Teacher Support Materials 2009

Maths GCE

Paper Reference MM03

Question 1

1 A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h.

It is thought that h depends on m, u, the acceleration due to gravity g, and a dimensionless constant k, such that

$$h = km^{\alpha}u^{\beta}g^{\gamma}$$

where α , β and γ are constants.

By using dimensional analysis, find the values of α , β and γ . *(5 marks)*

Student Response

1. height $= L$
(h)

mass $= M$
(m)

speed $= LT^{-1}$
(u)

gravity $= LT^{-2}$
(g)

$$h = Km^{\alpha}u^{\beta}g^{\gamma}$$

$$L = M^{\alpha}(LT^{-1})^{\beta}(LT^{-2})^{\gamma} \checkmark$$

$\alpha = 0$ ✓

compare L's : $1 = \beta + \gamma$ ✓

compare T's : $0 = -\beta - 2\gamma$ ✓

$\beta = -2\gamma$ ✓

$1 = -2\gamma + \gamma$ ✓

$1 = -\gamma$

$\gamma = -1$ ✓ $\quad 1 = \beta + (-1)$

$\beta = 2$ ✓

5

$\underline{\underline{\alpha = 0, \beta = 2, \gamma = -1}}$

Commentary

It is worth noting that this candidate presented their answer neatly and clearly.
The candidate first wrote down the dimensions of the height, mass, speed and gravitational acceleration before substituting these into the given relationship. The candidate then equated the corresponding indices on the right and the left of the relationship. The resulting equations were then solved simultaneously to arrive at the required values for α, β and γ .

Mark scheme

1	$L = M^{\alpha}(LT^{-1})^{\beta}(LT^{-2})^{\gamma}$	M1A1		
	$\beta + \gamma = 1$			
	$-\beta - 2\gamma = 0$			
	$\alpha = 0$	m1		Getting three equations
	$\gamma = -1$	m1		Solution
	$\beta = 2$	A1F	5	
	Total		**5**	

Question 2

2 A particle is projected from a point O on a horizontal plane and has initial velocity components of $2\,\mathrm{m\,s^{-1}}$ and $10\,\mathrm{m\,s^{-1}}$ parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

Student Response

Leave blank

2a $x = 2t \Rightarrow t = \frac{x}{2}$

$y = 10t + \frac{1}{2}(-g)t^2$

$y = \frac{10x}{2} + \frac{-gx^2}{2(2)^2}$

$y = 5x - \frac{gx^2}{8}$

4

b $1 = 5x - \frac{gx^2}{8} \Rightarrow \frac{gx^2}{8} - 5x + 1 = 0$

$\Rightarrow x = \frac{5 \pm \sqrt{(-5)^2 - 4(\frac{g}{8})(1)}}{(\frac{g}{4})}$

$= 3.87073565\ldots, \ 0.21089699\ldots$ m.

$\Rightarrow$ Distance travelled $= 3.8707\ldots - 0.2108\ldots$

$= 3.65983865\ldots$

$= 3.66$ (3SF) m.

4

c. $t = \frac{x}{2}$. Between times $\frac{0.210896\ldots}{2}$ and $\frac{3.870735\ldots}{2}$

$\Rightarrow 0.105 < t < 1.94$ Seconds

$\Rightarrow$ total time $= 1.829919\ldots$

$= 1.83$ Seconds (3SF)

