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Pure Core 4

Report on the Examination

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General

This paper proved to be quite demanding for the more able candidates with only about 15% of the candidates achieving 60 of the marks (80%) or more. However, the questions were generally accessible to all candidates with about 75% of the candidates achieving 30 marks (40%) or more. Some candidates achieved full marks over the whole paper. About 10% of the entry scored below 20 marks, showing little knowledge and understanding of the mathematics in the specification.

The first question was generally answered well, as were parts (a) of questions 2, 3, 4, 5 and 8. The exceptions were in questions 6 and 7. Question 7 proved to be the most difficult question with less than 5% of the candidates achieving full marks. Question 8(b) was also demanding with less than 5% of the candidates here achieving full marks, but candidates did score across the full mark range available for this question.

Question 1

Part (a) Virtually all candidates scored this one mark, and interpreted the result correctly for later in the question, although the interpretation was not required here. Even some of those who failed to give zero as the answer, used $(x+2)$ as a factor.

Part (b) Although most candidates knew they were to evaluate $\mathrm{f}\left(\frac{3}{2}\right)$ with only a few attempting $\mathrm{f}\left(\frac{2}{3}\right)$, only about half the candidates gave a convincing response. Some assumed the evaluation would come to zero without showing any arithmetic, whilst some other candidates didn't bring their evaluation to an appropriate conclusion.

Part (c) This was generally done well with correct factorisation being achieved by most candidates, although in cancelling down many then left their answer as $(2x-1)$ rather than its reciprocal. Methods of finding the third factor in the cubic expression varied from inspection, to long division to using undetermined coefficients; the latter usually requiring far more work to achieve the result and errors often being made in attempting long division. The factorisation of the quadratic expression in the numerator was often incorrect, with many candidates who made this mistake then being unclear as to what was cancelling with what.

Question 2

Part (a)(i) Virtually all candidates had this correct.
Part (a)(ii). Over 80% of the candidates were correct here, although a variety of methods were seen, ranging from taking the 25^{th} root of 25, to expressions involving logarithms. Those who failed to achieve both marks, usually started correctly, but did not continue to give an expression from which the correct value of k could be found.

Part (b) Most candidates were able to use logarithms correctly to solve their equation for t, with few using trial and improvement. However, many then interpreted the result incorrectly rounding down rather than up, not realising that as the result was over 55 it indicated the 56^{th} year and thus 2016. Some candidates used the wrong base year, 1985 being common, and some didn't interpret their answer as a year at all.

Question 3

Part (a)(i) About 75% of the candidates achieved both marks here. Some failed to achieve any marks through having the wrong sign on the second term, and some had the coefficient or sign wrong on the x^2 term.
Part (a)(ii) Most candidates knew they needed to take out a factor of 125, and of these most also knew it was to the power $\frac{1}{3}$ thus demonstrating where the given 5 in the expression came from. Those who failed to do this often just divided their expansion by 25 to obtain the given 5. Most went on to gain the method mark of using $\pm\frac{27}{125}$ in their expression from part (a)(i) or starting the binomial expansion again. Although most got the coefficients of 9 and 81 numerically correct, there were often sign errors. These might have become apparent in part (b). Very few candidates attempted to use the binomial expansion formula from the formula book, and those that did use it often made a mistake in evaluating the coefficients.

Part (b) Most candidates derived an appropriate value of x to use in their expression, but they couldn't achieve the expected answer from a wrong expression. Candidates should have expected to get a result close to 4.92 but this appeared not to have been considered by those who had an incorrect expansion, even by those who had a result over 5. Some candidates just used their calculator to find $\sqrt[3]{119}$ for no credit, whilst a few thought they were to substitute $\sqrt[3]{119}$ into their expansion.

Question 4

Most candidates scored the marks for the two derivatives with relatively few omitting the minus signs. Some candidates managed to drop the 2 from $\cos 2\theta$. Many candidates brought the commonly used parameter t into their expression using $\frac{\mathrm{d}x}{\mathrm{d}t} = -\sin\theta$ or similar, but were not penalised. Virtually all knew how to use the chain rule, but many got mixed up with the coefficients in both expanding $\sin 2\theta$ and simplifying the expression. The common error was to find $k = 3$. Some candidates got a result in terms of $\sin\theta$ and some got no result at all, but any value of k, however found or invented, was allowed in part (a)(ii).

