



Teacher Support Materials 2009

Maths GCE

Paper Reference MS2B

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Dr Michael Cresswell, Director General.

Question 1

A machine fills bottles with bleach. The volume, in millilitres, of bleach dispensed by the machine into a bottle may be modelled by a normal distribution with mean μ and standard deviation 8.

A recent inspection indicated that the value of μ was 768. Yvonne, the machine's operator, claims that this value has not subsequently changed.

Zara, the quality control supervisor, records the volume of bleach in each of a random sample of 18 bottles filled by the machine and calculates their mean to be 764.8 ml.

Test, at the 5% level of significance, Yvonne's claim that the mean volume of bleach dispensed by the machine has not changed from 768 ml. (6 marks)

Student Response

1.	$\mu = 768 \text{ ml}$ $\bar{x} = 764.8 \text{ ml}$ $n = 18$ $\sigma = 8$ Two-tailed test	5% level of significance
B1	$H_0: \mu = 768$ $H_1: \mu \neq 768$	
B0	$18 - 1 = 17$ degrees of freedom	
M1	$t = \frac{764.8 - 768}{(8/\sqrt{18})}$	
A1	$= \frac{-3.2}{(8/\sqrt{18})}$ $= -1.6971$	
	$-2.110 < t < 2.110$	
A0	$\therefore H_0$ accepted (H_1 rejected)	
E0	Mean volume of bleach dispensed by machine Evidence support H_0 , mean volume of bleach dispensed by machine not changed from 768 ml.	
	(3)	

Commentary

The candidates as a whole usually fell in to two categories on this question; those who correctly used z-values and consequently usually gained full 6 marks and those who incorrectly used t-values who usually gained 3 marks.

In the example given, this candidate has correctly written down the hypotheses, evaluated the test value correctly but then incorrectly uses a critical **t value** instead of the required critical z-value.

It should be noted that when the population is stated as being Normal and when the population standard deviation (or variance) is known then a z-test should be used.

Mark scheme

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 768$	B1		(Both)
	$H_1: \mu \neq 768$			
	Test statistic: $z = \frac{764.8 - 768}{\frac{8}{\sqrt{18}}}$	M1		
	$= -1.70$	A1		(-1.697)
	$z_{crit} = \pm 1.96$	B1		($z_{crit} = 1.96$ or $z_{crit} = -1.96$)
	\Rightarrow Accept H_0	A1		
	No evidence at the 5% level of significance, to deny Yvonne's claim.	E1	6	
	Total		6	

Question 2

John works from home. The number of business letters, X , that he receives on a weekday may be modelled by a Poisson distribution with mean 5.0.

The number of private letters, Y , that he receives on a weekday may be modelled by a Poisson distribution with mean 1.5.

(a) Find, for a given weekday:

(i) $P(X < 4)$; *(2 marks)*

(ii) $P(Y = 4)$. *(2 marks)*

(b) (i) Assuming that X and Y are independent random variables, determine the probability that, on a given weekday, John receives a **total** of more than 5 business and private letters. *(3 marks)*

(ii) Hence calculate the probability that John receives a **total** of more than 5 business and private letters on at least 7 out of 8 given weekdays. *(3 marks)*

(c) The numbers of letters received by John's neighbour, Brenda, on 10 consecutive weekdays are

15 8 14 7 6 8 2 8 9 3

(i) Calculate the mean and the variance of these data. *(2 marks)*

(ii) State, giving a reason based on your answers to part (c)(i), whether or not a Poisson distribution might provide a suitable model for the number of letters received by Brenda on a weekday. *(2 marks)*

Student response

$X \sim P_0(5.0)$
 $Y \sim P_0(1.5)$

2 ai $P(X < 4) = P(X \leq 3) = 0.2650 = 0.265$ 3sf ✓

0 aii $\left\{ \begin{aligned} P(Y=4) &= P(Y \leq 4) - P(Y \leq 3) \\ &= \frac{(0.9857 + 0.4763)}{2} - \frac{(0.9463 + 0.9212)}{2} \\ &= 0.46845 - 0.43375 = 0.0347 \end{aligned} \right\}$ No!!

