Teacher Support Materials 2009

Maths GCE

Paper Reference MD01

Question 1

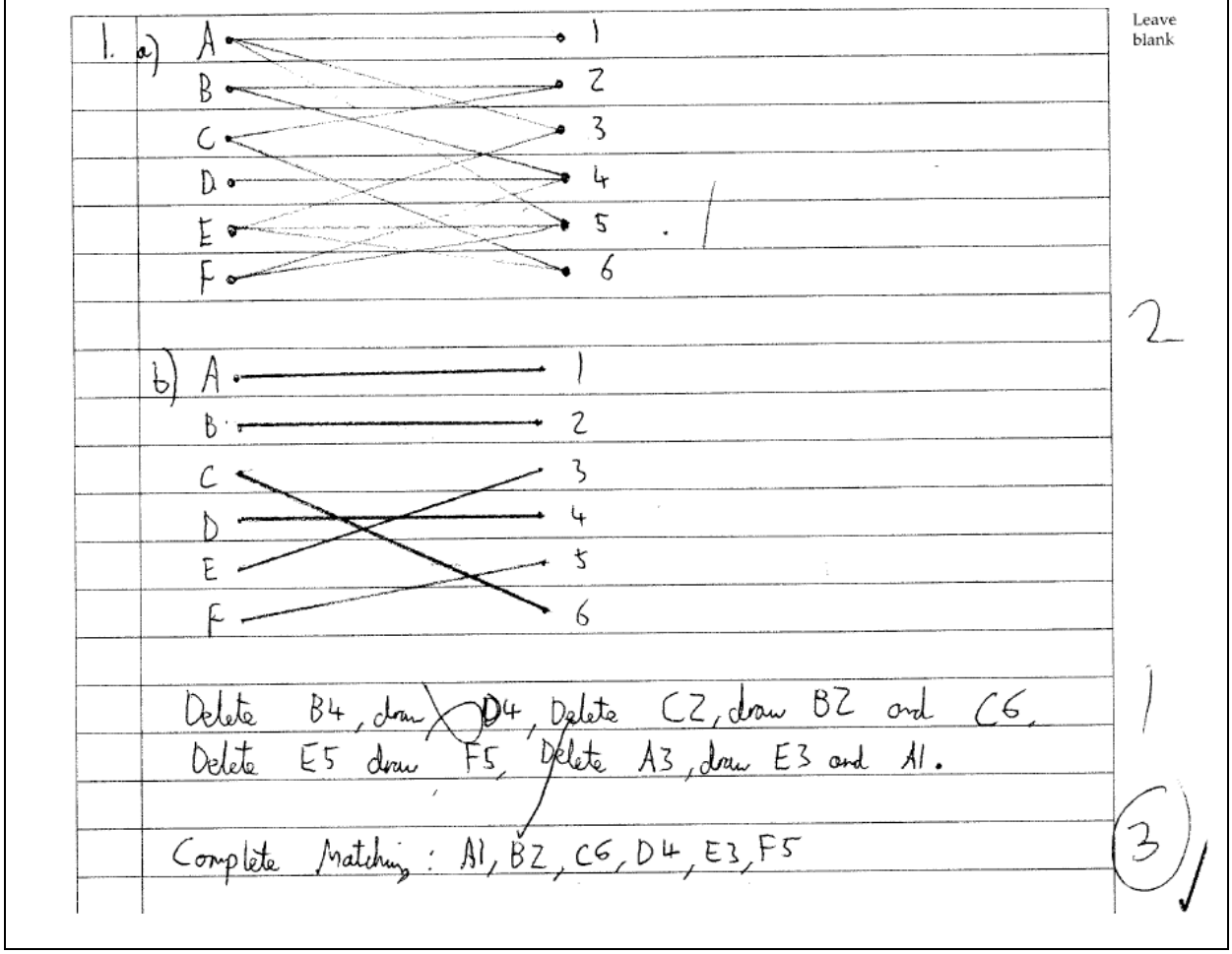
(a) Draw a bipartite graph representing the following adjacency matrix.

	1	2	3	4	5	6
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(2 marks)

(b) Initially, A is matched to 3, B is matched to 4, C is matched to 2, and E is matched to 5. Use the maximum matching algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

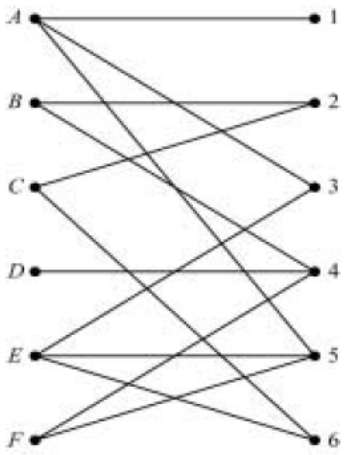
Student Response



Commentary

In recent years the standard of student responses on alternating paths has significantly improved. However there are still a number of candidates who fail to correctly apply the algorithm. From an initial match candidates **must** start with an unconnected vertex.
This candidates' response is a common incorrect approach. The candidate has started by deleting a random edge and then used 'intuition'. This will not score the marks. The candidate scored the final mark for a correct match. It must be stressed to students that although an exam problem could be solved by inspection, if there was a match involving 30 vertices inspection would not work and an algorithm is essential.