2

10

Commentary

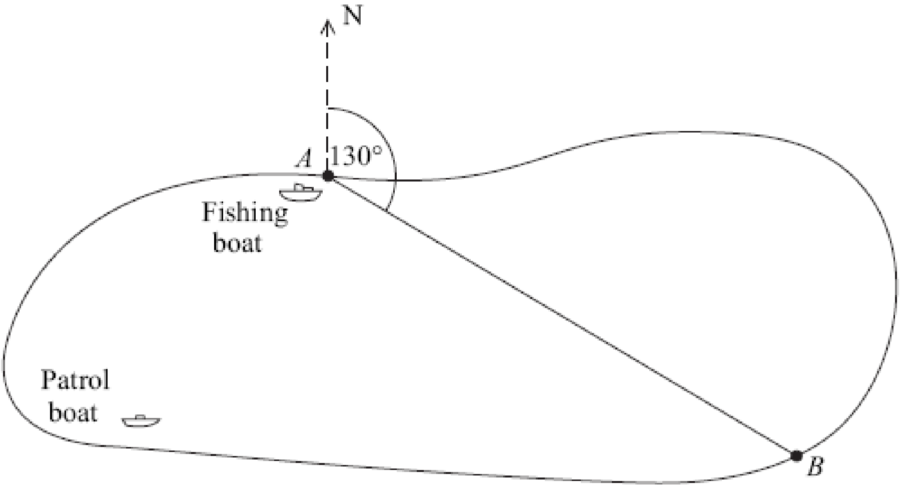
The candidate understands that the horizontal velocity of the projectile is constant and that the vertical velocity is subject to the force of gravity. The candidate is able to write the equations of motion of the particle parallel and perpendicular to the plane, substituting the given initial velocity components of 2 ms^{-1} and 10 ms^{-1}. The candidate then eliminates the time *t* from the equation for vertical motion to arrive at the required result. For part (b), the candidate finds the limits of the horizontal distance travelled by the particle whilst it is more than 1 m above the plane. Sensibly, the candidate does not round off these partial results to less than three significant figures and proceeds to finding the horizontal distance travelled and gives the required result to three significant figures. Similarly they find the corresponding time requested in part (c) to appropriate degree of accuracy.

Mark Scheme

2(a)	$x = 2t$	M1		
	$y = -\frac{1}{2}gt^2 + 10t$	M1		
	$t = \frac{x}{2}$			
	$y = -\frac{1}{2}g\left(\frac{x}{2}\right)^2 + 10\left(\frac{x}{2}\right)$	m1		
	$y = -\frac{g}{8}x^2 + 5x$	A1	4	AG
(b)	$1 = -\frac{g}{8}x^2 + 5x$	M1		
	$gx^2 - 40x + 8 = 0$			
	$x = \frac{40 \pm \sqrt{(-40)^2 - 4\times 8g}}{2g}$	M1		
	$x = 3.871,\ 0.211$	A1		A1 for both answers
	$\text{Distance} = 3.66\,\text{m}$	A1	4	
(c)	$t = \frac{3.66}{2}$	M1		
	$t = 1.83$ sec	A1	2	
	Total		**10**	

Question 3a

3 A fishing boat is travelling between two ports, A and B, on the shore of a lake. The bearing of B from A is 130°. The fishing boat leaves A and travels directly towards B with speed $2\,\text{m}\,\text{s}^{-1}$. A patrol boat on the lake is travelling with speed $4\,\text{m}\,\text{s}^{-1}$ on a bearing of 040°.

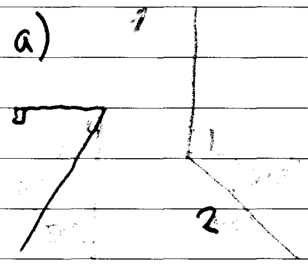


(a) Find the velocity of the fishing boat relative to the patrol boat, giving your answer as a speed together with a bearing. *(5 marks)*

Student Response (3a)

j ↑ → i

a)



$V_F = (2\cos 40)i + (-2\sin 40)j$ ✓

$\cancel{V_{BP} = (4\cos 50)i + P}$

$V_P = (4\sin 40)i + (4\cos 40)j$ ✓

${}_PV_F = (2\cos 40 - 4\sin 40)i + (-2\sin 40 - 4\cos 40)j$ ✓

$= -1.039i - 4.350j$ ✓

$\therefore |{}_PV_F| = \sqrt{(-1.039)^2 + (-4.350)^2} = \underline{\underline{\sqrt{20}}}\ \text{ms}^{-1}$ ✓