bi $X+Y \sim P_0(6.5)$

} $\begin{aligned} P(X+Y > 5) &= 1 - P(X+Y \leq 5) \\ &= 1 - 0.369 \\ &= 0.631 \end{aligned}$ ✓

bii $P((X+Y) > 5)^7 (P(X+Y < 5) + P(X+Y > 5))^8$

M1 $= 10.631^7 \times 0.369 + 0.631^8$ ✓
 A0 $= 0.6147 + 0.0251$
 A0 $= 0.6398$

B0 c. $\bar{x} = 9 \times \frac{\sum x}{n}$
 $s^2 = \frac{\sum (xc - \bar{x})^2}{n-1}$

B0 $= \frac{242}{9}$
 $= 26.889 = 26.9$ 3sf ✗

Commentary

Candidates are encouraged to use the tables provided where ever this is appropriate. However, when the value of λ does not appear in the tables then use of the formula is required. Unfortunately, in part (a)(ii), this candidate thought incorrectly that, when a value of 1.5 (not in the tables) was given, the use of the average of the values found under $\lambda = 1.4$ and $\lambda = 1.6$ was an appropriate course of action. In part (b)(ii) very few correct expressions were seen. As with this candidate, many omitted the binomial coefficient whilst others only gave 0.631^8 or two non-binomial terms.

Mark Scheme

2(a)(i)	$X \sim \text{Po}(5.0)$ $\Rightarrow P(X < 4) = P(X \leq 3)$ $= 0.265$	B2	2	(0.440 to 0.441) for B1 CAO
(ii)	$Y \sim \text{Po}(1.5)$ $\Rightarrow P(Y = 4) = \frac{e^{-1.5} \times (1.5)^4}{4!}$ $= 0.0471$	M1 A1	2	(0.047 to 0.0471)
2(b)(i)	$T = X + Y \sim \text{Po}(6.5)$ $\Rightarrow P(T > 5) = 1 - P(T \leq 5)$ $= 1 - 0.369$ $= 0.631$	B1 B1		(1 - 0.2237) or (1 - 0.5265)
(ii)	$p = {}^8C_7 (0.631)^7 (0.369) +$ $(0.631)^8$ $p = 0.11758 + 0.02513$ $= 0.143$	M1ft A1ft A1	3	ft on their p from (b)(i) Either part attempted (both parts correct) AWFW 1.142 to 0.143 (CAO)
(c)(i)	Mean = 8 Variance = $s^2 = 16.9$ (sample variance = 15.2)	B1 B1	2	CAO (AWRT)
(ii)	Poisson not a good model for data Mean \neq Variance	B1dep B1	2	
Total			14	

Question 3

A sample survey, conducted to determine the attitudes of residents to a proposed reorganisation of local schools, gave the following results.

		Against reorganisation	Not against reorganisation
Age of resident	16–17	9	2
	18–21	17	10
	22–49	115	90
	50–65	41	34
	Over 65	3	4

Use a χ^2 test, at the 5% level of significance, to determine whether there is an association between the ages of residents and their attitudes to the proposed reorganisation of local schools.

(12 marks)

Student Response

Q3.		Against		Not Against		Total
	16-17	9	6.26	2	4.74	11
	18-21	17	15.37	10	11.63	27
	22-49	115	116.69	90	88.31	205
	50-65	41	42.69	34	32.31	75
	65+	3	3.99	4	3.01	7
	Total	185		140		325

Lea
blar

H_0 (Hypoth)
Some θ E-values < 5

B1		Against		Not Against		Total
M1	16-21	26	21.63	12	16.37	38
A1	22-49	115	116.69	90	88.31	205
M1 A1	50+	44	46.68	38	35.32	82
M1 A1	Total.	185		140		325

$\frac{(O-E)^2}{E}$

A1 $= (0.88289) + (0.02448) + (0.15386) + (1.1666) + (0.03234) + (0.20335)$

$= 2.46356$

$= 2.46$

B1 $v = (3-1)(2-1)$

$= 2$ at 5%

B1 $= 5.991$

A0 $\therefore 2.46356 < 5.991$

\therefore there is ^{significance between} ~~no association between~~ ?

