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General Certificate of Education (A-level)
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Mathematics MFP3

(Specification 6360)

Further Pure 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1	$k_1 = 0.1\times\left(3+\sqrt{4}\right)$ (=0.5)	M1		
	$k_2 = 0.1\,\mathrm{f}\,(3.1, 4.5)$	M1		
	$k_2 = 0.1\times\left(3.1+\sqrt{4.5}\right) = 0.522132...$	A1		PI accept 3dp or better
	$y(3.1) = y(3) + \frac{1}{2}[k_1 + k_2]$ $= 4 + 0.5 \times 1.022132....$	m1		Dep on previous two Ms and numerical values for k's
	$y(3.1) = 4.511$	A1	5	Must be 4.511
	Total		**5**	
2(a)	$p\cos x - q\sin x + 5p\sin x + 5q\cos x = 13\cos x$	M1		Differentiation and subst. into DE
	$p + 5q = 13$; $5p - q = 0$	m1		Equating coeffs.
	$p = \frac{1}{2}$; $q = \frac{5}{2}$	A1	3	OE Need both
(b)	Aux. eqn. $m + 5 = 0$	M1		PI. Or solving $y'(x)+5y=0$ as far as $y=$
	$(y_{CF} =)A\mathrm{e}^{-5x}$	A1		OE
	$(y_{GS} =)A\mathrm{e}^{-5x} + \frac{1}{2}\sin x + \frac{5}{2}\cos x$	B1F	3	c's CF + c's PI with exactly one arbitrary constant OE
	Total		**6**	
3(a)	$r + r\cos\theta = 2$	M1		
	$r + x = 2$	B1		$r\cos\theta = x$ stated or used
	$r = 2 - x$	A1		
	$x^2 + y^2 = (2-x)^2$	M1		$r^2 = x^2 + y^2$ used
	$y^2 = 4 - 4x$	A1	5	Must be in the form $y^2 = \mathrm{f}(x)$ but accept ACF for f(x).
(b)	Equation of line: $r\cos\theta = \frac{3}{4} \Rightarrow x = \frac{3}{4}$	M1		Use of $r\cos\theta = x$
		A1		$4x=3$ OE
	$y^2 = 4 - 4\left(\frac{3}{4}\right) = 1 \Rightarrow y = \pm 1$; [Pts $\left(\frac{3}{4}, \pm 1\right)$]	M1		
	Distance between pts (0.75, 1) and (0.75,−1) is 2	A1	4	
	Altn:			
	At pts of intersection, $r = \frac{5}{4}$ and $\cos\theta = \frac{3}{5}$ OE	(M1A1)		(M1 elimination of either r or θ) (For A condone slight prem approx.)
	Distance $PQ = 2r\sin\theta$	(M1)		Or use of cosine rule or Pythag.
	$= 2\times\frac{5}{4}\times\frac{4}{5} = 2$	(A1)		Must be from exact values.
	Total		**9**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
4	IF is $e^{\int -\frac{2}{x}\,dx}$	M1		Award even if negative sign missing
	$= e^{-2\ln(x)\ (+c)} = e^{\ln(x)^{-2}\ (+c)}$	A1		OE Condone missing *c*
	$= (k)x^{-2}$	A1F		Ft earlier sign error
	$x^{-2}\frac{dy}{dx} - 2x^{-3}y = 2xe^{2x}$			
	$\frac{d}{dx}\left(x^{-2}y\right) = 2x\,e^{2x}$	M1		LHS as d/dx(y×IF) PI
	$x^{-2}y = \int 2x\,e^{2x}\,dx$			
	$= \int x\ d(e^{2x}) = x\,e^{2x} - \int e^{2x}\,dx$	M1 A1		Integration by parts in correct dirn
	$x^{-2}y = xe^{2x} - \frac{1}{2}e^{2x}\ (+c)$	A1		ACF
	When $x = 2$, $y = e^4$ so $\frac{1}{4}e^4 = 2e^4 - \frac{1}{2}e^4 + c$	m1		Boundary condition used to find *c* after integration.