$\tan\theta = \frac{1.039}{4.350} = 0.239$ ✓

$\theta = \tan^{-1}(0.239) = 13.4^\circ$ ✓

$\therefore$ Bearing $= 180^\circ + 13.4^\circ = \underline{\underline{193.4^\circ}}$ ✓

5

Commentary 3a

Evidently, the candidate does not realise that the velocity triangle for part (a) is a right-angled triangle and the use of Pythagoras's theorem would be a less time-consuming approach here. However, the candidate uses the correct resolution of the velocities of the fishing boat and the patrol boat to write down the velocity of the former relative to the latter. This is then used to find the required speed. The candidate uses the tangent ratio to find the angle between the direction of the relative velocity and the southerly direction. The required bearing is then found by adding 180° to this angle.

Mark Scheme 3a

3(a)	${}_P v_F = \sqrt{4^2 + 2^2}$	M1		
	$= 4.47\ \text{m s}^{-1}$ or $2\sqrt{5}\ ms^{-1}$ or $\sqrt{20}\ ms^{-1}$	A1		
	$\theta = \tan^{-1}\frac{2}{4}$	M1		
	$\theta = 26.6°$	A1F		
	Bearing $= 40° + 180° - 26.6°$ $= 193°$	A1F	5	
	Alternative: Comp. due west $= 4\sin 40° - 2\sin 50° = 1.04\ \text{m s}^{-1}$ Comp. due south $= 2\cos 50° + 4\cos 40° = 4.35\ \text{m s}^{-1}$	(M1)		OE; resolving in two directions
	${}_P v_F = \sqrt{1.04^2 + 4.35^2} = 4.47\ \text{m s}^{-1}$	(A1)		
	$\theta = \tan^{-1}\frac{1.04}{4.35}$ or $\tan^{-1}\frac{4.35}{1.04}$	(M1)		
	$\theta = 13.4°$ or $76.6°$	(A1F)		
	Bearing $= 13.4° + 180°$ or $270° - 76.6°$ $= 193°$	(A1F)		
	Alternative: Correct triangle	(M1)		Any orientation
	${}_P v_F = \sqrt{1.04^2 + 4.35^2} = 4.47\text{ms}^{-1}$	(A1)		
	Rel. Vel. Triangle angle $26.6°$ or $63.4°$	(A1)		
	Bearing $= 40° + 180° - 26.6°$ or $63.4° + 40° + 90°$	(M1)		
	$= 193°$	(A1F)		

Question 3b

(b) When the patrol boat is 1500 m due west of the fishing boat, it changes direction in order to intercept the fishing boat in the shortest possible time.

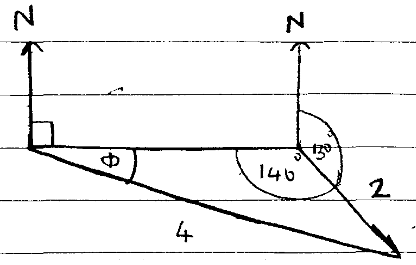
(i) Find the bearing on which the patrol boat should travel in order to intercept the fishing boat. *(4 marks)*

(ii) Given that the patrol boat intercepts the fishing boat before it reaches *B*, find the time, in seconds, that it takes the patrol boat to intercept the fishing boat after changing direction. *(4 marks)*

(iii) State a modelling assumption necessary for answering this question, other than the boats being particles. *(1 mark)*