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)		M1 A1	 2	Bipartite graph, 2 sets of (some) vertices labelled, 6+ edges
(b)	$A3, B4, C2, E5$			
	$D-4+B,\quad 6-C+2,\quad 6-E+5$ $F-5+E,\quad 1-A+3,\quad F-4+B$	M1 M1		1 correct 1 correct
	$D-4+B-2+C-6$ $F-5+E-3+A-1$ ignore extra paths attempted	A1 A1		Or reverse Or reverse
	OR			
	$F-4+B-2+C-6$ $D-4+F-5+E-3+A-1$ ignore extra paths attempted	(A1) (A1)		Or reverse Or reverse
	$A1, B2, C6, D4, E3, F5$	B1	5	Must be list, not diagram
	Watch for correct method using unusual notation One continuous path scores M1A1M0 eg $D-4+B-2+C-6+F-5+E-3+A-1$ If working on diagram(s) **only** then max M1A0 M1A0 for each M1: must have start point labelled and a **clear** path (numerically labelled or coloured) of at least left to right to left (or reverse)			
	Total		7	

Question 2

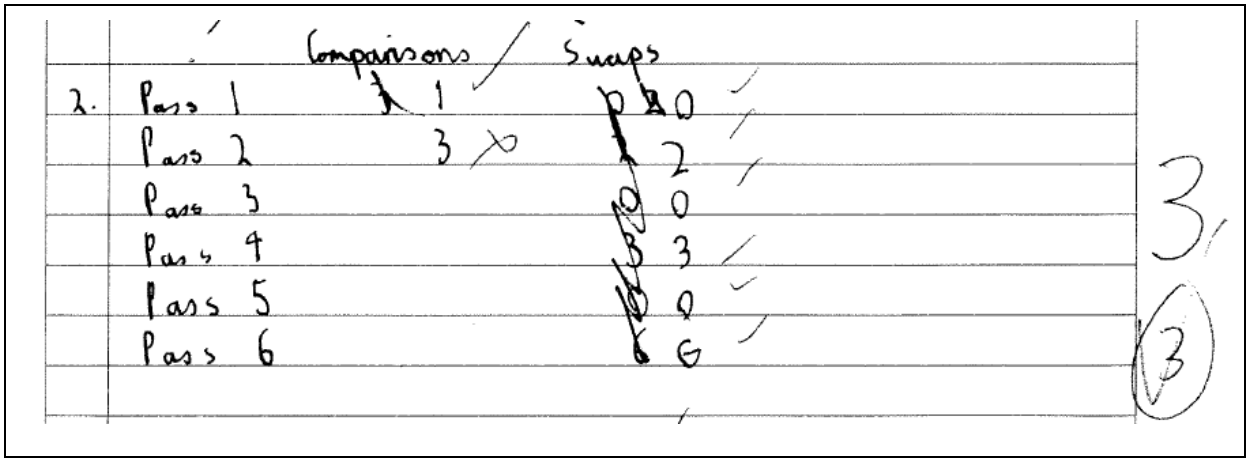
A student is using a shuttle sort to rearrange a set of numbers into ascending order.

Her correct solution is as follows.

Initial list	5	6	3	9	4	13	1
After 1st pass	5	6	3	9	4	13	1
After 2nd pass	3	5	6	9	4	13	1
After 3rd pass	3	5	6	9	4	13	1
After 4th pass	3	4	5	6	9	13	1
After 5th pass	3	4	5	6	9	13	1
After 6th pass	1	3	4	5	6	9	13

Write down the number of comparisons and swaps on each of the passes. *(6 marks)*

Student response



Commentary

Although there were many fully correct responses to this question, there was a significant number who failed to write down the correct number of comparisons.
The number of swaps was well done, as this candidate demonstrated, but there is clearly a lack of understanding of when comparisons are being made. It is good practise for candidates to record every comparison as each pass is being completed.

Mark Scheme

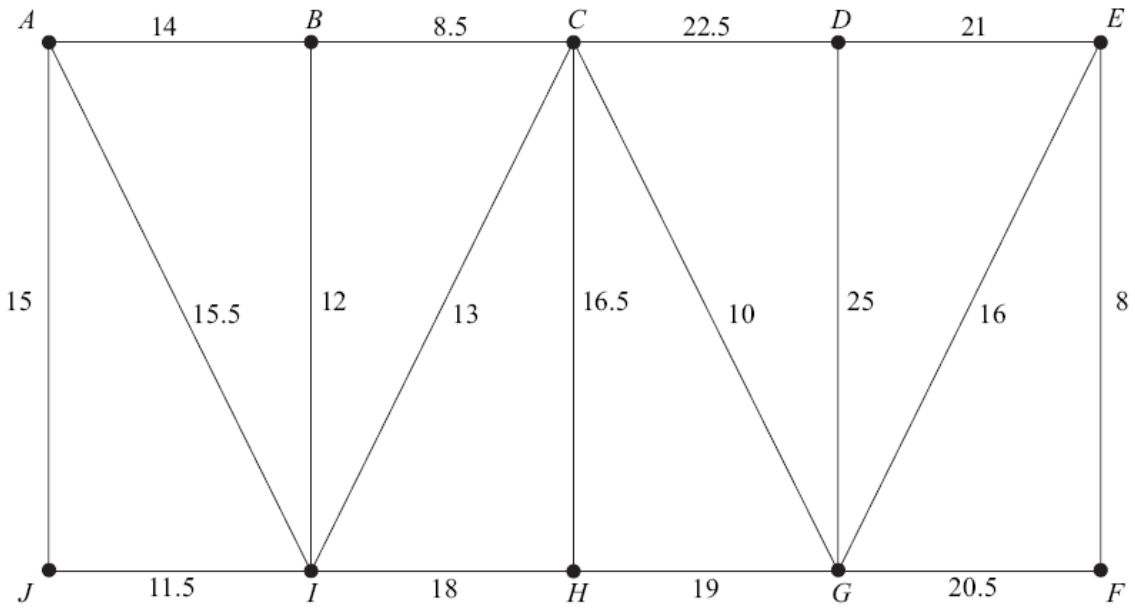
Q	Solution	Marks	Total	Comments
2	<table><tr><th></th><th>C</th><th>S</th></tr><tr><td>1st</td><td>1</td><td>0</td></tr><tr><td>2nd</td><td>2</td><td>2</td></tr><tr><td>3rd</td><td>1</td><td>0</td></tr><tr><td>4th</td><td>4</td><td>3</td></tr><tr><td>5th</td><td>1</td><td>0</td></tr><tr><td>6th</td><td>6</td><td>6</td></tr></table>	B6 (B5) (B4) (B3) (B2) (B1)	6	All 12 correct 10 correct 8 correct 7 correct 6 correct 5 correct Tallies can only score max B2 for three 1s and three 0s (not blanks)
	Total		**6**	

