Mark	Total	Comment
2(a)	Volume = $\pi \int_0^a a^2 x \, dx$	**M1**		Use of correct formula seen
	$= \pi \left[\frac{a^2 x^2}{2}\right]_0^a$	**A1**		Correct integration
	$= \frac{1}{2}\pi a^4$	**A1**	3	**CSO** - Printed answer
(b)	$\pi \int_0^a xy^2 \, dx = \pi \int_0^a a^2 x^2 \, dx$			
	$= \pi \left[\frac{a^2 x^3}{3}\right]_0^a$	**M1**		Correct use of formula - integrating kx^2
	$= \frac{1}{3}\pi a^5$	**A1**		**CAO**
	$\bar{x} = \frac{\frac{1}{3}\pi a^5}{\frac{1}{2}\pi a^4}$	**M1**		Their answer for $\pi \int_0^a xy^2 \, dx$ ÷ part (a)
	$= \frac{2a}{3}$	**A1**	4	**CAO**
(c)	$\tan 30^0 = \frac{distance\ of\ c\ of\ m\ from\ base}{radius\ of\ base}$	**M1**		Forming an equation using tan 30^0 or tan 60^0
	$\tan 30^0 = \frac{\frac{a}{3}}{a^{\frac{3}{2}}}$	**A1**		Fully correct equation formed – correct expressions for distances seen – accept equivalent, unsimplified expressions
	$a^{-\frac{1}{2}} = \sqrt{3}$	**m1**		Solving their equation using correct value for tan 30^0 or tan 60^0 - dependent on first **M1**
	$a = \frac{1}{3}$	**A1F**	4	Correct solution to **their** equation – must have both **M**s above
	Total		**11**	

Q	Solution	Mark	Total	Comment
3	MI for particle at B = $2(0.3)^2 = 0.18$ MI for particle at D = $m(0.4)^2 = 0.16m$ MI for particle at C = $6(0.5)^2 = 1.5$	**M1** **A1** **A1**		Use of md^2 at least once Correct MI for both B and D Correct MI for particle C All terms can be unsimplified – **M0** if formulas for rods are used
	Forming an equation, $1.68 + 0.16m = 2.24$	**m1**		Forming an equation to find *m* – dependent on first **M1**
	Hence $m = \frac{0.56}{0.16} = 3.5$	**A1**	5	Correct value of *m* obtained **- CAO**
	Total		**5**	

Q	Solution	Mark	Total	Comment
4 (a)	Consider forces horizontally at A,			
	$T_{AB} = T_{AE}\cos 60^0 = 50\cos 60^0 = 25$ N	**B1**		**CAO**
	AB is in tension	**E1**	2	**CAO**
(b)	Consider forces vertically at A,			
	$R_A = T_{AE}\sin 60^0 = 50\sin 60^0 = 25\sqrt{3}$	**B1**	1	**CAO** – accept any equivalent (eg 43.3)
(c)	Resolving vertically for whole framework, $x = R_A + R_C$			
	By symmetry, $R_A = R_C$	**M1**		Resolving and use of symmetry seen
	$x = 2R_A = 50\sqrt{3}$	**A1F**	2	Follow through part b) - x = twice their answer to part b) accept any equivalents (eg 86.6)
(d)	$T_{ED} = T_{AE}\cos 60^0 + T_{EB}\cos 60^0$	**M1**		**M1** Resolving forces horizontally at E (or D) – at least one correct term seen on right hand side
	Considering vertical forces at E, Rod EB must be in tension with magnitude 50N			
	Hence $T_{ED} = 50$N	**A1**		**CAO**
	ED is in compression	**E1**	3	**CAO**
	Total		**8**	