Student Response 3b

b)



$$\frac{2}{\sin \Phi} = \frac{4}{\sin 140}$$

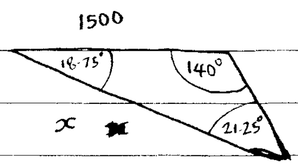
$$\sin \Phi = \frac{\sin 140^\circ}{2}$$

$$\Phi = 18.75^\circ$$

Bearing = 109°

The patrol boat should travel on a bearing of 109°. 4

(ii)



$$\frac{x}{\sin 140^\circ} = \frac{1500}{\sin 21.25^\circ}$$

$$x = 1500 \frac{\sin 140^\circ}{\sin 21.25^\circ}$$

$$x = 2660 \text{ m}$$

Time taken = $\frac{\text{Distance}}{\text{Speed}}$ = 665 seconds 4

(iii) No curents in the water. 1

Commentary 3b

The candidate draws a neat velocity diagram indicating the angles correctly. They then use the sine rule to find the angle between the direction of the velocity of the patrol boat and the easterly direction. The result is stated with an appropriate degree of accuracy. The candidate writes down the correct bearing on which the patrol boat should travel in order to intercept the fishing boat. This result can clearly be gleaned from the candidate's diagram. The candidate then refines their diagram in order to answer part (ii). The distance travelled by the patrol boat on the new bearing is found by using the sine rule again. The time taken for intercepting the fishing boat is found by dividing this distance by the patrol boat's constant speed. The candidate gains a further mark by stating the acceptable for part (iii).

Mark Scheme 3b

(b)(i)	$v_F = v_P + {}_Pv_F$			
	$\dfrac{\sin\alpha}{2} = \dfrac{\sin 140°}{4}$	M1A1		
	$\alpha = 18.7°$	A1F		
	Bearing $= 90° + 18.7°$			
	$= 109°$	A1F	4	
	Alternative:			
	$2\sin 40° = 4\sin\alpha$	(M1)		
	$\alpha = \sin^{-1}\left(\dfrac{1}{2}\sin 40°\right)$	(A1)		
	$\alpha = 18.7°$	(A1F)		
	Bearing $= 109°$	(A1F)		
3(b)(ii)	$\beta = 180° - (140° + 18.7°)$	B1F		
	$= 21.3°$			
	$\dfrac{{}_Pv_F}{\sin 21.3°} = \dfrac{4}{\sin 140°}$	M1		
	${}_Pv_F = 2.2568\,\text{m s}^{-1}$	A1F		
	$t = \dfrac{1500}{2.2568}$			
	$= 665$ sec	A1F	4	
	Alternative:			o.e. resolving in two directions
	${}_Fv_P = 4\cos 18.7 - 2\cos 40 = 2.2568$	(M1) (A2,1,0)		
	$t = \dfrac{1500}{2.2568} = 665$ sec	(A1F)		
(iii)	No cross wind, calm lake, instantaneous change of direction by the patrol boat	B1	1	Any sensible assumption
	Total		**14**	

Question 4

4 A particle of mass 0.5 kg is initially at rest. The particle then moves in a straight line under the action of a single force. This force acts in a constant direction and has magnitude $(t^3 + t)$ N, where t is the time, in seconds, for which the force has been acting.

(a) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 0$ and $t = 4$. *(3 marks)*

(b) Hence find the speed of the particle when $t = 4$. *(2 marks)*

(c) Find the time taken for the particle to reach a speed of $12\,\text{m}\,\text{s}^{-1}$. *(5 marks)*

Student Response

4a. Impulse $= \int_0^4 t^3 + t \, dt$

$= \int_0^4 t^3 + t \, dt$ ✓

$= \left[\frac{t^4}{4} + \frac{t^2}{2}\right]_0^4$ ✓

$= \frac{4^4}{4} + \frac{4^2}{2} - 0$ ✓

$= 64 + 8$

$= 72$ ✓

3

b. Impulse = Change in momentum

$= m(v-u) \qquad = Ft$

$72 = 0.5(v-u)$

$72 = 0.5(v-0)$ ✓

$\frac{72}{0.5} = v$

$v = 144 ms^{-1}$ ✓

2

c. $v = 12 ms^{-1}$

$I = 0.5 \times (12-0)$

$= 6$

$\frac{t^4}{4} + \frac{t^2}{2} = 6$ ✓

$t^4 + 2t^2 = 24$ ✓

$t^4 + 2t^2 - 24 = 0$

$(t^2 - 4)(t^2 + 6) = 0$

$t^2 = 4$ ✓

$t = 2 s$ ✓

5

10

Commentary

Unlike some other candidates who mistakenly treated the force as constant by multiplying it by *t*, this candidate understood that the impulse of the variable force should be found by integration. The candidate uses the correct limits to evaluate the integral.