For residents and attitudes at 5% significance level.

(9)

Commentary

This question is traditionally one which provides the candidates with a good source of marks, whatever their standard.

Unfortunately, as with this candidate, there are some who fail to state the null hypothesis which is the minimum requirement here. However, they still feel that they are justified in conclusion to state 'Accept H_0 ' having not stated what H_0 actually is!

Others simply interchange the null and alternative hypotheses.

Mark Scheme

Q	Solution	Marks	Total	Comments																																																																																												
3	<p>H_0 : no association between age and attitude to school reorganisation H_1 : association between age and attitude to school reorganisation</p> <table border="1"> <thead> <tr> <th>Age</th> <th colspan="2">Against</th> </tr> <tr> <td></td> <th>O_i</th> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>16 - 17</td> <td>9</td> <td>$6\frac{17}{65}$</td> </tr> <tr> <td>18 - 21</td> <td>17</td> <td>$15\frac{24}{65}$</td> </tr> <tr> <td>22 - 49</td> <td>115</td> <td>$116\frac{9}{13}$</td> </tr> <tr> <td>50 - 65</td> <td>41</td> <td>$42\frac{9}{13}$</td> </tr> <tr> <td>> 65</td> <td>3</td> <td>$3\frac{64}{65}$</td> </tr> <tr> <td>Total</td> <td>185</td> <td>185</td> </tr> </tbody> </table> <table border="1"> <thead> <tr> <th>Age</th> <th colspan="2">Not Against</th> </tr> <tr> <td></td> <th>O_i</th> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>16 - 17</td> <td>2</td> <td>$4\frac{48}{65}$</td> </tr> <tr> <td>18 - 21</td> <td>10</td> <td>$11\frac{4}{65}$</td> </tr> <tr> <td>22 - 49</td> <td>90</td> <td>$88\frac{4}{13}$</td> </tr> <tr> <td>50 - 65</td> <td>34</td> <td>$32\frac{4}{13}$</td> </tr> <tr> <td>> 65</td> <td>4</td> <td>$3\frac{1}{65}$</td> </tr> <tr> <td>Total</td> <td>140</td> <td>140</td> </tr> </tbody> </table> <p>Row totals: $\overline{11.27}$ 205, $\overline{75.7}$ (325) Column totals: 185, 140 (325) $E_i's < 5$ \therefore combine cells 16 – 17 and 18 – 21 also 50 – 65 and 'over 65' to give:</p> <table border="1"> <thead> <tr> <th>O_i</th> <th>E_i</th> <th>$\alpha = O_i - E_i$</th> <th>$\frac{\alpha^2}{E_i}$</th> </tr> </thead> <tbody> <tr> <td>26</td> <td>21.63</td> <td>4.369</td> <td>0.8825</td> </tr> <tr> <td>115</td> <td>116.69</td> <td>-1.692</td> <td>0.0245</td> </tr> <tr> <td>44</td> <td>46.68</td> <td>-2.677</td> <td>0.1535</td> </tr> <tr> <td>12</td> <td>16.37</td> <td>-4.369</td> <td>1.1662</td> </tr> <tr> <td>90</td> <td>88.31</td> <td>1.692</td> <td>0.0324</td> </tr> <tr> <td>38</td> <td>35.32</td> <td>2.677</td> <td>0.2029</td> </tr> <tr> <td>325</td> <td>325</td> <td></td> <td>2.462</td> </tr> </tbody> </table> <p>$X^2 = 2.462$ $\nu = 2$ $\chi^2_{\nu=2} (0.95) = 5.991$ Accept H_0 No real evidence at 5% level of significance to suggest any association between age and attitude to school reorganisation.</p>	Age	Against			O_i	E_i	16 - 17	9	$6\frac{17}{65}$	18 - 21	17	$15\frac{24}{65}$	22 - 49	115	$116\frac{9}{13}$	50 - 65	41	$42\frac{9}{13}$	> 65	3	$3\frac{64}{65}$	Total	185	185	Age	Not Against			O_i	E_i	16 - 17	2	$4\frac{48}{65}$	18 - 21	10	$11\frac{4}{65}$	22 - 49	90	$88\frac{4}{13}$	50 - 65	34	$32\frac{4}{13}$	> 65	4	$3\frac{1}{65}$	Total	140	140	O_i	E_i	$\alpha = O_i - E_i$	$\frac{\alpha^2}{E_i}$	26	21.63	4.369	0.8825	115	116.69	-1.692	0.0245	44	46.68	-2.677	0.1535	12	16.37	-4.369	1.1662	90	88.31	1.692	0.0324	38	35.32	2.677	0.2029	325	325		2.462	<p>B1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>ml</p> <p>A1 B1 B1ft A1ft</p> <p>E1ft</p>	<p>12</p> <p>12</p>	<p>E's attempted correctly (at least 6 E's)</p> <table border="1"> <thead> <tr> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>6.262</td> </tr> <tr> <td>15.369</td> </tr> <tr> <td>116.692</td> </tr> <tr> <td>42.692</td> </tr> <tr> <td>3.985</td> </tr> </tbody> </table> <table border="1"> <thead> <tr> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>4.738</td> </tr> <tr> <td>11.631</td> </tr> <tr> <td>88.308</td> </tr> <tr> <td>32.308</td> </tr> <tr> <td>3.015</td> </tr> </tbody> </table> <p>Totals correct</p> <p>Attempt at combining rows Correctly</p> <p>Final column attempted (dep M1)</p> <p>2.4 to 2.5</p> <p>On their ν</p> <p>(context)</p>	E_i	6.262	15.369	116.692	42.692	3.985	E_i	4.738	11.631	88.308	32.308	3.015
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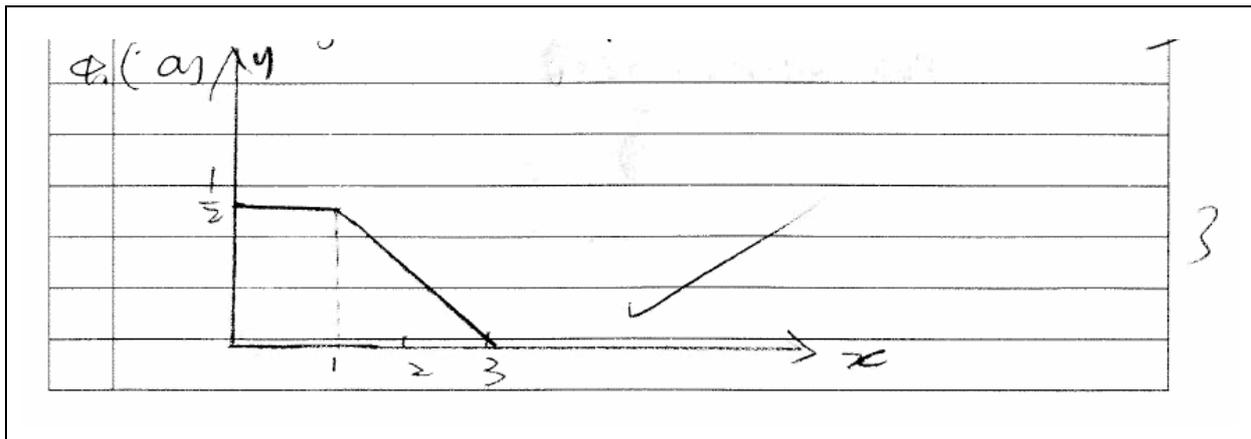
Question 4

