Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM04

Unit Mechanics 4

Thursday 24 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

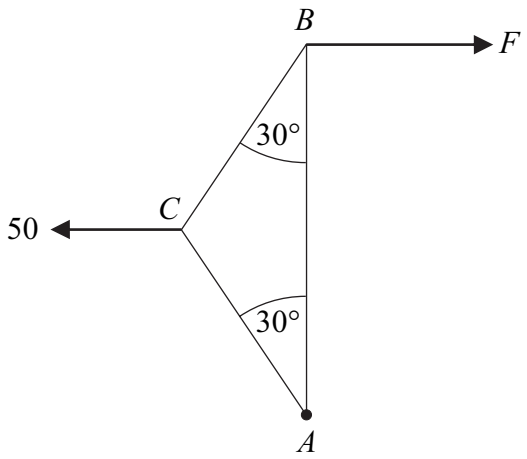
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A framework consists of three light inextensible smoothly jointed rods AB, BC and CA. Rods BC and CA each have length 2 metres and angle BAC = angle $ABC = 30°$. The framework is freely pivoted to a fixed support at A. Two horizontal forces, of magnitudes 50 newtons and F newtons, act on the framework. The system is in equilibrium in a vertical plane with AB vertical, as shown in the diagram.



(a) By taking moments about A, find F. *(2 marks)*

(b) State the magnitude and direction of the reaction force acting on the framework at A. *(2 marks)*

(c) (i) Find the magnitude of the force in the rod BC. *(2 marks)*

(ii) Find the magnitude of the force in the rod AB. *(2 marks)*

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2 Stephanie is practising a ballet dancing routine. As part of the routine, she rotates about a vertical axis through her centre of mass.

(a) When both her arms are fully extended, her moment of inertia about her axis of rotation is $0.6\,\text{kg}\,\text{m}^2$ and her angular speed is $3\,\text{rad}\,\text{s}^{-1}$. Find her angular momentum. *(2 marks)*

(b) Stephanie now lowers her arms until they are vertical. Her moment of inertia in this position is $0.45\,\text{kg}\,\text{m}^2$. Find her angular speed when her arms are vertical. *(2 marks)*

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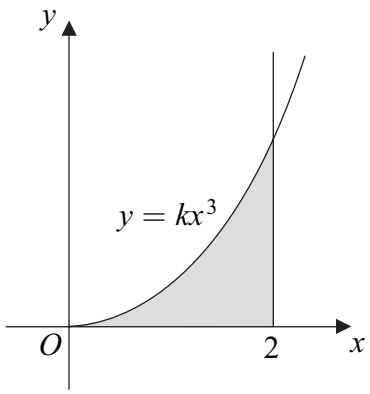
3 A uniform lamina is bounded by the curve $y = kx^3$, the line $x = 2$ and the x-axis, as shown in the diagram.



(a) Find an expression for the area of the lamina in terms of k. *(2 marks)*

(b) Find the x-coordinate of the centre of mass of the lamina. *(4 marks)*

(c) The y-coordinate of the centre of mass of the lamina is 8.

(i) Determine the value of k. *(4 marks)*

(ii) The lamina is freely suspended from the corner at the origin O. Find the acute angle between the straight edge at the point of suspension and the vertical. *(3 marks)*

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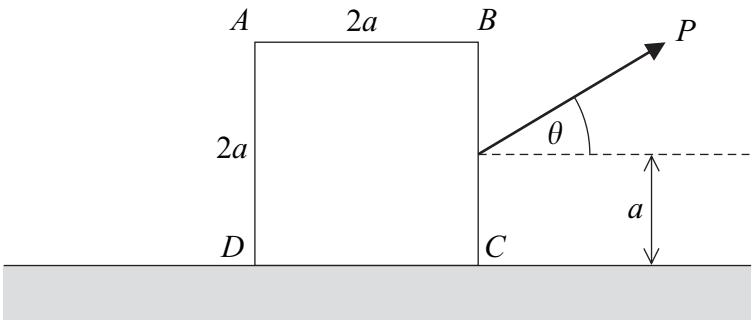
4 A uniform cube, of side $2a$ and mass m, rests on a rough horizontal plane. The diagram shows a vertical cross-section $ABCD$ through the centre of mass of the cube.