For part (b) the candidate uses the impulse/momentum principle to find the speed of the particle when *t*=4.

The candidate calculates the impulse needed for the particle to reach a speed of $12\ \text{ms}^{-1}$. They then equate this impulse with the integral of the variable force to form a quadratic equation in t^2. The equation is factorised and solved correctly to find the required time.

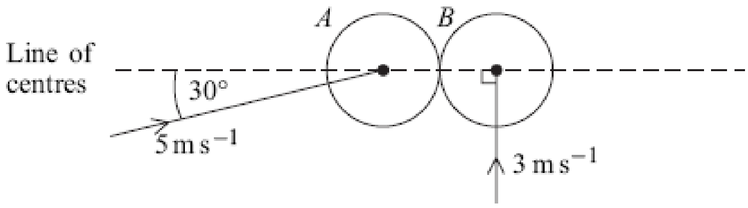
Mark Scheme

4(a)	$I = \int_0^4 (t^3 + t)\ dt$	M1		
	$= \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^4$	m1		
	$= 72$ Ns	A1	3	
(b)	$72 = 0.5v - 0.5(0)$	M1		Condone $-5(0)$
	$v = 144$	A1F	2	
(c)	$\int_0^T (t^3 + t)\ dt = 0.5(12) - 0.5(0)$	M1		Condone $-5(0)$
	$\left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^T = 6$			
	$T^4 + 2T^2 - 24 = 0$	A1		
	$T^2 = \frac{-2 \pm \sqrt{2^2 - 4(1)(-24)}}{2(1)}$	m1 A1F		
	or $(T^2 - 4)(T^2 + 6) = 0$			
	$T^2 = 4$			
	$T = 2$	A1F	5	
	Total		**10**	

Question 5

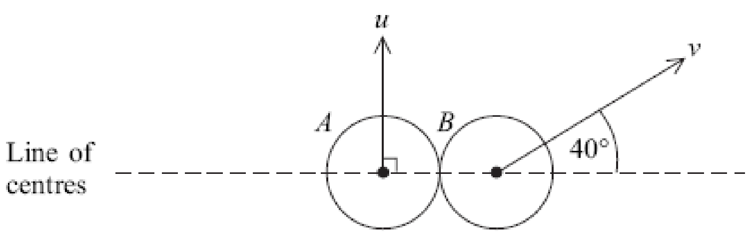
5 Two smooth spheres, A and B, of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed $5\,\text{m}\,\text{s}^{-1}$ at an angle of $30°$ to the line of centres of the spheres, and B is moving with speed $3\,\text{m}\,\text{s}^{-1}$ perpendicular to the line of centres, as shown in the diagram below.



Before collision

Immediately after the collision, A and B move with speeds u and v in directions which make angles of $90°$ and $40°$ respectively with the line of centres, as shown in the diagram below.



After collision

(a) Show that $v = 4.67\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(3 marks)*

(b) Find the coefficient of restitution between the spheres. *(3 marks)*

(c) Given that the mass of A is 0.5 kg, show that the magnitude of the impulse exerted on A during the collision is 2.17 N s, correct to three significant figures. *(3 marks)*

(d) Find the mass of B. *(3 marks)*

Student Response

Leave blank

5a ~~C.o.m. (parallel loc): $5\cos 30° = V\cos 40°$~~

~~$\Rightarrow V = \frac{5\cos 30°}{\cos 40°} =$~~

C.o.m (perp loc): $3 = \cancel{V\cos} \; V\sin 40°$

$V = \frac{3}{\sin 40°} = 4.6671764...$

$= 4.67 \text{ ms}^{-1}$ (3SF) 3

b $e(5\cos 30°) = V\cos 40°$

$e = \frac{(4.667...)\cos 40°}{5\cos 30°} = 0.825671...$

$= 0.826$ (3SF) 3

c Impulse acts parallel to loc.

