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General Certificate of Education (A-level)
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Mathematics MS04

(Specification 6360)

Statistics 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS04

Q	Solution	Marks	Total	Comments
1	$\mathrm{H}_0 : \sigma = 0.7 \qquad \mathrm{H}_1 : \sigma \neq 0.7$	B1		Both
	$s = 0.3147$	B1		$\sum(x-\bar{x})^2 = 1.0891\dot{6}$ or $s^2 = 0.0990$
	$\nu = 11$	B1		
	$\chi^2_{\text{crit}} = 2.60$, 26.76	B1		Accept 2.60 only
	Under H_0			
	$\dfrac{11\times 0.3147^2}{0.49} = 2.22$	M1A1		
	$2.22 < 2.60 \Rightarrow$ Reject H_0			
	Evidence to suggest that $\sigma \neq 0.7$ at 1% level	E1ft	7	
		Total	**7**	
2 (a)	d: 2 5 1 0 4 -6 3 4	M1		At most two errors
	$\bar{d} = 1.625$	B1		
	$s = 3.503$	B1		
	$\nu = 7$	B1		
	$t = 3.499$	B1		
	Confidence Limits are			
	$1.625 \pm \dfrac{3.499\times 3.503}{\sqrt{8}}$	M1A1 ft		ft on $\bar{d}$, s, and t
	Confidence interval is $(-2.71, 5.96)$	A1	8	
(b)	Normally distributed	E1	1	
(c)	$5 \in$ CI	E1ft		ft on (a)
	$\Rightarrow$ Not unreasonable that $\mu_D = 5$.	E1ft	2	Dependent on previous E1
		Total	**11**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
3 (a)	$s_X{}^2 = 15300 \qquad s_Y{}^2 = 4760$	M1A1	2	M1 One A1 both (15334) AWRT (4760) S.C. B1 $\geq$ 1 S.D.
(b)(i)	$\nu_1 = 9 \qquad \nu_2 = 7$	B1		Both
	$F_{9,7} = 6.719 \qquad F_{7,9} = 5.613$	B1B1		
	$F_{\text{calc}} = \dfrac{15334}{4764} = 3.219$	M1		
	$\dfrac{1}{6.719} \leq \dfrac{\text{VR}}{3.219} \leq 5.613$	M1A1 ft		ft on variances and F values
	$\Rightarrow 0.479 \leq \text{VR} \leq 18.1$	A1	7	
(ii)	$1 \in$ CI	E1ft		ft on (b)(i)
	$\Rightarrow$ No significant evidence of a difference between 'Killrust' and 'Stoprust'	E1ft	2	Dependent on previous E1
		Total	**11**	
4(a)(i)	F(t) : 0.0272 0.1792 0.4752 0.8192 1	M1A1		Table function on GDC allowed
(ii)	p_i : 0.0272 0.1520 0.2960 0.3440 0.1808	M1A1	4	Differences $\Rightarrow$ M1
(b)	H_0 : Claim is correct	B1		
	O_i : 2 9 12 22 5 E_i : 1.36 7.60 14.80 17.20 9.04	M1A1		
	Combine first two classes: $\begin{pmatrix} 11 \\ 8.96 \end{pmatrix}$	M1		
	$\chi^2_{\text{calc}} = \sum \left\{ \dfrac{(\text{O}-\text{E})^2}{\text{E}} \right\} = 4.139$	M1A1		Accept 4.234 based on $\nu = 5 - 1 = 4$
	$\nu = 4 - 1 = 3$	B1ft		
	$\chi^2_{\text{crit}} = 7.815$	B1ft		ft on $\nu = 4$
	$4.139 < 7.815 \Rightarrow$ Accept H_0 at 5% level Evidence to suggest claim is correct	E1ft	9	ft $\chi^2_{\text{crit}} = 9.488$
		Total	**13**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	X~ Geo (p)			
	$\text{E}(X) = \sum x\text{P}(X = x)$			
	$= p + 2qp + 3q^2 p + \ldots$	M1		Where $p + q = 1$
	$= p(1 + 2q + 3q^2 + \ldots.)$	A1		
	$= p(1 - q)^{-2}$			
	$= \frac{p}{p^2}$			
	$= \frac{1}{p}$	A1	3	CSO AG
(b)	By part (a) expect 6 throws	B1		
	$6x - 10 = 1$	M1A1		Where x is the price per throw
	$\Rightarrow x = £1.84$ for profit	A1	4	Accept £1.83
		Total	**7**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mu = \mathrm{E}(X) = \int_0^\infty kx\mathrm{e}^{-kx}\mathrm{d}x$	M1		
	$= \left[-x\mathrm{e}^{-kx}\right]_0^\infty + \int_0^\infty \mathrm{e}^{-kx}\mathrm{d}x$	M1		
	$= 0 + \left[\frac{\mathrm{e}^{-kx}}{-k}\right]_0^\infty$	A1		
	$= \frac{1}{k}$		3	AG
(ii)	$\int_0^m k\mathrm{e}^{-kx}\mathrm{d}x = \frac{1}{2}$	M1		**Or** $\mathrm{F}(x) = 1 - \mathrm{e}^{-kx}$
	$\left[-\mathrm{e}^{-kx}\right]_0^m = \frac{1}{2}$			
	$-\mathrm{e}^{-km} + 1 = \frac{1}{2}$	A1		$\frac{1}{2} = 1 - \mathrm{e}^{-km}$
	$\mathrm{e}^{-\mathrm{km}} = \frac{1}{2} \Rightarrow km = \ln 2 \Rightarrow m = \frac{\ln 2}{k}$	M1A1		
	$\ln 2 < 1 \Rightarrow m < \mu$	A1	5	
(b)(i)	$\mathrm{P}(T = 0) = \mathrm{e}^{-\frac{t}{\lambda}}$	B1	1	
(ii)(A)	$\mathrm{P}(T > t) = 1 - \mathrm{F}(t) = \mathrm{e}^{-\frac{t}{\lambda}}$	M1		
	$\mathrm{P}(T < t) = \mathrm{F}(t) = 1 - \mathrm{e}^{-\frac{t}{\lambda}} \quad t \ge 0$	A1	2	AG
(B)	$\mathrm{f}(t) = \mathrm{F}'(t) = \frac{1}{\lambda}\mathrm{e}^{-\frac{t}{\lambda}} \quad t \ge 0$	M1		
	$\Rightarrow$ Exponential distribution	A1	2	
		Total	**13**	