Part (a)(ii) This part was generally done well with most candidates demonstrating they both knew and could use the relation between the gradients of the tangent and the normal, although a few found an equation of the tangent. The evaluation using $\theta = \frac{\pi}{3}$ was generally accurately done; however candidates had to have derived $k = 6$ correctly to gain the final mark for the equation of the normal.

Part (b) It was expected here that candidates would note the whole question was about the use of double angle formulae, and many did. Most got as far as at least trying to express $\sin^2 x$ in term of $\cos 2x$ and of those, many did it correctly, even if sometimes there was a mix of variables. The integration of $\cos 2x$ was usually accurate, with the common error being to double $\sin 2x$ rather than halve it. There were few sign errors. Virtually all candidates who achieved an integral knew how to use the limits, but the real test here was in handling $\sin 2\left(-\frac{\pi}{4}\right)$; many candidates lost credit through not showing their evaluation or getting the sign wrong.

However, many candidates, about 25% of the entry, went on to obtain the correct answer and give it in an exact form, although some did make a mistake with the signs, particularly the three minus signs involved with the lower limit. Of the few candidates who attempted the integral using parts most made little progress beyond the first line of working, not realising that in order to proceed with the integral they needed to change $\cos^2 x$ back to $\sin^2 x$, and manipulate the resulting expression.

Question 5

Part (a) Most candidates had the line correct in terms of a point on the line and its direction vector but the notation was often poor, with many not giving an equation by omitting $\mathbf{r} =$ or stating line = or similar. Some made sign errors in finding their direction vector.

Part (b)(i) Most candidates knew they needed to equate the two vector equations of the lines and solve simultaneously for λ and μ. Some candidates failed to change parameter and had three equations in μ and could make no further progress. Most candidates noticed that the equations had been set up so that it was easy to find λ first, and although they could calculate the intersection point just from the value of λ, most did find μ before doing so.

Most candidates attempted to check that all the equations were satisfied but many failed to bring this to a conclusion, just saying they were all satisfied, or even just $12 = 12$ from the third equation, without stating the implication; it was necessary to say 'therefore the lines intersect', or similar, to gain the mark. Most did find the intersection point correctly, although several made sign errors. The few who had an error in their solution to the simultaneous equations, appeared not to question their solution when finding the third equation didn't check.

Part (b)(ii) Attempts at this part of the question ranged from no attempt, or just a superficial diagram with no progress, to about 15% of the candidates giving a fully correct solution. Most knew they were to equate a scalar product to zero, but many chose at least one wrong vector. Those who drew a diagram usually chose the right vectors, but often used $\overrightarrow{OC}$ instead of $\overrightarrow{BC}$ in their calculation. Many who did attempt to calculate $\overrightarrow{BC}$ correctly made a sign or coefficient error which meant they could get neither the correct value of μ nor the correct coordinates for point C. No credit was given for expanding the scalar product until it was in terms of one parameter and thus soluble. A few candidates equated each term of their scalar product to zero, in order to find values for x, y and z or whatever they had called the coordinates of point C. The other fairly common misunderstanding was just to use a parameter p, as in $\overrightarrow{BC} = \begin{pmatrix} p-4 & p+1 & p-3 \end{pmatrix}^T$ rather than the equation of the line, to parameterise the point C. A few candidates attempted to use PythagorasTheorem to solve this problem, with little success, although some did give a fully correct solution.

Question 6

Part (a) About a third of the candidates could not evaluate the expression in e correctly, the term $\mathrm{e}^2\left(\frac{1}{\mathrm{e}}\right)^2$ causing many problems.