Question 3

3 (a) (i) State the number of edges in a minimum spanning tree for a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, ..., J. The number on each edge represents the distance between a pair of adjacent vertices.



(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

Student Response

Question number

Leave blank

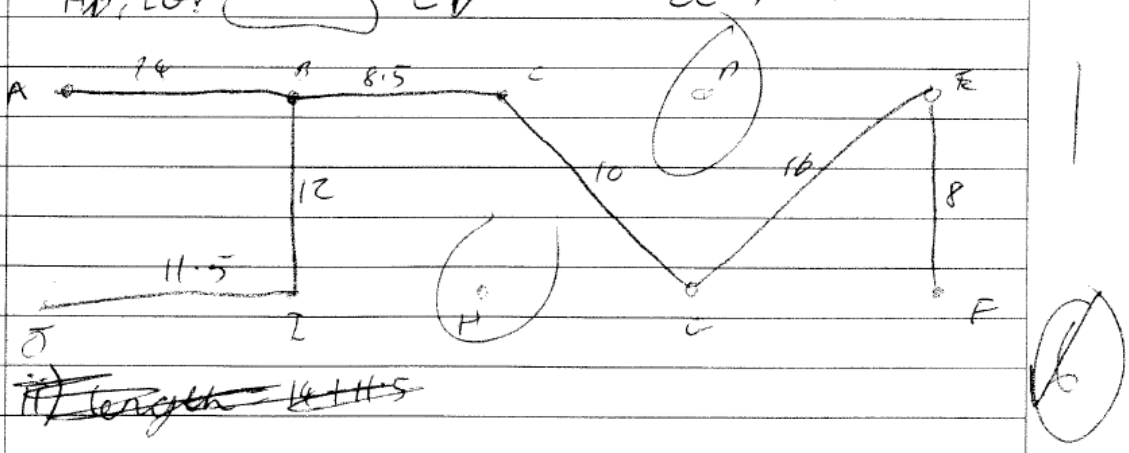
3 a) i) $n-1 = 10-1 = 9$ ✓

ii) $n-1$ ✓ 1

b) i) ranked — ~~EF~~ — length

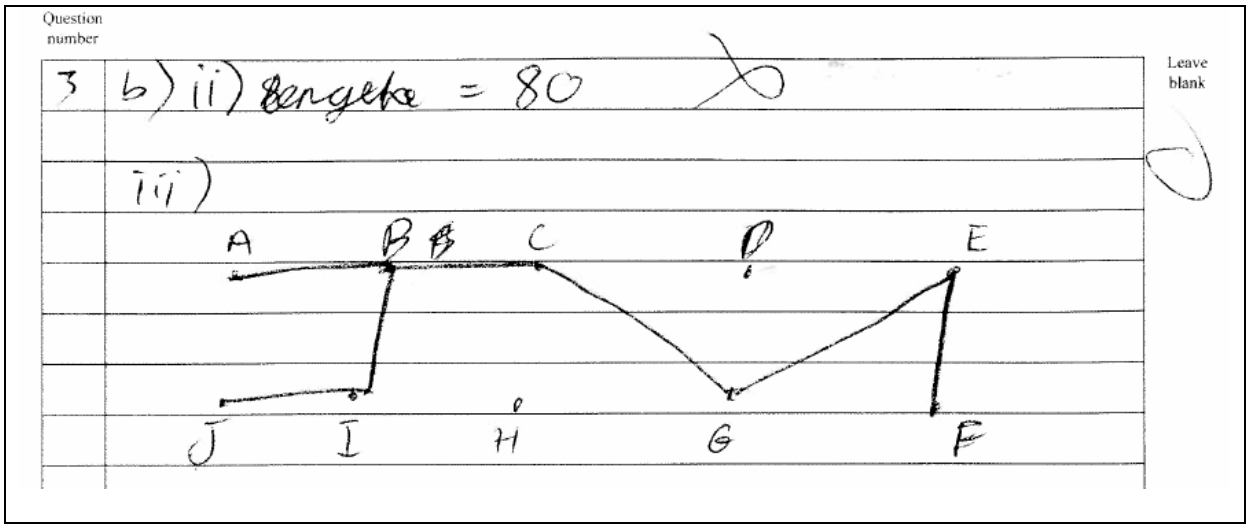
EF	8
BC	8.5
CG	10
JI	11.5
BI	12
~~CI~~	~~13~~
AB	14
~~AJ~~	~~15~~
~~AI~~	~~15.5~~
EG	16
CH	16.5
IH	18
HG	19
GF	20.5
DE	21
CD	22.5