MS04 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\mathrm{E}(T)=\theta$	B1	1	
(b)(i)	$\mathrm{E}(T_1)=\mu$			
	$\Rightarrow \mathrm{E}(4\bar{X}-a\bar{Y})=4\mathrm{E}(\bar{X})-a\mathrm{E}(\bar{Y})=\mu$	M1		
	$\Rightarrow 4\mu - a\dfrac{\mu}{3}=\mu$	A1		
	$\Rightarrow 3=\dfrac{a}{3}\Rightarrow a=9$	A1	3	
(ii)	$\mathrm{Var}(T_1)=16\mathrm{Var}(\bar{X})+a^2\mathrm{Var}(\bar{Y})$	M1		
	$=16\dfrac{\sigma^2}{n}+81b\dfrac{\sigma^2}{n}$	M1		
	$=\dfrac{\sigma^2}{n}(16+81b)$	A1		AG
	$\mathrm{Var}(T_2)=\dfrac{64}{81}\times\dfrac{\sigma^2}{n}+\dfrac{9}{81}\times\dfrac{b\sigma^2}{n}$	M1		
	$=\dfrac{\sigma^2}{81n}(64+9b)$	A1	5	
(iii)	$\mathrm{RE}(T_2\,\mathrm{wrt}\,T_1)=\dfrac{\{\mathrm{Var}(T_2)\}^{-1}}{\{\mathrm{Var}(T_1)\}^{-1}}=\dfrac{\mathrm{Var}(T_1)}{\mathrm{Var}(T_2)}$	M1		
	$=\ 81\times\dfrac{(16+81b)}{(64+9b)}$	A1ft		ft on $\mathrm{Var}(T_2)$
	$81\times16+81^2b>64+9b$			
	$(\because \{9,16,64,81\}\subset Z^+$ and $b>0$ [given]$)$	E1ft		
	$\Rightarrow \mathrm{RE}(T_2\,\mathrm{wrt}\,T_1)>1$			
	$\Rightarrow T_2$ more efficient than T_1.	E1ft	4	Dependent on previous E1
		Total	**13**	
	TOTAL		**75**	