Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MM04

(Specification 6360)

Mechanics 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

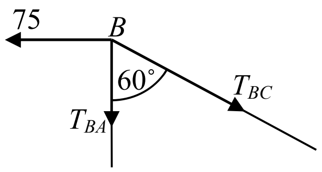
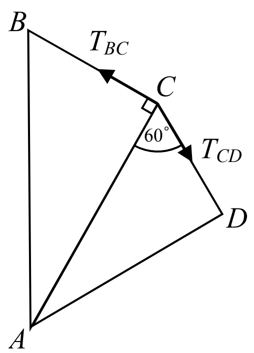
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

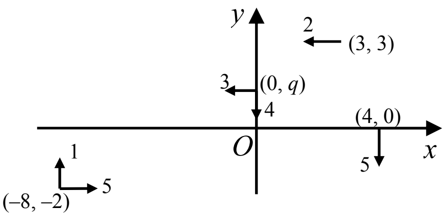
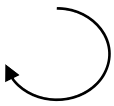
MM04

Q	Solution	Marks	Total	Comments
1(a)	$M = \left(\frac{4-2}{2}, \frac{-1+1}{2}, \frac{4+6}{2}\right) = (1, 0, 5)$	B1		mid-point found
	$\overrightarrow{PM} = -\begin{pmatrix}-2\\-1\\4\end{pmatrix} + \begin{pmatrix}1\\0\\5\end{pmatrix} = \begin{pmatrix}3\\1\\1\end{pmatrix}$	B1	2	AG
	alternative			
	$\overrightarrow{PQ} = \begin{pmatrix}4\\1\\6\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix} = \begin{pmatrix}6\\2\\2\end{pmatrix}$	(B1)		
	$\overrightarrow{PM} = \frac{1}{2}\begin{pmatrix}6\\2\\2\end{pmatrix} = \begin{pmatrix}3\\1\\1\end{pmatrix}$	(B1)	(2)	AG
(b)	Moment = $\mathbf{r} \times \mathbf{F}$			
	$= \begin{vmatrix}\mathbf{i} & 3 & a\\ \mathbf{j} & 1 & 1\\ \mathbf{k} & 1 & -2\end{vmatrix}$	M1		attempt at $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$
	$= \begin{pmatrix}-3\\a+6\\3-a\end{pmatrix}$	A2,1	3	one component correct $\Rightarrow$ A1 $\mathbf{F} \times \mathbf{r}$ attempt $\Rightarrow$ M1A1A0
(c)	Magnitude $= \sqrt{(-3)^2 + (a+6)^2 + (3-a)^2}$	M1		attempt at magnitude of their moment
	Hence $9 + (a+6)^2 + (3-a)^2 = 50$	A1F		forms equation magnitude^2 = 50
	$a^2 + 3a + 2 = 0$ $(a+2)(a+1) = 0$	m1		attempts to solve a quadratic – real roots
	$a = -2$ or -1	A1	4	both values obtained; CAO No further penalty for $\mathbf{F} \times \mathbf{r}$ attempt which is correct ie (3, – a – 6, a – 3) as components
	Total		**9**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Take moments at A	M1		evidence of force $\times$ perpendicular distance
	$2lP = \frac{200\sqrt{3}}{3}\left(\frac{3}{2}l\cos 30°\right)$	A1		correct equation
	$P = 75\text{N}$	A1	3	AG
	alternative			
	At B, perpendicular to AB $\quad P = T_{BC}\cos 30°$			
	At C, parallel to BC $\quad T_{BC} = T_{CD}\cos 30°$	(M1)		Sufficient equations to find P
	At D, parallel to CD $\quad T_{CD} = \frac{200\sqrt{3}}{3}\cos 30°$	(A1)		All correct
	$\Rightarrow P = \frac{200\sqrt{3}}{3} \times (\cos 30°)^3 = 75\text{N}$	(A1)	(3)	AG
(b)				
	At B, resolve horizontally			
	$T_{BC}\cos 30° = 75$	M1		Equation involving T_{BC}
	$\Rightarrow T_{BC} = 86.6\text{N}$	A1		or $50\sqrt{3}$
	BC in tension	E1		
	Resolve vertically			
	$T_{BA} + T_{BC}\cos 60° = 0$	M1		Equation involving T_{BA}
	$\Rightarrow T_{BA} = -T_{BC}\cos 60°$			
	$\Rightarrow \lvert T_{BA}\rvert = 43.3\text{N}$	A1F		or $25\sqrt{3}$
	BA in compression	E1	6	ft their T_{BC}
(c)				
	Resolve perpendicular to AC			
	$T_{BC} = T_{CD}\cos 30°$	M1		Equation involving T_{CD}
	$\Rightarrow T_{CD} = \frac{86.6...}{\cos 30°} = 100\text{N}$	A1F	2	ft their T_{BC}
(d)	CD in tension			
	AC in compression			B1 two correct
	AD in compression	B2,1	2	B2 all correct
	Total		**13**	

