General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MS2B

Unit Statistics 2B

Wednesday 20 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Roger claims that, on average, his journey time from home to work each day is greater than 45 minutes.

The times, x minutes, of 30 randomly selected journeys result in $\bar{x} = 45.8$ and $s^2 = 4.8$.

Investigate Roger's claim at the 1% level of significance. *(5 marks)*

2 The error, in minutes, made by Paul in estimating the time that he takes to complete a college assignment may be modelled by the random variable T with probability density function

$$\mathrm{f}(t) = \begin{cases} \frac{1}{30} & -5 \leqslant t \leqslant 25 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find:

(i) $\mathrm{E}(T)$; *(1 mark)*

(ii) $\mathrm{Var}(T)$. *(1 mark)*

(b) Calculate the probability that Paul will make an error of magnitude at least 2 minutes when estimating the time that he takes to complete a given assignment. *(3 marks)*

3 Lorraine bought a new golf club. She then practised with this club by using it to hit golf balls on a golf range.

After several such practice sessions, she believed that there had been no change from 190 metres in the mean distance that she had achieved when using her old club.

To investigate this belief, she measured, at her next practice session, the distance, x metres, of each of a random sample of 10 shots with her new club. Her results gave

$$\sum x = 1840 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 1240$$

Investigate Lorraine's belief at the 2% level of significance, stating any assumption that you make. *(7 marks)*

4 Julie, a driving instructor, believes that the first-time performances of her students in their driving tests are associated with their ages.

Julie's records of her students' first-time performances in their driving tests are shown in the table.

Age	**Pass**	**Fail**
17–18	28	20
19–30	2	14
31–39	12	33
40–60	6	5

(a) Use a χ^2 test at the 1% level of significance to investigate Julie's belief. *(9 marks)*

(b) Interpret your result in part (a) as it relates to the 17–18 age group. *(1 mark)*

5 (a) In a remote African village, it is known that 70 per cent of the villagers have a particular blood disorder. A medical research student selects 25 of the villagers at random.

Using a binomial distribution, calculate the probability that more than 15 of these 25 villagers have this blood disorder. *(3 marks)*

(b) (i) In towns and cities in Asia, the number of people who have this blood disorder may be modelled by a Poisson distribution with a mean of 2.6 per 100 000 people.

A town in Asia with a population of 100 000 is selected. Determine the probability that at most 5 people have this blood disorder. *(1 mark)*

(ii) In towns and cities in South America, the number of people who have this blood disorder may be modelled by a Poisson distribution with a mean of 49 per **million** people.

A town in South America with a population of 100 000 is selected. Calculate the probability that exactly 10 people have this blood disorder. *(3 marks)*

(iii) The random variable T denotes the **total** number of people in the two selected towns who have this blood disorder.

Write down the distribution of T and hence determine $\mathrm{P}(T > 16)$. *(3 marks)*

6 (a) Ali has a bag of 10 balls, of which 5 are red and 5 are blue. He asks Ben to select 5 of these balls from the bag at random.

The probability distribution of X, the number of red balls that Ben selects, is given in **Table 1**.

Table 1

x	0	1	2	3	4	5
$\mathrm{P}(X = x)$	$\frac{1}{252}$	$\frac{25}{252}$	$\frac{100}{252}$	a	$\frac{25}{252}$	$\frac{1}{252}$

(i) State the value of a. *(1 mark)*

(ii) Hence write down the value of $\mathrm{E}(X)$. *(1 mark)*

(iii) Determine the standard deviation of X. *(5 marks)*

(b) Ali decides to play a game with Joanne using the same 10 balls. Joanne is asked to select 2 balls from the bag at random.

Ali agrees to pay Joanne 90p if the two balls that she selects are the same colour, but nothing if they are different colours. Joanne pays 50p to play the game.

The probability distribution of Y, the number of red balls that Joanne selects, is given in **Table 2**.

Table 2

y	0	1	2
$\mathrm{P}(Y = y)$	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$

(i) Determine whether Joanne can expect to make a profit or a loss from playing the game once.

(ii) Hence calculate the expected size of this profit or loss after Joanne has played the game 100 times. *(3 marks)*

7 Jim, a mathematics teacher, knows that the marks, X, achieved by his students can be modelled by a normal distribution with unknown mean μ and unknown variance σ^2.

Jim selects 12 students at random and from their marks he calculates that $\bar{x} = 64.8$ and $s^2 = 93.0$.

(a) (i) An estimate for the standard error of the sample mean is d. Show that $d^2 = 7.75$. *(2 marks)*

(ii) Construct an 80% confidence interval for μ. *(3 marks)*

(b) (i) Write down a confidence interval for μ, based on Jim's sample of 12 students, which has a width of 10 marks. *(1 mark)*

(ii) Determine the percentage confidence level for the interval found in part (b)(i). *(4 marks)*

8 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}(x^2 + 1) & 0 \leqslant x \leqslant 1 \\ (x-2)^2 & 1 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(3 marks)*

(b) Calculate $P(X \leqslant 1)$. *(3 marks)*

(c) Show that $E(X^2) = \dfrac{4}{5}$. *(5 marks)*

(d) (i) Given that $E(X) = \dfrac{19}{24}$ and that $\text{Var}(X) = \dfrac{499}{k}$, find the numerical value of k. *(3 marks)*

(ii) Find $E(5X^2 + 24X - 3)$. *(2 marks)*

(iii) Find $\text{Var}(12X - 5)$. *(2 marks)*

END OF QUESTIONS

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