Q	Solution	Mark	Total	Comment
5(a)	$\begin{pmatrix} 6-2p \\ p-3 \end{pmatrix}$	**B1**	1	
(b)	Moments about O = -1(3) – 5(2) – 5(4) + p(3) = 3p -33 or 3(p-11)	 **M1** **A1** **A1** **A1**	 4	 Two pairings correct – force x distance All pairings correct All signs consistent c = 3 (or -3 if directions interchanged)
(c)	$6-2p=0$ and $p-3=0$ Hence $p=3$ (is consistent solution) Magnitude = 24 Sense is clockwise No linear resultant and non-zero moment therefore is a couple	**M1** **A1** **B1** **B1** **E1**	 5	Setting components/resultant force equal to zero from part a) Correct and consistent p value obtained Correct magnitude of couple stated Correct sense of couple stated Fully correct deduction concerning couple
(d)(i)	**F** = $\begin{pmatrix} -16 \\ 8 \end{pmatrix}$	**M1** **A1**	 2	**M1** -Substitution of their p value in a) **A1** Correct **F** seen - **CAO**
(ii)	When p = 11, resultant moment about O is 0 hence line goes through origin Line L is $y=-\frac{1}{2}x$	**M1** **A2**	 3	Use of zero moment seen or comment made **CAO**
	Total		**15**	

Alternative for b)

$$\begin{vmatrix} \boldsymbol{i} & 0 & 1 \\ \boldsymbol{j} & 3 & 2 \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$$

M1 – One determinant attempted – at least two components correct

$$\begin{vmatrix} \boldsymbol{i} & 2 & -2p \\ \boldsymbol{j} & 0 & -5 \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -10 \end{pmatrix}$$

A1 – two determinants evaluated correctly

$$\begin{vmatrix} \boldsymbol{i} & 3 & 5 \\ \boldsymbol{j} & 4 & p \\ \boldsymbol{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3p-20 \end{pmatrix}$$

A1 – All three correct

Total = $\begin{pmatrix} 0 \\ 0 \\ 3p-33 \end{pmatrix}$

Hence c = 3

A1 – correct sum obtained or c = 3 stated from fully correct working

Q	Solution	Mark	Total	Comment
6(a)	$T_1 - 2mg = 2ma$ $4mg - T_2 = 4ma$	M1		Forming two correct equations of motion for particles
	$2(T_1 - 2mg) = 4mg - T_2$	m1		Correct elimination of a (acceleration)
	Hence $2T_1 + T_2 = 8mg$	A1	3	Fully shown – printed answer - **CSO**
(b)	Using $C = I\ddot{\theta}$ $rT_2 - rT_1 - \frac{1}{2}mgr = 6mr^2\ddot{\theta}$	M1A1		Correct use of equation of motion for the pulley. **M1** for LHS with inclusion of all turning forces. **A1** Fully correct
	Cancelling r gives $T_2 - T_1 - \frac{1}{2}mg = 6mr\ddot{\theta}$	A1	3	**CSO** – printed answer
(c)	From a) adding equations gives $T_1 - T_2 + 2mg = 6ma$			
	In b) $T_2 - T_1 - \frac{1}{2}mg = 6ma$			
	Hence $T_2 - T_1 - \frac{1}{2}mg = T_1 - T_2 + 2mg$	M1A1		**M1** Use of elimination to obtain a second equation involving T_1 and T_2 **A1** Correct equation obtained
	Gives $T_2 - T_1 = \frac{5mg}{4}$			
	Solving with a) $2T_1 + T_2 = 8mg$	m1		Solving a pair of equations to obtain expressions for T_1 and T_2. Dependent on first **M1**
	Gives $T_1 = \frac{9mg}{4}$ and $T_2 = \frac{7mg}{2}$	A1	4	Both T_1 and T_2 correct - **CSO**
(d)	Using $T_2 - T_1 - \frac{1}{2}mg = 6mr\ddot{\theta}$	M1		Substituting their T_2 and T_1 and rearranging to obtain expression for $\ddot{\theta}$
	$\ddot{\theta} = \frac{g}{8r}$	A1F	2	Must involve g and r only
	Total		**12**	