MM04 (cont)

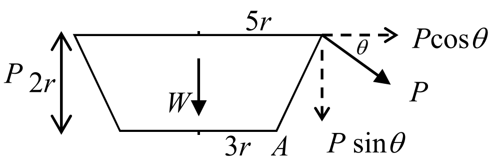
Q	Solution	Marks	Total	Comments
3(a)	$\begin{pmatrix}-2\\0\end{pmatrix}+\begin{pmatrix}0\\-5\end{pmatrix}+\begin{pmatrix}p\\1\end{pmatrix}+\begin{pmatrix}-3\\-4\end{pmatrix}=\begin{pmatrix}p-5\\-8\end{pmatrix}$	B1	1	
(b)(i)	Parallel to y-axis $\Rightarrow p-5=0$	M1		set **i** component = 0 (seen or implied)
	$p=5$	A1	2	
(ii)				
	Moments about O for given system $-5(4)+2(3)+3q+5(2)-1(8)$	M1		F × d for at least four components
	$=3q-12$	A2,1F		–1 each type of error, ft (a), (b)(i) (12 – 3q scores M1A2)
	Moments about O for equivalent system $=-8(3)$ $=-24$	B1		±24 seen ft (a) allow ± 3 × their **j** component
	Hence $3q-12=-24$	M1		attempt at moment equation – must see clear use of Force × distance on RHS
	$3q=-12$ $q=-4$	A1F	6	ft error with p from (b)(i)
(c)	$\lvert C\rvert=24$	B1F		Should match part (b) – must be positive
		B1	2	accept 'clockwise'
	Total		**11**	

MM04 (cont)

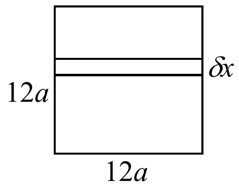
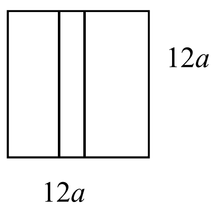
Q	Solution	Marks	Total	Comments
4(a)	G = mid-point of BC			
	$PG = \sqrt{2}l$ or $PG^2 = 2l^2$	B1		correct distance, seen/used
	$\text{MI}_P = \text{MI}_G + m\,PG^2$	M1		use of parallel axis theorem
	$= \frac{ml^2}{3} + m(2l^2)$	A1		$\frac{ml^2}{3}$ used
	$= \frac{7ml^2}{3}$	A1	4	AG
(b)	$\text{MI}_{particles} = 3ml^2 + 3ml^2 + 4m(5l^2)$	M1		MI of three particles
		A1		$3ml^2$ seen
	$= 26ml^2$	A1		use of $5l^2$ with $4m$
	$\text{MI}_{rods} = \frac{ml^2}{3} + \frac{7ml^2}{3}$	M1		MI of two rods (a) + (b)
	$= \frac{8ml^2}{3}$			
	$\text{MI}_{system} = 26ml + \frac{8ml^2}{3}$			
	$= \frac{86ml^2}{3}$	A1F	5	ft error in (a)
(c)	Gain in KE $= \frac{1}{2}\text{I}\dot{\theta}^2$			
	$= \frac{1}{2}\left(\frac{86}{3}ml^2\right)\dot{\theta}^2$	B1F		use of KE formula with MI from (b)
	$= \frac{43}{3}ml^2\dot{\theta}^2$			
	Loss in PE for rod BC only = mgh	M1		use of mgh seen
	$= 2mgl$	A1		loss for one rod only
	Loss in PE for $4m$ particle = $4mg(3l)$			
	$= 12mgl$	A1		loss for $4m$ particle
	Gain for $3m$ particle at A = loss for $3m$ particle at B = $3mgl$			
	(System) total loss of PE = $14mgl$	A1		total loss for system
	$\therefore \frac{43}{3}ml^2\dot{\theta}^2 = 14mgl$	m1		conservation of energy equation – dependent on use of KE, PE for rod <u>and</u> particles
	$\dot{\theta} = \sqrt{\frac{42g}{43l}}$	A1F	7	ft error in (a) or (b) Condone $\dot{\theta}^2 = \frac{42g}{43l}$
	Total		**16**	