MST = EF, BC, CG, JI, BI, AB, EG ✓ 3



ii) ~~length~~ 14 + 11.5 ✓ 1

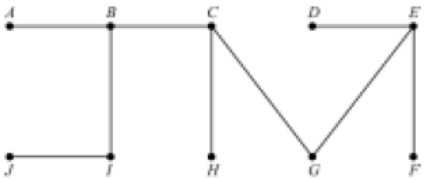
b) ✓



Commentary

Candidates were given a piece of bookwork at the start of this question to help with the network given in part (b). This candidate correctly stated that there were 9 edges in a minimum spanning tree for a network with 10 vertices. The network in part (b) had 10 vertices. The candidate correctly listed the edges in order, Kruskal's algorithm, but then only deleted three of these edges, and then wrote down that the spanning tree had seven edges. Candidates will normally be required to draw their spanning tree. This candidate has correctly drawn the 10 vertices but failed to notice that two of the vertices have remained unconnected. It is good practise for candidates to check that their spanning tree has the correct number of edges in their final diagram.

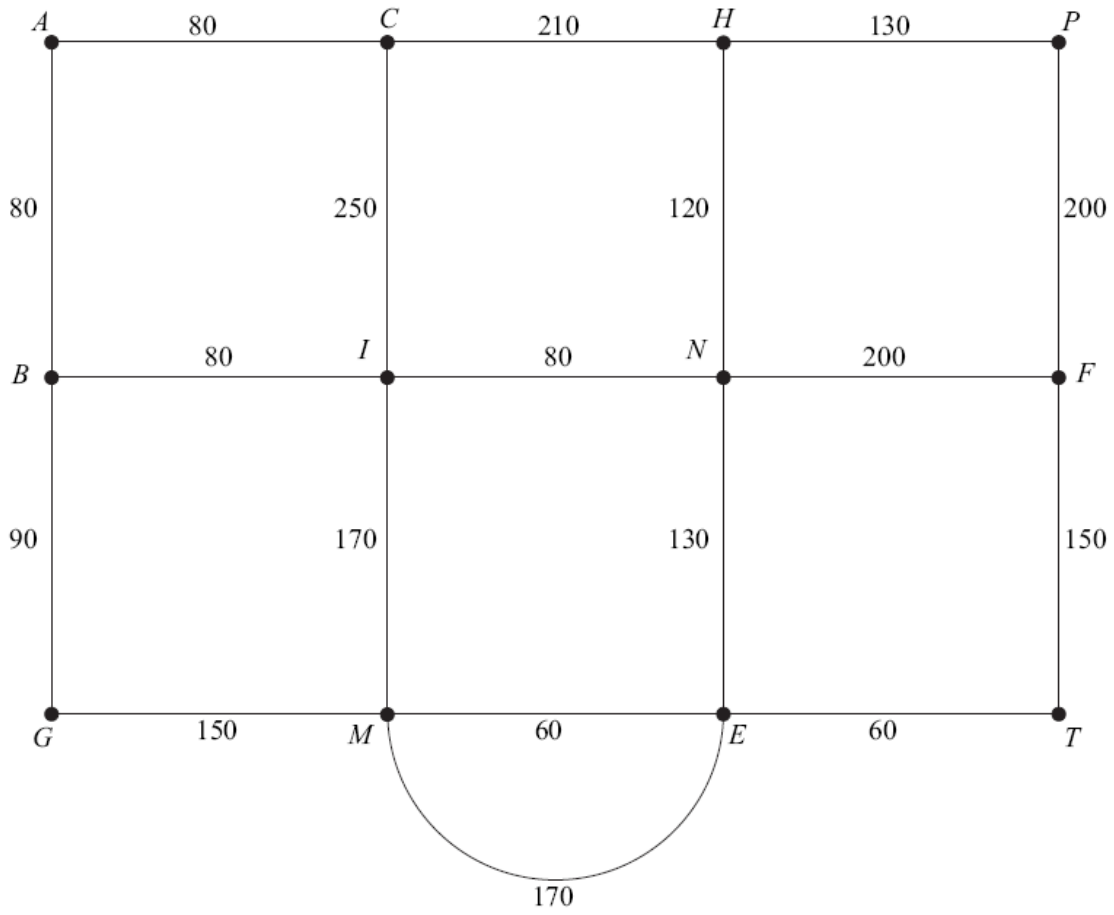
Mark Scheme

3(a)(i)	9	B1	1	
(ii)	$n-1$	B1	1	
(b)(i)	*EF* 8 *BC* 8.5 *CG* 10 *JI* 11.5 *BI* 12 *AB* 14 *GE* 16 *CH* 16.5 *DE* 21	M1		SCA minimum spanning tree, 7+ **edges** (not cycles), must be in ascending order and edges required (not lengths alone)
		A1		*BC* 2nd
		A1		*JI* 4th
		B1		9 **edges** (not lengths alone) – may be earned in (b)(iii)
		A1	5	All correct
(ii)	117.5	B1	1	
(iii)				
		M1		7+ edges, minimum spanning tree
		A1	2	Correct, including labelling
	Total		**10**	

Question 4a

4 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a network of roads on a housing estate. The number on each edge is the length, in metres, of the road.