$\Rightarrow I = 0.5(5\cos 30°)$

$= 2.1650635...$

$= 2.17$ NS (3SF) 3

d $I = m(V\cos 40°)$

$\Rightarrow m = \frac{2.165...}{(4.667...)\cos 40°} = 0.6055679...$

$= 0.606$ (3SF) kg. 3

12

Commentary

The candidate knows that the momentum of the sphere *B* perpendicular to the line of centres is not changed by the collision. The resolution of the velocity perpendicular to the line of centres is correct and leads to the given answer.

For part (b), the candidate applies the law of restitution along the line of centres using the correct resolutions of the velocities. Unlike some other candidates, this candidate is able to use the calculater correctly to work out the value of the fraction.

The candidate understands that the magnitude of the impulse exerted on *A* is equivalent to the change of the magnitude of the momentum of *A* along the line of centres. Hence, the candidate is able to show the result for part (c).

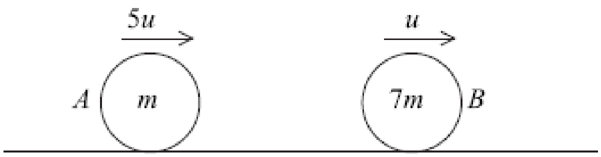
To find the mass of *B* requested in part (d), the candidate uses the unrounded value of the impulse found in part (c). They understand that the loss of momentum of *A* along the line of centres is equal to the gain of momentum by *B* along the line of centres. The mass is given correct to three significant figures.

Mark Scheme

5(a)	Momentum of B perpendicular to the line of centres is unchanged $m_B v \sin 40° = 3m_B$ $v = 4.667 \text{ m s}^{-1} = 4.67 \text{ ms}^{-1}$ (3sf)	M1A1 A1	3	AG
(b)	$e = \dfrac{4.67\cos 40°}{5\cos 30°}$ $e = 0.826$	M1A1 A1F	3	
(c)	Impulse on A = change in momentum of A along the line of centres $= 0.5 \times 5\cos 30° = 2.165$ $= 2.17$ Ns	M1A1 A1	3	AG
(d)	$2.165 = m_B(4.667)\cos 40°$ $m_B = 0.6056 = 0.606$ kg (3sf)	M1A1 A1F	3	Condone use of premature rounding giving 0.605kg or 0.607 kg
	Total		**12**	

Question 6

6 A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.



Before collision

(a) Show that the speed of B after the collision is $\frac{u}{2}(e+3)$. *(5 marks)*

(b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. *(4 marks)*

(c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. *(4 marks)*

Student Response

Leave blank

6 $m_A = m$ $m_B = 7m$ e

$u_A = 5u$ $u_B = u$

a) Newton's co-efficient of restitution; $v = eu$

$v_B - v_A = e(u_A - u_B)$

$v_B - v_A = e(5u - u)$ ① $4eu = v_B - v_A$

Conservation of momentum;