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . (3 marks)
- (b) Explain why the value of η , the median of X , is 1. (2 marks)
- (c) Show that the value of μ , the mean of X , is $\frac{13}{12}$. (4 marks)
- (d) Find $P(X < 3\mu - \eta)$. (3 marks)

Student Response



0
↓

$$(b) f(n) = \frac{3-n}{4} = 0.5$$

$$3-n=2$$

$$\therefore n=1$$

$$(c) F_{\text{mean}} = \int_0^1 \frac{x}{4} dx + \int_1^3 \frac{(3-x)x}{4} dx$$

~~$$= \left[\frac{x^2}{8} \right]_0^1 + \left[\frac{3x^2}{8} - \frac{x^3}{12} \right]_1^3$$~~

~~$$= \frac{1}{8} + \left(\frac{3 \times 3^2}{8} - \frac{3^3}{12} - \frac{1}{8} + \frac{1}{12} \right)$$~~

~~$$= \frac{1}{8} + \left(\frac{27}{8} - \frac{27}{4} - \frac{1}{8} + \frac{1}{12} \right)$$~~

$$= \int_0^1 \frac{x}{4} dx + \int_1^3 \frac{3x-x^2}{4} dx$$

$$= \left[\frac{x^2}{8} \right]_0^1 + \left[\frac{3x^2}{8} - \frac{x^3}{12} \right]_1^3$$

$$= \frac{1}{8} + \left[\frac{3 \times 3^2}{8} - \frac{3^3}{12} - \frac{1}{8} + \frac{1}{12} \right]$$

$$= \frac{1}{8} + \frac{27}{8} - \frac{27}{4} - \frac{1}{8} + \frac{1}{12}$$

$$= \frac{13}{12} \text{ as required}$$

~~$$(d) P(X < 3U - 1) = P(X < 3 \times \frac{13}{12} - 1)$$~~

~~$$= P(X < \frac{13}{4})$$~~

~~$$= \frac{1}{8} + \left(\frac{27}{8} - \frac{27}{4} - \frac{1}{8} + \frac{1}{12} \right)$$~~

~~$$= \frac{1}{8} + \left(\frac{27}{8} - \frac{27}{4} - \frac{1}{8} + \frac{1}{12} \right)$$~~

~~$$= \frac{1}{8} + \frac{13}{12}$$~~

~~$$= \frac{13}{12}$$~~

$$P(X < 3U - 1) = P(X < 3 \times \frac{13}{12} - 1)$$

$$= P(X < \frac{13}{4})$$

$$\text{when } x = \frac{13}{4}, P(x) = \frac{3 - \frac{13}{4}}{4} = \frac{3}{16}$$

$$P(X < 3U - 1) = \frac{1}{8} + \frac{3}{16} + \left(\frac{1}{2} \right) \times \left(\frac{13}{4} - 1 \right)$$

$$= \frac{1}{8} + \frac{55}{128}$$

~~$$= \frac{1}{8} + \frac{55}{128} = 0.6875$$~~

$$= \frac{1}{8} + \frac{55}{128} = \frac{19}{128} \approx 0.1484$$

4

3

10

Commentary

Most candidates were able to draw the sketch in part (a).

In part (b), many candidates, like this candidate, simply showed that $f(1) = 0.5$ or solved the equation $f(x) = 0.5$ and then stated that the median = 1. This did not answer the question.

Candidates should have attempted to use $F(\eta) = 0.5$ or relate area and probability to gain these marks.

Part (c) was usually attempted correctly.

In part (d), candidates used integration to assist them in gaining the correct answer.

However, although many achieved the right result by doing so, this is not the most efficient way of tackling a question where the graph of f is made up from straight lines. This candidate used relevant areas (and no integration) to achieve the required answer.

Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)	Sketch: 	B3	3	1 for straight line $0 \leq x \leq 1$ from (0, 0.5) to (1, 0.5) 1 for straight line $1 \leq x \leq 3$ from (1, 0.5) to (3, 0) 1 for axes [must have at least (0,0.5) (1,0) and (3,0) labelled]
(b)	$P(X \leq \eta) = F(\eta) = 0.5$ $(\Rightarrow \eta = 1 \text{ (from graph)})$	M1 A1	2	AG
(c)	$\mu = E(X) = \int_0^1 \left(\frac{x}{2}\right) dx + \int_1^3 x \left(\frac{3-x}{4}\right) dx$ $= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{4} \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$ $= \frac{1}{4} + \frac{1}{4} \left[\left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - \frac{1}{3}\right) \right]$ $= \frac{1}{4} + \frac{5}{6} \quad (0.25 + 0.83\bar{3})$ $= 1 \frac{1}{12}$	M1 A1 ml A1	4	Both integrals stated Either Correct limits used on both integrals +combined dep M1 (CAO)
(d)	Area of Δ $= P\left(X > 2\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4}$ $= \frac{3}{32} \times \frac{3}{4} = \frac{9}{128}$ $\therefore P\left(X < 2\frac{1}{4}\right) = 1 - \frac{9}{128}$ $= \frac{119}{128} (0.9296875)$	M1ft M1ft A1	3	Alternative: For $1 \leq x \leq 3$ $F(x) = 1 - \frac{1}{8}(3-x)^2$ \Downarrow $F\left(2\frac{1}{4}\right) = 1 - \frac{1}{8} \times \frac{9}{16}$ $= \frac{119}{128}$ CAO

4(d)	or		Alternative
	$\int_{2\frac{1}{4}}^3 \frac{3-x}{4} dx \left(= \frac{9}{128} \right)$ <p style="text-align: right;">M1 ft</p> $= 1 \int_{2\frac{1}{4}}^3 \frac{3-x}{4} dx$ <p style="text-align: right;">M1 ft</p> $= 1 - \frac{1}{4} \left[3x - \frac{x^2}{2} \right]_{2\frac{1}{4}}^3$ $= 1 - \frac{1}{4} \left[9 - \frac{9}{2} - \frac{27}{4} + \frac{81}{32} \right]$ $= 1 - \frac{1}{4} \times \frac{9}{32} = \frac{119}{128}$ <p style="text-align: right;">A1</p> <p>or $(1 - 0.0703125 = 0.9296875)$</p>		$f\left(2\frac{1}{4}\right) = \frac{3}{16} = 0.1875$ $P(X < 3\mu - \eta) = P\left(X < 2\frac{1}{4}\right)$ $= \frac{1}{2} + \frac{1}{2} \left(\frac{3}{16} + \frac{1}{2} \right) \times 1 \frac{1}{4}$ <p style="text-align: right;">M1ft</p> $= \frac{1}{2} + \frac{55}{128} (0.4296875)$ <p style="text-align: right;">M1ft</p> $= \frac{119}{128} (0.930)$ <p style="text-align: right;">A1</p>
	Total		12

Question 5

Joanne has 10 identically-shaped discs, of which 1 is blue, 2 are green, 3 are yellow and 4 are red. She places the 10 discs in a bag and asks her friend David to play a game by selecting, at random and without replacement, two discs from the bag.

(a) Show that:

- (i) the probability that the two discs selected are the same colour is $\frac{2}{9}$; (2 marks)
- (ii) the probability that exactly one of the two discs selected is blue is $\frac{1}{5}$. (2 marks)

(b) Using the discs, Joanne plays the game with David, under the following conditions:

If the two discs selected by David are the same colour, she will pay him 135p.
If exactly one of the two discs selected by David is blue, she will pay him 145p.
Otherwise David will pay Joanne 45p.