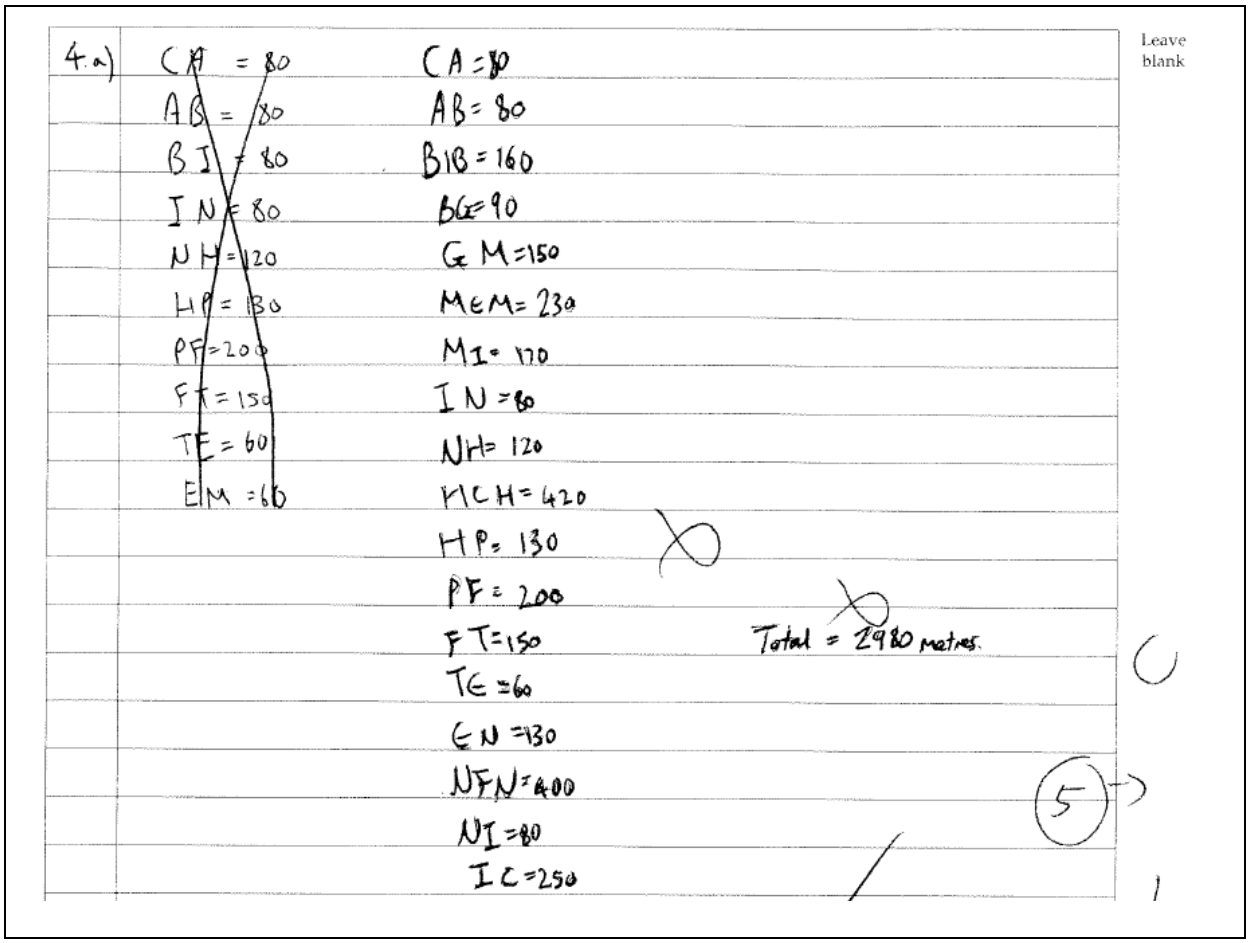


Total length of roads = 2410 metres

Joe is starting a kitchen-fitting business.

(a) Joe delivers leaflets advertising his business. He walks along all of the roads at least once, starting and finishing at C. Find the length of an optimal Chinese postman route for Joe. *(6 marks)*

Student Response



Commentary

Questions that are set on Chinese postman problem require candidates to demonstrate that they have a complete understanding of the algorithm. Candidates must state the odd vertices and then find the sum of the 3 possible pairings of these odd vertices. In this script the candidate has simply tried to find a route around the network without applying the algorithm. This is very time consuming and, in this case, incorrect. If the final total had been 2890 then the candidate would have scored some marks.