Part (b) Most candidates scored the marks for the implicit differentiation, with many using the product rule correctly. The common errors were to keep the y in the derivative, of $2y$, or not to have two terms in the product or to drop a 2. Although virtually all candidates differentiated x^2 correctly, the constant in terms of e caused problems for many; they "differentiated" it despite being told it was constant in the question. Most attempted to solve their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and of the many who had incorrectly written $\frac{\mathrm{d}y}{\mathrm{d}x} =$ in their opening line of implicit differentiation, most ignored it to get a correct expression, and thus it was condoned. Many were unable to obtain a fully correct solution as they still had their constant term, *C* in terms of e, in their expression.

Part (c) Candidates had to have the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ correct from part (b) in order to achieve both marks. Most substituted the given values of x and y into their expression and knew it should come to zero, and stated it was zero even if it clearly wasn't, rather than looking for an error. About 30% of the candidates gave a fully correct solution to the whole question. A few who otherwise had correct solutions, failed to bring the solution to a conclusion, although most did state a stationary point required the derivative to be zero, or equivalent.

Question 7

Part (a) Most candidates at least used $\frac{\mathrm{d}A}{\mathrm{d}t}$ in their attempt at a differential equation although some did use other letters. Very few equated to a constant of either sign. The term "constant rate" as given in the question just wasn't understood, with most taking the rate of change to be proportional to the surface area, or to time or functions of these such as an exponential function. Only about 5% of the candidates set up the correct equation, which meant they could achieve little in part (b)(i).

Part (b)(i) Those who had set up a correct equation in either $\pm k$ usually completed correctly, using integration and the conditions given. Some did attempt to integrate their differential equation, and this was give credit ; many did get as far as a value for k, in which algebraic and numerical errors were condoned for the method mark, but somewhat less than 15% of the candidates did get this far.
Part (b)(ii) Most candidates were able to interpret the given solution in part (b)(i) to answer this correctly with over 60% of the candidates gaining this 1 mark.

Question 8

Part (a) A repeated term partial fractions expression such as this is the most complex form in the specification, but most candidates approached it with confidence and demonstrated they knew how to find the coefficients. Most chose to use values of x to find C and A in that order, and then either a third value of x, or equated coefficients in order to find a value for B. This approach was generally more successful than those who equated coefficients to set up and solve three simultaneous equations. Most had the value of C correct, and similarly A although $\frac{1}{4}A = 1$ often resulted in $A = \frac{1}{4}$. However, about 60 % of the candidates had all of A, B and C correct.

Part (b) Many candidates ignored the partial fractions they had just found, and attempted all sorts of nonsensical ways of "integrating" the right hand side, some before even attempting to separate the variables, using what appeared to be an attempt to use parts. Some who did separate the y term to the left hand side, then attempted to multiply out the right hand side and integrate term by term, taking no notice of denominators. About 40% of the candidates gained no credit in part (b). However, many did know the first step in solving the differential equation was to separate the variables and use the partial fractions. The notation was often poor as was the algebra in handling the $2\sqrt{y}$, the latter often appearing as such on the left hand side and not as its reciprocal. Many mistakes were made in attempting to integrate the various terms. Indices caused problems for many with the term $\frac{1}{\sqrt{y}}$ being expressed as both $y^{\frac{1}{2}}$ and $y^{-\frac{1}{2}}$. Credit was given for integrating $\frac{k}{\sqrt{y}}$ to $2k\sqrt{y}$, for any k. In the two log integrals on the right hand side coefficient and sign errors were common and many thought the third term had a logarithmic integral as well. Others assumed this term integrated to $\frac{x}{1-x}$ as given in the answer. Some candidates did integrate this term correctly by inspection, whereas others used substitution, also correctly. Most candidates included a constant and tried to find it using the boundary conditions given, but they were only credited with the correct value if was exact, and found from a correct expression. The penultimate mark was for correctly combining the logarithm terms in a candidate's solution; many failed to do this correctly, multiplying terms when they should have been dividing or vice versa depending on signs. Those who had made errors and so didn't have all the coefficients equal to 2, did make some valiant efforts using powers to combine their terms, and got the credit if it was done correctly. Everything had to be correct to gain the last mark, and this was achieved by rather less than 5% of the candidates.

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