	$c = -\frac{5}{4}e^4$			
	$y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} - \frac{5}{4}x^2e^4$	A1	9	Must be in the form $y = f(x)$
	Total		**9**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{12x+8-12x-3}{(4x+1)(3x+2)} = \frac{5}{(4x+1)(3x+2)}$	B1	1	Accept $C = 5$
(b)	$\int \frac{10}{(4x+1)(3x+2)}\,\mathrm{d}x = 2\int\left(\frac{4}{4x+1} - \frac{3}{3x+2}\right)\mathrm{d}x$	M1		
	$= 2\left[\ln(4x+1) - \ln(3x+2)\right]$ $(+c)$	A1		OE
	$\mathrm{I} = \lim_{a\to\infty} \int_1^a \left(\frac{10}{(4x+1)(3x+2)}\right)\,\mathrm{d}x$	M1		∞ replaced by a and $\lim_{a\to\infty}$ (OE)
	$=2\lim_{a\to\infty}\left[\ln(4a+1) - \ln(3a+2)\right] - (\ln 5 - \ln 5)$			
	$=2\lim_{a\to\infty}\left[\ln\left(\frac{4a+1}{3a+2}\right)\right] = 2\lim_{a\to\infty}\left[\ln\left(\frac{4+\frac{1}{a}}{3+\frac{2}{a}}\right)\right]$	m1,m1		Limiting process shown. Dependent on the previous M1M1
	$= 2\ln\frac{4}{3} = \ln\frac{16}{9}$	A1	6	CSO
	Total		**7**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
6	Area = $\frac{1}{2}\int\left(2\sin 2\theta\sqrt{\cos\theta}\right)^2 \, \mathrm{d}\theta$	M1		Use of $\frac{1}{2}\int r^2 \, \mathrm{d}\theta$
	$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(4\cos\theta \sin^2 2\theta\right) \, \mathrm{d}\theta$	B1		$r^2 = 4\cos\theta\sin^2 2\theta$ or better
		B1		Correct limits
	$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(16\sin^2\theta \cos^3\theta\right) \, \mathrm{d}\theta$	M1		$\sin^2 2\theta = k\sin^2\theta\cos^2\theta$ (k>0)
	$=\int_0^{\frac{\pi}{2}}\left(8\sin^2\theta \text{ (1-}\sin^2\theta)\right) \, \mathrm{d}\sin\theta$	m1		Substitution or another valid method to integrate $\sin^2\theta\cos^3\theta$
	$=\left[\frac{8\sin^3\theta}{3}-\frac{8\sin^5\theta}{5}\right]_0^{\frac{\pi}{2}}$	A1F		Correct integration of $p\sin^2\theta\cos^3\theta$
	$=\left(\frac{8}{3}-\frac{8}{5}\right)-0=\frac{16}{15}$	A1	7	CSO AG
	<u>Alternatives for the last four marks</u>			
	Area = $\int_0^{\frac{\pi}{2}}\left(\cos\theta-\cos 4\theta\cos\theta\right) \, \mathrm{d}\theta$	(M1)		$2\cos\theta\sin^2 2\theta = \lambda\cos\theta+\mu\cos 4\theta\cos\theta$ $(\lambda,\mu\neq 0)$
	$\int\left(\cos 4\theta\cos\theta\right) \, \mathrm{d}\theta$	(m1)		Integration by parts twice or use of $\cos 4\theta\cos\theta=\frac{1}{2}(\cos 5\theta+\cos 3\theta)$
	$=-\frac{1}{15}(\cos 4\theta\sin\theta-4\sin 4\theta\cos\theta)$	(A1F)		Correct integration of $p\cos 4\theta\cos\theta$ [eg $p\left[\frac{1}{10}\sin 5\theta+\frac{1}{6}\sin 3\theta\right]$]
	Area = (1−0) + $\frac{1}{15}[(1-0)-(0)]=\frac{16}{15}$	(A1)		CSO AG $\{1-\frac{1}{10}+\frac{1}{6}=\frac{16}{15}\}$
	Total		**7**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\cos x + \sin x = 1 + x - \frac{1}{2}x^2 - \frac{1}{6}x^3$	B1	1	Accept coeffs unsimplified, even 3! for 6.
(ii)	$\ln(1+3x) = 3x - \frac{1}{2}(3x)^2 + \frac{1}{3}(3x)^3 = 3x - \frac{9}{2}x^2 + 9x^3$	B1	1	Accept coeffs unsimplified
(b)(i)	$y = e^{\tan x}, \quad \frac{dy}{dx} = \sec^2 x\ e^{\tan x}$	M1 A1		Chain rule ACF eg $y\sec^2 x$
	$\frac{d^2y}{dx^2} = 2\sec^2 x \tan x\ e^{\tan x} + \sec^4 x\ e^{\tan x}$	m1 A1		Product rule OE ACF
	$= \sec^2 x\ e^{\tan x}(2\tan x + \sec^2 x)$			
	$= \frac{dy}{dx}(2\tan x + 1 + \tan^2 x)$			
	$\frac{d^2y}{dx^2} = (1+\tan x)^2 \frac{dy}{dx}$	A1	5	AG Completion; CSO any valid method.