- (i) When a game is played, X is the amount, in pence, **won by David**. Construct the probability distribution for X , in the form of a table. (2 marks)
- (ii) Show that $E(X) = 33$. (2 marks)
- (c) Joanne modifies the game so that the amount per game, Y pence, that **she wins** may be modelled by

$$Y = 104 - 3X$$

- (i) Determine how much Joanne would expect to win if the game is played 100 times. (3 marks)
- (ii) Calculate the standard deviation of Y , giving your answer to the nearest 1p. (4 marks)

Student Response

Leave blank

5. (a) i) $P(2 \text{ same } \del{27/100} \text{ colours})$
 $= P(2 \text{ green or } 2 \text{ yellow or } 2 \text{ red})$
 $= \frac{2}{10} \times \frac{1}{9} + \frac{2}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9} = \frac{2}{9} \text{ as required}$ 2

ii) $P(1 \text{ blue}) = P(1 \text{ blue } \cap \text{ other colour})$
 $+ P(\text{other colour } \cap \text{ 1 blue})$
 $= \frac{1}{10} \times \frac{9}{9} + \frac{9}{10} \times \frac{1}{9} = \frac{1}{5} \text{ as required}$ 2

(b) i)

money owned by David	135p	145p	-45p
Probability	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$

 2

ii) $E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} - 45 \times \frac{26}{45}$
 $= 59 - 26$
 $= 33$ 2

(c) i) $E(Y) = 104 - 3 \times 33 = 5$ 3
 Expect to win $= 5 \times 100 = 500$ pence.

ii) Joanne would expect to win if the game is played 100 times.

iii) $VAR(X) = E(X^2) - \mu^2$
 $= 9425 - 33^2 = 8336$ ✓
 $VAR(Y) = 9 \times 8336 = 75024$ ✓
 Standard deviation $= \sqrt{75024} = 273.905931 \dots$ (15)
 ≈ 274 pence

Commentary

The basic probability in part (a) was not done as well as expected. The answers are given and a lot of attempts were not at all convincing. This candidate, however, did tackle the question in the correct way.

This candidate did actually produce a fully correct solution to the whole question.

Mark Scheme

5(a)(i)	$P(\text{GG or YY or RR})$ $= \frac{2}{10} \times \frac{1}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9}$ $= \frac{2}{9}$	M1 A1	2	(AG)												
(ii)	$P(\text{B}\bar{\text{B}} \text{ or } \bar{\text{B}}\text{B}) = \frac{1}{10} \times \frac{9}{9} + \frac{9}{10} \times \frac{1}{9}$ $= \frac{1}{5}$	M1 A1	2	$\frac{1}{10} + \frac{9}{10} \times \frac{1}{9}$ (AG)												
(b)(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Same</th> <th>1 Blue</th> <th>Neither</th> </tr> </thead> <tbody> <tr> <th>x</th> <td>135</td> <td>145</td> <td>-45</td> </tr> <tr> <th>$P(X=x)$</th> <td>$\frac{2}{9}$</td> <td>$\frac{1}{5}$</td> <td>$\frac{26}{45}$</td> </tr> </tbody> </table>		Same	1 Blue	Neither	x	135	145	-45	$P(X=x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$	B1 B1	2	
	Same	1 Blue	Neither													
x	135	145	-45													
$P(X=x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$													
(ii)	$E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} + (-45) \times \frac{26}{45}$ $= 29 + 30 - 26$ $= 33 \text{ pence}$	M1 A1	2	Multiply two rows of their table from (b)(i) AG												
(c)(i)	$E(Y) = 104 - 3E(X)$ $= 104 - 3 \times 33$ $= 5 \text{ pence}$ <p>\therefore Joanne would expect to win £5</p>	M1 A1 A1	3	OE (eg 500p)												
5(c)(ii)	$E(X^2) = 9425$ $\text{Var}(X) = 9425 - 33^2 = \mathbf{8336}$ $\text{Var}(Y) = 9 \times \text{Var}(X)$ $= 9 \times 8336$ $= 75024$ <p>\Rightarrow standard deviation (Y) = 274 pence</p>	B1 B1 M1 A1	4	(4205 + 4050 + 1170) $sd(X) = 91.30$ $9 \times (\text{their Var}(X) > 0)$ or $3 \times (\text{their } sd(X))$ 273.9p or £2.74												
	Total		15													

Question 6

Bishen believes that the mean weight of boxes of black peppercorns is 45 grams. Abi, thinking that this is not the case, weighs, in grams, a random sample of 8 boxes of black peppercorns, with the following results.