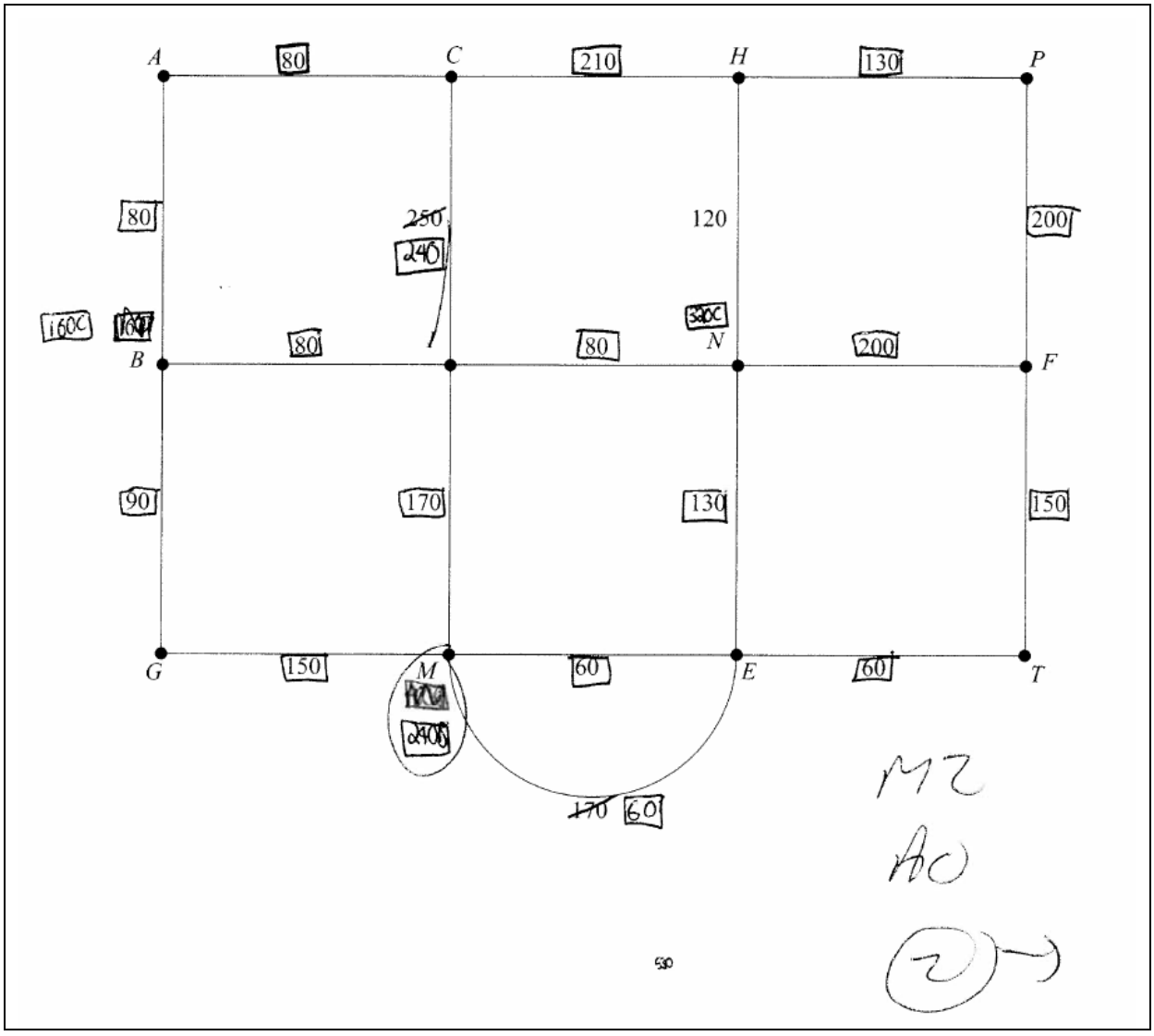
Mark Scheme

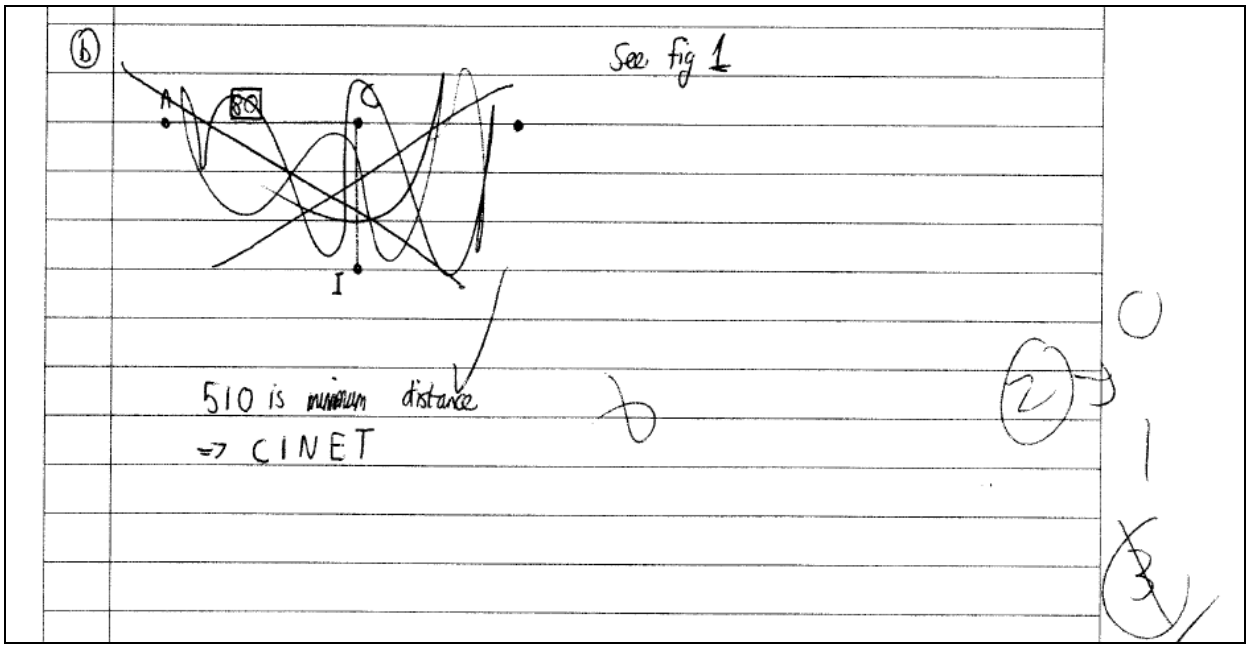
4(a)	Odds B, C, H, F	E1		PI (must be these 4 vertices - CAO)
	$BC + HF = 160 + 320$ or 480 $BH + CF = 280 + 520$ or 800 $BF + CH = 360 + 210$ or 570	M1 A2,1,0		3 sets of pairs A2 for all 3 correct, A1 for 2 correct
	(Total =)(2410 + 480) = 2890	A1F B1	6	2410 + their shortest pairing (PI) **SC** 2890 with no working or 2890 with one route listed scores 2/6 Route listed not 2890 scores 0/6