$m \times 5u + 7m \times u = m \times v_A + 7m \times v_B$

~~12mu~~ ② $12u = v_A + 7v_B$

② + ① $12u + 4eu = 8v_B$

$3u + eu = 2v_B$ $v_B = \frac{u}{2}(3+e)$

5

b) $v_A < 0$ $4eu = \frac{u}{2}(3+e) - v_A$

$v_A = \frac{u}{2}(3+e) - 4eu$

$v_A = \frac{u}{2}[3 + e - 8e] = \frac{u}{2}(3 - 7e)$

as v_A is negative as reversed

$v_A < 0$ $\frac{u}{2}(3-7e) < 0$

$3 - 7e < 0$

$3 < 7e$

$\frac{3}{7} < e$

4

c) Collision; $\omega_B = e v_B$

$\omega_B = -\frac{1}{2} \times \frac{u}{2}(e+3)$

$\omega_B = -\frac{u}{4}(e+3)$ for B to collide with A

$|v_A| < |\omega_B|$

~~$\frac{u}{4}(e+3) > \frac{u}{2}(3-7e)$~~ $|v_A| = -1 \times \frac{u}{2}(3-7e) = \frac{u}{2}(7e-3)$

~~$e+3 > 2(3-7e)$~~

~~$e+3 > 6-14e$~~ $\frac{u}{2}(7e-3) < \frac{u}{4}(e+3)$

~~$15e > 3$~~ $2(7e-3) < e+3$

$14e - 6 < e + 3$

$13e < 9$ $e < \frac{9}{13}$

4

(13)

Commentary

The candidate applies Newton's law of restitution correctly. They use the principle of conservation of linear momentum to write the second equation involving the velocities of the spheres *A* and *B*. The candidate is consistent in the use of signs for the velocities. The two equations are solved simultaneously to give the required result.

For part (b), the candidate finds the velocity of *A* after the first collision in terms of *u* and *e* in simplified form. Having taken the left-to-right as positive direction, the candidate uses the given fact that the direction of motion of *A* is reversed to form an inequality. A simple manipulation of the inequality is used to show the required result.

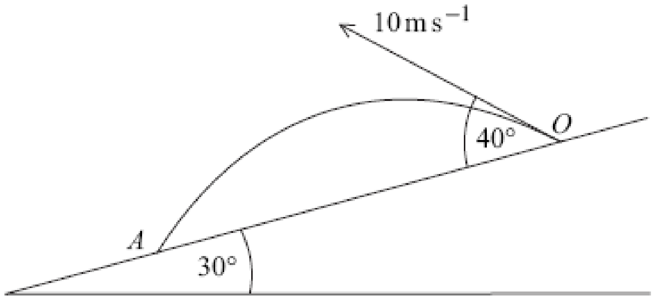
The candidate finds the velocity of *B* after collision with the wall. The candidate recognises that as the spheres move in the same direction, the speed of *B* should be greater than the speed of *A* for a further collision. The candidate states this requirement by writing the inequality. The candidate solves the inequality to show the required result.

Mark Scheme

6(a)	$5mu + 7mu = mv_A + 7mv_B$	M1A1		Allow consistent use of positive or negative sign for v_A.
	$12u = v_A + 7v_B$			
	$e = \frac{-v_A + v_B}{4u}$	M1		
	$-v_A + v_B = 4eu$			
	$8v_B = 12u + 4eu$	m1		
	$v_B = \frac{u}{2}(e+3)$	A1	5	AG
(b)	$v_A = \frac{u}{2}(e+3) - 4eu$	M1		
	$v_A = \frac{u}{2}(3-7e)$	A1F		
	$\frac{u}{2}(3-7e) < 0$	M1		
	$3 - 7e < 0$			
	$e > \frac{3}{7}$	A1	4	AG
(c)	$w_B = \frac{u}{4}(e+3)$	M1		
	$\frac{u}{2}(7e-3) < \frac{u}{4}(e+3)$	M1		
	$2(7e-3) < e+3$			
	$13e < 9$	m1		
	$e < \frac{9}{13}$	A1	4	AG
	Total		**13**	

Question 7

7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity $10\ \text{m s}^{-1}$ at an angle of 40° above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

$$\frac{20\sin 40°}{g\cos 30°}$$

(3 marks)

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5 . Find the speed of the particle as it rebounds from the slope. *(4 marks)*