44 44 43 46 42 40 43 46

- (a) (i) Construct a 95% confidence interval for the mean weight of boxes of black peppercorns, stating any assumption that you make. *(6 marks)*
- (ii) Comment on Bishen's belief. *(2 marks)*
- (b) (i) Abi claims that the mean weight of boxes of black peppercorns is less than 45 grams. Test this claim at the 5% level of significance. *(6 marks)*
- (ii) If Bishen's belief is true, state, with a reason, what type of error, if any, may have occurred when conclusions to the test in part (b)(i) were drawn. *(2 marks)*

Student Response

6) $\bar{x} = 43.5$ $\hat{\sigma} = 2$ $n < 30 \therefore$ use t value
 4) $t_{crit} = 2.365$ ✓
 $v = 7$
 $x = \cancel{43.5} \pm 2.365 \times \frac{2}{\sqrt{8}}$ ✓
 $= (41.8, 45.2)$ ✓
 Assuming the parent population is normal

2) i) The boxes of peppercorns are probably not 45g because ~~the~~ 45g is only just inside the upper limit of the confidence interval.

b) i) $H_0: M = 45$ / one tailed test
 $H_1: M < 45$
 $\bar{x} = 43.5$ $\hat{\sigma} = 2$
 test statistic = $\frac{\bar{x} - \mu}{\frac{\hat{\sigma}}{\sqrt{n}}}$
 $= \frac{43.5 - 45}{\frac{2}{\sqrt{8}}}$
 $= -2.12$ ✓
 $t_{crit} = -1.895$
 $-2.12 < -1.895$ hence we accept H_1 , reject H_0 ✓ there is sufficient evidence to suggest that ~~M < 45~~ $M < 45$, our result is significant (at the 5% level)

2) ii) A type 1 error because we have rejected H_0 when it is true, i.e. we have rejected Bishar's belief $M = 45$ when M does = 45.

(16)

Commentary

In part (a)(i) many candidates either ignored the request for an assumption or stated one which was inappropriate. The most common incorrect statements were: 'It is normally distributed' **or** 'The sample is normally distributed'. This candidate, however managed to complete part (a)(i) correctly including a correctly stated assumption.
 In part (a)(ii), far too many of the statements were too positive. 'Confidence interval contains 45 so Bishen's belief **is** true' was not thought to be an appropriate response.
 In part (b) the wrong tail was often used with $t_{crit} = +1.895$ often seen.
 This candidate completed the solution correctly.

Mark Scheme

6(a)(i)	$\bar{x} = 43.5$ $s = 2 \quad (s^2 = 4)$ Assumption: Weights of boxes are normally distributed $t_{0.975} = 2.365$ 95% CI for μ : $43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$ 43.5 ± 1.6723 $\Rightarrow (41.8, 45.2)$	B1 B1 B1 B1 M1 A1	6	(AWRT)
(ii)	CI contains mean (45) Bishen's belief probably justified or [Since 45 within CI] but close to upper limit, there is some evidence that Bishen's Belief is untrue [but the evidence is not significant at 5%.] (75% of sample less than 45grams)	B1 dep B1 dep (B1)	2	Must be clear use of 45 and not 43.5
6(b)(i)	$H_0: \mu = 45$ $H_0: \mu < 45$ Test statistic: $t = \frac{43.5 - 45}{\frac{2}{\sqrt{8}}}$ $= -2.12$ $\nu = 7 \Rightarrow t_{crit} = -1.895$ \Rightarrow Reject H_0 Evidence at the 5% level of significance . to support Abi's claim that mean content < 45 grams	B1 M1 A1 B1 A1 E1	6	(both) $P(t_7 < -2.12.) = 0.035791 < 0.05$
(ii)	Type I error have/may have rejected H_0 when H_0 true or No error have/may have accepted H_0 when H_0 true	B1 B1 (B1) (B1)	2	Clear statement Clear statement
Total			16	