Q	Solution	Marks	Total	Comments
4(c)	**Alternative 1**			
	PE before motion = $mgl + 4mg(2l)$ $= 9mgl$	(M1) (A1)		mgh used total PE correct
	PE after motion $= -3mgl - mgl - 4mgl + 3mgl$ $= -5mgl$	(A1)		total PE correct
	KE before = 0			
	KE after $= \frac{43}{3}ml^2\dot{\theta}^2$	(B1F)		use of KE formula with MI from (b)
	C of E ⇒ $9mgl = \frac{43}{3}ml^2\dot{\theta}^2 - 5mgl$	(M1)		attempt at C of E equation
	⇒ $\frac{43}{3}ml^2\dot{\theta}^2 = 14mgl$	(A1)		correct equation
	⇒ $\dot{\theta} = \sqrt{\frac{42g}{43l}}$	(A1F)	(7)	
	Alternative 2			
	Centre of mass of system at $\left(\frac{17}{12}l, \frac{3}{4}l\right)$	(M1)		Centre of mass attempted
	Change in height of centre of mass = $\frac{3}{4}l + \left(\frac{17}{12}l - l\right) = \frac{7}{6}l$	(A1)		Change in height seen/used
	Total PE loss = $12mg\left(\frac{7}{6}l\right) = 14mgl$	(A1) (A1)		mgh used Total loss found
	KE gain $= \frac{43}{3}ml^2\dot{\theta}^2$	(B1F)		use of KE formula with MI from (b)
	C of E ⇒ $\frac{43}{3}ml^2\dot{\theta}^2 = 14mgl$	(M1)		C of E equation formed
	$\dot{\theta} = \sqrt{\frac{42g}{43l}}$	(A1F)	(7)	