Student Response

7. a) parallel

$u = 10\cos 40$

$v = ?$

$a = g\sin 30$

$s = x$

$t = ?$

perpendicular

$u = 10\sin 40$

$v = ?$

$a = -g\cos 30$

$s = y$

$t = ?$

$y = ut + \frac{1}{2}at^2$

$\cancel{y = 10\cos 40 t + \frac{g}{2}\sin 30 t^2}$

$\cancel{y = t(10\cos 40 + \frac{g}{2}}$

$y = 10\sin 40 t - \frac{g}{2}\cos 30 t^2$ ✓

$y = t\left(10\sin 40 - \frac{g}{2}\cos 30 t\right)$ ✓

for OA

$t = \dfrac{10\sin 40}{\frac{g}{2}\cos 30}$ ✓

$t = \dfrac{20\sin 40}{g\cos 30}$ ✓

b) $v = u + at$

parallel: $v = 10\cos 40 + g\sin 30 \times \frac{20\sin 40}{g\cos 30}$ ✓

$v = 10\cos 40 + 20\sin 40 \tan 30$ ✓

$v = 15.1 \text{ ms}^{-1}$ (3sf) ✓

perpendicular

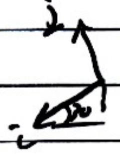
$v = 10\sin 40 - \frac{\cancel{g}\cos 30 \times 20\sin 40}{\cancel{g}\cos 30}$ ✓

$v = 10\sin 40 - 20\sin 40$ ✓

$v = -10\sin 40$

$v = -6.43 \text{ ms}^{-1}$ (3sf) ✓

so if



$\underline{v} = (15.1\underline{i} - 6.43\underline{j}) \text{ ms}^{-1}$

c) the $\underline{i}$ component remains as 15.1 ✓

$\underline{j}$ comp: $V = -eU$

$V = 6.43 \times 0.5$ ✓

$V = 3.213$ ✓

$\underline{v} = 15.1\underline{i} + 3.21\underline{j}$ ✓

$|v| = \sqrt{15.1^2 + 3.21^2}$ ✓

using exact values

$|v| = 15.4 \ ms^{-1}$ ✓ (3sf). just speed

Commentary

The candidate lists the components of the initial velocity of projection, the acceleration and the displacement parallel and perpendicular to the inclined plane. They write down the equation of motion perpendicular to the incline plane. They recognise that at the point *A*, the vertical displacement is zero and this is substituted into the equation for *y*. A simple manipulation of the equation leads to the required result for the time of the flight from *O* to *A*.

The candidate finds the components of the velocity of the particle parallel and perpendicular to the slope as it hits the point *A* by substituting the time found in part (a) in the velocity equations. The work is accurate and the candidate gives results to three significant figures.

For part (c) of the question, the candidate recognises that the parallel component of the velocity is not changed by the collision, whereas the perpendicular component is subject to restitution. The candidate then uses these to calculate the speed of the particle as it rebounds from the slope. Again, the result is given to appropriate degree of accuracy.

Mark Scheme

7(a)	$y = 10t\sin 40° - \frac{1}{2}gt^2\cos 30°$	M1A1		
	$y = 0 \Rightarrow t = \frac{20\sin 40°}{g\cos 30°}$	A1	3	AG
(b)	$\dot{x} = 10\cos 40° + g\sin 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{x} = 15.08\ \mathrm{m\,s^{-1}}$	A1		
	$\dot{y} = 10\sin 40° - g\cos 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{y} = -6.427\ \mathrm{m\,s^{-1}}$	A1	4	Allow 3 sf
(c)	$\dot{x}$ will be unchanged	B1		
	Rebound $\dot{y} = 6.427 \times 0.5 = 3.214$	M1		Allow using 3 sf
	Rebound speed $= \sqrt{15.08^2 + 3.214^2}$	m1		
	$= 15.4\ \mathrm{m\,s^{-1}}$	A1F	4	
	Total		**11**	
	TOTAL		**75**	