Question 4b

(b) Joe gets a job fitting a kitchen in a house at T. Joe starts from C and wishes to drive to T. Use Dijkstra's algorithm on **Figure 1** to find the minimum distance to drive from C to T. State the corresponding route. *(7 marks)*

Student Response

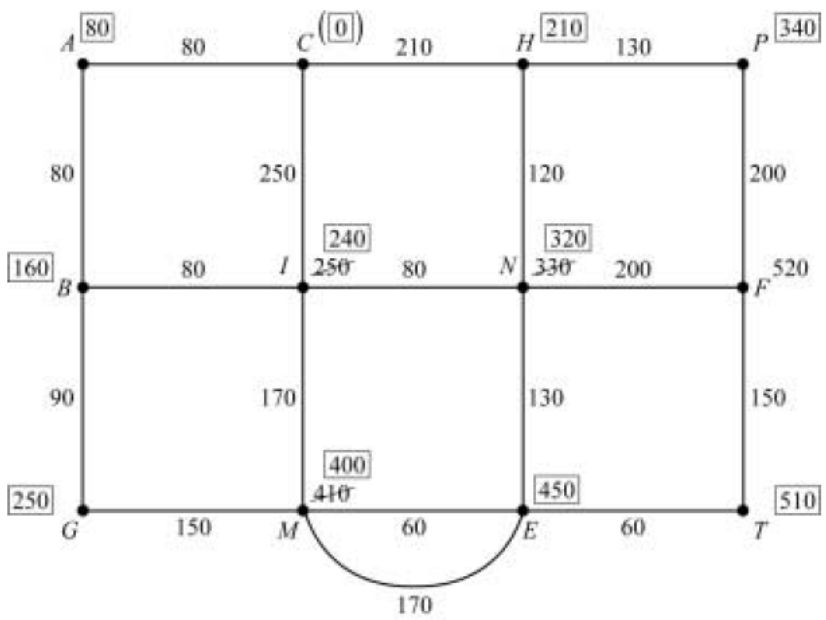




Commentary

Questions that are set on finding 'minimum' distance/time through a network will be based on Dijkstra's algorithm. That means that a candidate must show all working – even if they could answer the question by inspection. This candidate has not applied the algorithm throughout the network. A common mistake candidates make is to start using Dijkstra's algorithm and then to complete the network by inspection. In addition this candidate has 'boxed' totals on the edges and not at the vertices.

Mark Scheme

(b)				
		M1		SCA; cancelling required at I or N
		m1		2 values at I
		m1		2 values at M
		m1		2 values at N
		A1		All correct – no extra values Condone 520 boxed at F and condone final values at each vertex unboxed
		B1		510 at T (diagram takes precedence over answer book)
	Route $CABINET$	B1	7	Or reverse
	Total		**13**	

Question 5

Angelo is visiting six famous places in Palermo: A, B, C, D, E and F. He intends to travel from one place to the next until he has visited all of the places before returning to his starting place. Due to the traffic system, the time taken to travel between two places may be different dependent on the direction travelled.

The table shows the times, in minutes, taken to travel between the six places.

From \ To	***A***	***B***	***C***	***D***	***E***	***F***
A	–	25	20	20	27	25
B	15	–	10	11	15	30
C	5	30	–	15	20	19
D	20	25	15	–	25	10
E	10	20	7	15	–	15
F	25	35	29	20	30	–