(ii)	$\frac{d^3y}{dx^3} = 2(1+\tan x)\sec^2 x \frac{dy}{dx} + (1+\tan x)^2 \frac{d^2y}{dx^2}$	M1		
	When $x = 0$, $\frac{d^3y}{dx^3} = 2(1)(1)(1)+(1)(1) = 3$	A1	2	CSO
(iii)	$y(0) = 1;\ y'(0) = 1;\ y''(0) = 1;\ y'''(0) = 3;$ $y(x) \approx y(0) + x\,y'(0) + \frac{1}{2}x^2 y''(0) + \frac{1}{3!}x^3 y'''(0)$	M1		
	$e^{\tan x} \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3$	A1	2	CSO AG
(c)	$\lim_{x\to 0}\left[\frac{e^{\tan x} - (\cos x + \sin x)}{x\ln(1+3x)}\right]$			
	$= \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + \frac{x^3}{2} - 1 - x + \frac{x^2}{2} + \frac{x^3}{6}}{x\left(3x - \frac{9}{2}x^2 + \ldots\right)}$	M1		Using series expns.
	$= \lim_{x\to 0}\left[\frac{x^2 + \frac{2}{3}x^3 + ..}{3x^2 - \frac{9}{2}x^3 \ldots}\right] = \lim_{x\to 0}\left[\frac{1 + \frac{2}{3}x + ..}{3 - \frac{9}{2}x \ldots}\right]$	m1		Dividing numerator and denominator by x^2 to get constant terms. OE following a slip.
	$= \frac{1}{3}$	A1	3	
	Total		**14**	

MFP3(cont)

Q	Solution	Marks	Total	Comments
8(a)	$\frac{dx}{dt}\frac{dy}{dx} = \frac{dy}{dt}$	M1		Chain rule
	$e^{t}\frac{dy}{dx} = \frac{dy}{dt} \Rightarrow \quad x\frac{dy}{dx} = \frac{dy}{dt}$	A1	2	CSO AG
(b)	$\frac{d}{dt}\left(x\frac{dy}{dx}\right) = \frac{d^2y}{dt^2}$; $\frac{dx}{dt}\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{d^2y}{dt^2}$	M1		OE $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{dt}{dx}\frac{d^2y}{dt^2}$
	$\frac{dx}{dt}\left(\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right) = \frac{d^2y}{dt^2}$	m1		Product rule (dep on previous M)
	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{d^2y}{dt^2}$	A1		OE
	$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2\ln x$ becomes			
	$\frac{d^2y}{dt^2} - x\frac{dy}{dx} - 3x\frac{dy}{dx} + 4y = 2\ln x$			
	$\Rightarrow \frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2\ln e^t$ (using (a))	m1		
	$\Rightarrow \frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = 2t$	A1	5	CSO AG
(c)	Auxl eqn $m^2 - 4m + 4 = 0$	M1		PI
	$(m-2)^2 = 0, \quad m = 2$	A1		PI
	CF: $(y_C =) \quad (At + B)e^{2t}$	M1		Ft wrong value of m provided equal roots and 2 arb. constants in CF. Condone x for t here
	PI Try $(y_P =) \quad at + b$	M1		If extras, coeffs. must be shown to be 0.
	$-4a + 4at + 4b = 2t \Rightarrow a = b = \frac{1}{2}$	A1		Correct PI. Condone x for t here
	GS $\{y\} = (At + B)e^{2t} + 0.5(t + 1)$	B1F	6	Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
(d)	$\Rightarrow y = (A\ln x + B)x^2 + 0.5(\ln x + 1)$	M1		
	$y = 1.5$ when $x = 1 \Rightarrow B = 1$	A1F		Ft one earlier slip
	$y'(x) = (A\ln x + B)\,2x + Ax + 0.5x^{-1}$	m1		Product rule
	$y'(1) = 0.5 \Rightarrow A = -2$	A1F		Ft one earlier slip
	$y = (1 - 2\ln x)x^2 + \frac{1}{2}(\ln x + 1)$	A1	5	ACF
	Total		**18**	
	TOTAL		**75**	