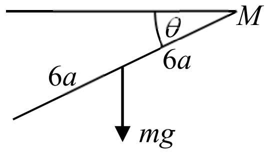
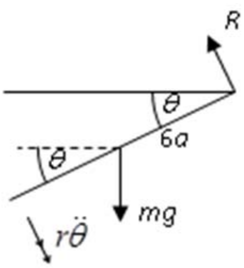
MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\pi\int xy^2\,dx = \pi\int_0^{2r} x(x+3r)^2\,dx$	M1		(use of π must be consistent) attempt to integrate $\int xy^2\,dx$ - must involve three terms
	$=\pi\int_0^{2r}(x^3+6rx^2+9r^2x)\,dx$			
	$=\pi\left[\frac{x^4}{4}+2rx^3+\frac{9r^2x^2}{2}\right]_0^{2r}$	A1		correct integration
	$=\pi\left[4r^4+16r^4+18r^4\right]$	m1		Limits used correctly
	$=38r^4\pi$	A1		correctly evaluated in terms of r^4
	$\bar{x}=\frac{38r^4\pi}{98\pi r^3/3}=\frac{57r}{49}$	M1		use of $\frac{\pi\int xy^2\,dx}{\text{volume}}$
		A1F	6	ft 'their' $\int xy^2\,dx$
(b)(i)				
	Moments at A: $W(3r)=P\sin\theta(2r)+P\cos\theta(2r)$	M1A1 A1		M1 attempt at moments — evidence of force × perpendicular distance A1 two terms correct A1 all terms correct
	$3W=2P(\cos\theta+\sin\theta)$			
	$\frac{3W}{2(\cos\theta+\sin\theta)}=P$	A1	4	AG Must see evidence of factorising
(ii)	Min value of P is when $\cos\theta+\sin\theta$ is at a maximum.	M1		Attempt to maximise denominator
	Max value of $\cos\theta+\sin\theta$ is $\sqrt{2}$	A1		$\sqrt{2}$ seen
	Max P value = $\frac{3W}{2\sqrt{2}}$	A1	3	Or equiv eg $\frac{6\sqrt{2}W}{8}$, $\frac{3}{4}\sqrt{2}W$ etc
(iii)	$\theta=45°$	B1	1	
	Total		**14**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)				
	$m = 144a^2\rho$ $\Rightarrow \rho = \dfrac{m}{144a^2}$	B1		seen anywhere – connection between ρ and m
	Mass of strip $= 12a\delta x\rho$ $\text{MI}_{square} = \sum 12a\delta x\rho x^2$	M1		Use of $\sum mx^2$
	$= \int_0^{12a} 12ax^2 \dfrac{m}{144a^2}\mathrm{d}x$	A1		Correct integral formed
	$= \int_0^{12a} \dfrac{mx^2}{12a}\mathrm{d}x$	M1		attempt at integration – must be of the form $\int kx^2\mathrm{d}x$
	$= \left[\dfrac{mx^3}{36a}\right]_0^{12a}$			
	$= 48ma^2$	A1	5	AG
	Alternative			
				
	$m = 144a^2\rho \ \Rightarrow \rho = \dfrac{m}{144a^2}$	(B1)		seen anywhere – connection between ρ and m
	Mass of strip $= 12a\rho\delta x$ MI of strip about end $= \sum \dfrac{4}{3}(12a\rho dx)(6a)^2$	(M1)		Use of $\dfrac{4}{3}ml^2$
	$= \sum 576\rho a^3\delta x$			
	$= \int_0^{12a} \dfrac{576a^3m}{144a^2}\mathrm{d}x = \int_0^{12a} 4am\mathrm{d}x$	(A1)		Correct integral
	$= \left[4amx\right]_0^{12a}$	(M1)		Attempt at integration
	$= 48ma^2$	(A1)	(5)	AG

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(i)				
	Using $C = I\dot{\theta}$			
	$mg\,6a\cos\theta = 48ma^2\ddot{\theta}$	M1 A1		Attempt at equation - one side correct both sides correct
	$\ddot{\theta} = \dfrac{g\cos\theta}{8a}$	A1	3	AG
	Alternative			
	PE lost = $6mag\sin\theta$			
	KE gained = $\frac{1}{2}(48ma^2)\dot{\theta}^2$			
	Conservation of energy $\Rightarrow$			
	$\frac{1}{2}(48ma^2)\dot{\theta}^2 = 6mag\sin\theta$	(M1)		Attempt at KE gained = PE lost to find $\dot{\theta}^2$
	$\dot{\theta}^2 = \dfrac{g\sin\theta}{4a}$			
	Differentiate			
	$2\dot{\theta}\ddot{\theta} = \dfrac{g\cos\theta}{4a}\dot{\theta}$	(A1)		Differentiating
	$\Rightarrow \ddot{\theta} = \dfrac{g\cos\theta}{8a}$	(A1)		AG
(b)(ii)				
	Using NSL			
	$mg\cos\theta - R = m(6a)\ddot{\theta}$	M1		attempt at $F = ma$
	$R = mg\cos\theta - \dfrac{6mg}{8}\cos\theta$	A1		fully correct
	$= \dfrac{mg\cos\theta}{4}$	A1	3	substituting $\ddot{\theta}$ to obtain answer
6(b)(iii)	Consider frictional forces/resistances	E1	1	Any sensible modelling comment
	Total		**12**	
	TOTAL		**75**	