A force, of magnitude P, is applied at the mid-point of BC. This force acts in the plane $ABCD$ and makes an angle θ with the horizontal. The coefficient of friction between the cube and the plane is μ.

(a) In the case where the cube does not slide but is on the point of toppling about the edge through C, find an expression for P in terms of m, g and θ. *(3 marks)*

(b) In the case where the cube remains upright but is on the point of sliding along the plane, show that $P = \dfrac{\mu mg}{\cos\theta + \mu \sin\theta}$. *(4 marks)*

(c) Find an inequality that μ must satisfy if the cube slides before it topples. *(3 marks)*

(d) Would your answer in part **(c)** change if the mass of the cube were doubled? Explain why. *(2 marks)*

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5 A pulley Q is fixed to the edge of a smooth horizontal table. The pulley can rotate freely in a vertical plane about a horizontal axis through its centre.

A light inextensible string runs over the pulley, connecting a block R, of mass m, to a block P, of mass $3m$. The block R is held at rest on the table with block P hanging freely, as shown in the diagram.

Model the pulley as a uniform disc of mass $12m$ and radius r. Model the blocks as particles.

(a) Write down the moment of inertia of the pulley about the horizontal axis through its centre and perpendicular to its plane. *(1 mark)*

(b) Block R is released. In the subsequent motion, R moves on the table. The string between P and Q is vertical and has tension T_1. The string between Q and R is horizontal and has tension T_2. The pulley has angular acceleration $\ddot{\theta}$. Assume that the string does not slip and that R does not reach the pulley.

(i) Show that $T_1 - T_2 = 6mr\ddot{\theta}$. *(3 marks)*

(ii) Show that $\ddot{\theta} = \dfrac{3g}{10r}$. *(6 marks)*

(iii) Find T_1 and T_2 in terms of m and g. *(3 marks)*

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6 Two forces, $2\mathbf{i} + a\mathbf{k}$ and $-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, act at the points whose coordinates are $(1, 0, 3)$ and $(–1, 2, 0)$ respectively.

(a) Show that the resultant moment of these forces about the origin is $6\mathbf{i} + (9 - a)\mathbf{j} + 3\mathbf{k}$. *(5 marks)*

(b) This system is equivalent to a force $\mathbf{F}$ that acts at the origin together with a couple of magnitude 7.

(i) Show that one possible value of a is 7 and find the other possible value of a. *(4 marks)*

(ii) In the case where $a = 7$, find $\mathbf{F}$. *(2 marks)*

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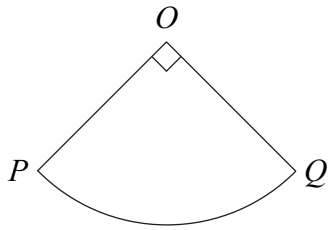
7 (a) Prove by integration that the moment of inertia of a uniform rod, of mass m and length $2a$, about an axis through one end of the rod and perpendicular to the rod is $\frac{4}{3}ma^2$. *(4 marks)*

(b) The diagram shows a simple model of a theme park swingboat ride.



The model consists of two uniform rods, OP and OQ, and a seat in the form of a circular arc PQ with centre O. Each rod has mass m and length $2a$. The seat is of mass $4m$ and angle $POQ = 90°$. The rods and the seat are rigidly fixed together and the model is free to rotate about a horizontal axis through O. The axis is perpendicular to the plane of OPQ.

(i) Show that the moment of inertia of the model about this axis is $\frac{56ma^2}{3}$. *(4 marks)*

(ii) The centre of mass of the model is at a distance of approximately $1.44a$ from the point O. The model is rotated until OQ is horizontal, with P vertically below O, and is then released from rest.

In the case where $a = 1.5$, find the greatest angular speed during the subsequent motion. *(6 marks)*

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END OF QUESTIONS