Q	Solution	Mark	Total	Comment
7(a)(i)	$m = \pi a^2 h\rho$ or $\rho = \frac{m}{\pi a^2 h}$	B1		Correct equation connecting m and ρ seen
	Mass of thin disc = $\pi a^2 dx\rho$ MI of thin disc = $\frac{1}{2}(\pi a^2 dx\rho)a^2$	M1		Formulating MI of elemental disc
	MI of cylinder $= \int_0^h \frac{1}{2}\pi a^4 \rho dx = \left[\frac{1}{2}\pi a^4 x\rho\right]_0^h$	A1		Integrating correctly
	$= \frac{1}{2}\pi a^4 \frac{m}{\pi a^2 h} h$	m1		Substituting correct limits and p – dependent on first **M1**
	$= \frac{1}{2}ma^2$	A1	5	Obtaining printed answer - **CSO**
	Alternative for (a)(i) $m = \pi a^2 h\rho$ or $\rho = \frac{m}{\pi a^2 h}$	(B1)		Correct equation connecting m and ρ seen
	MI of cylindrical shell = $2\pi rh\rho r^2 dr$	(M1)		Formulating MI of elemental shell
	MI of cylinder $= \int_0^a 2\pi rh\rho r^2 dr = \left[\frac{2\pi r^4 h\rho}{4}\right]_0^a$	(A1)		Integrating correctly
	$= \frac{2}{4}\pi a^4 \frac{m}{\pi a^2 h} h$	(m1)		Substituting correct limits and p – dependent on first **M1**
	$= \frac{1}{2}ma^2$	(A1)	(5)	Obtaining printed answer - **CSO**
(b)	Given cylinder MI = $\frac{1}{2}(2m)(3a)^2$	B1		Correct adjustment of formula in part a) - can be unsimplified
	Using parallel axis theorem $I_{AXIS} = \frac{1}{2}(2m)(3a)^2 + 2m(3a)^2$	M1A1		**M1** Correct structure of parallel axis theorem **A1** Distance and masses correct
	$= 27ma^2$	A1	4	**CAO**
(c)(i)	Forming an energy equation $\frac{1}{2}(27ma^2)\dot{\theta}^2 = (2mg)(3a\sin\theta)$	M1 A1F A1		**Follow through part b) throughout** Forming an equation using KE gain = PE loss Their correct kinetic energy used (LHS) Correct potential energy used (RHS)
	Hence $\dot{\theta}^2 = \frac{4g\sin\theta}{9a}$	A1F		Rearranging to get $\dot{\theta}^2$ - can be unsimplified
	Differentiating gives $2\dot{\theta}\ddot{\theta} = \frac{4g\cos\theta}{9a}\dot{\theta}$	M1		Differentiating correctly
	Hence $\ddot{\theta} = \frac{2g\cos\theta}{9a}$	A1F	6	Rearranging to get $\ddot{\theta}$ - can be unsimplified

Q	Solution	Mark	Total	Comment
c(ii)	Using F= ma tangentially $2mg\cos\theta - Y = (2m)(3a)\ddot{\theta}$ $Y = 2mg\cos\theta - (2m)(3a)\left(\frac{2g\cos\theta}{9a}\right)$ $Y = \frac{2mg\cos\theta}{3}$	**M1A1** **A1F**	 3	**M1** Use of F = ma - one side correct **A1** Both sides correct Can be unsimplified – follow through substitution of their $\ddot{\theta}$ from c)i)
(c)(i) **(c)(ii)**	**Alternative for part (c)(i) – final two marks** $I\ddot{\theta} = (2mg)(3a\cos\theta)$ Hence $\ddot{\theta} = \frac{2g\cos\theta}{9a}$ **Alternative to part (c)(ii)** Take moments about axis of cylinder gives $(3a)Y = \frac{1}{2}(2m)(3a)^2\ddot{\theta}$ Hence $Y = \frac{2mg\cos\theta}{3}$	 **(M1)** **(A1F)** **(M1A1)** **(A1F)**	 (3)	**Follow through part b) throughout** Correct use of C = $I\ddot{\theta}$ Rearranging to get $\ddot{\theta}$ - can be unsimplified Use of moments – one side correct for **M1** Both sides correct for **A1** Can be unsimplified – follow through substitution of their $\ddot{\theta}$ from c)i)
	Total		**18**	
	TOTAL		**75**	