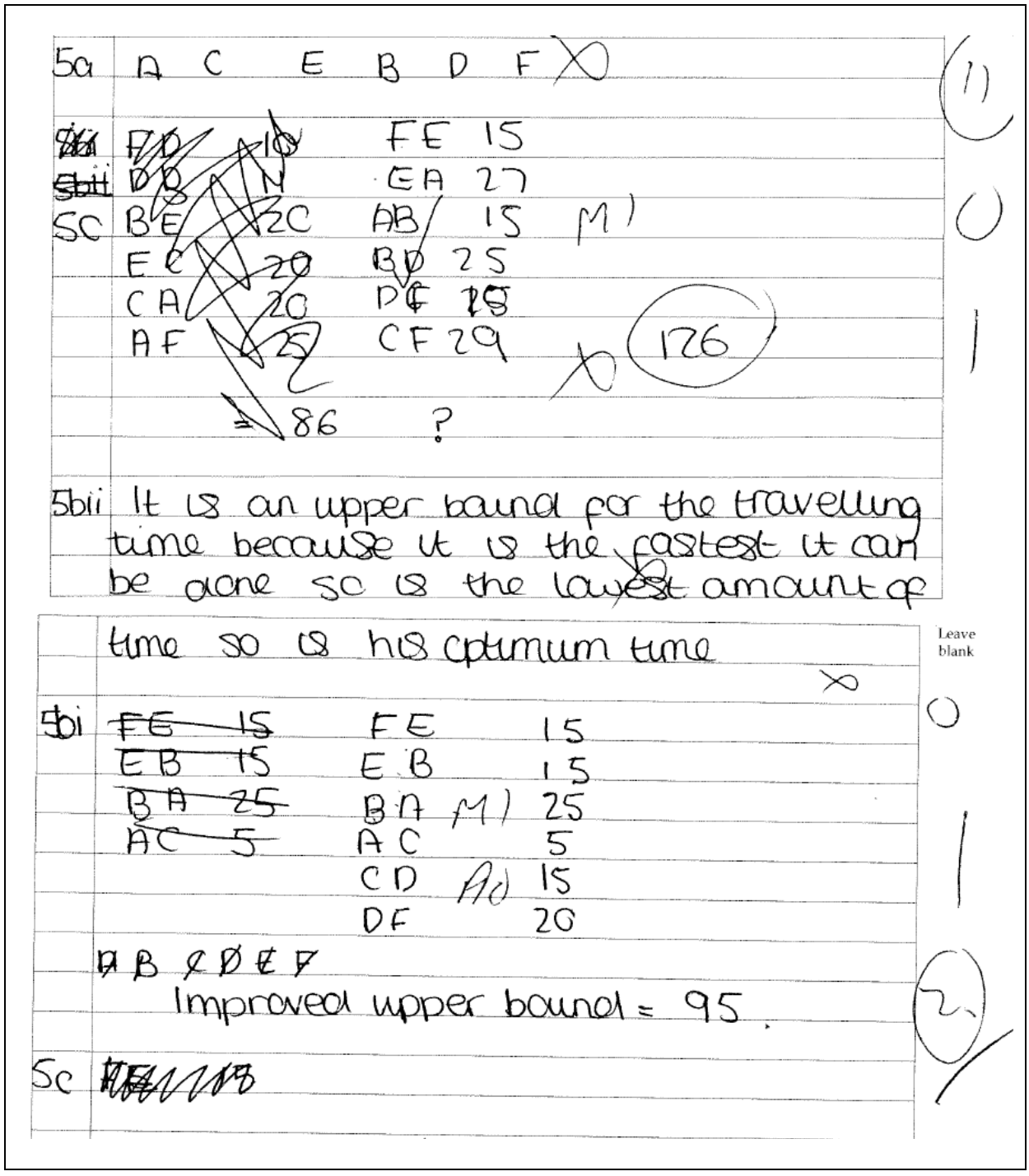
(a) Give an example of a Hamiltonian cycle in this context. *(2 marks)*

(b) (i) Show that, if the nearest neighbour algorithm starting from F is used, the total travelling time for Angelo would be 95 minutes. *(3 marks)*

(ii) Explain why your answer to part (b)(i) is an upper bound for the minimum travelling time for Angelo. *(2 marks)*

(c) Angelo starts from F and visits E next. He also visits B before he visits D. Find an improved upper bound for Angelo's total travelling time. *(3 marks)*

Student Response



5a A C E B D F ✗ (1)

FE 15
EA 27
AB 15 M1
BD 25
DF 15
CF 29

126 ✗

86 ?

5bii It is an upper bound for the travelling time because it is the fastest it can be done so is the lowest amount of time so is his optimum time ✗

Leave blank

5bi

~~FE 15~~	FE		15
~~EB 15~~	EB		15
~~BA 25~~	BA	M1	25
~~AC 5~~	AC		5
	CD	A1	15
	DF		20

A B ~~C~~ D E F

Improved upper bound = 95.

5c

Commentary

Upper and lower bounds are conceptually difficult. Candidates are normally well trained on finding upper bounds as they can follow the logic of the nearest neighbour algorithm, but they struggle with lower bounds. However this candidate in part (a) has made the mistake of visiting all vertices **but not** returning to the start vertex. This is a common mistake. As a check candidates should always ensure that the number of edges in any tour is the same as the number of vertices in the network.

Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	eg *A B C D E F A*	M1		Any tour *ABA* or better, any start vertex but not revisiting a vertex May be shown in a labelled diagram of a cycle (eg triangle *ABC*)
		A1	2	With all vertices visited May be shown in a labelled diagram of a cycle
(b)(i)	*F D C A B E F* (20) (15) (5) (25) (15) (15) (= 95) **AG**	M1 m1 A1	3	Any tour, start/finish at *F* Visits all vertices Correct order If solution shown solely on matrix, then order of selection of vertices must be shown
(ii)	Tour	E1		"It's an answer", "a cycle", "it works", "it's possible …"
	<u>May</u> be improved on	E1	2	"Can't be worse", "not necessarily best", "could be improved" Not "can be improved"
(c)	*F E C A B D F*	M1		Tour *FE*(*ABCD* in any order with *B* before *D*)*F*
	(30) (7) (5) (25) (11) (10)	A1		Correct order
	= 88	B1	3	If solution shown solely on matrix, order of selection of vertices must be shown
	Total		**10**	

Question 6a

[**Figure 2**, printed on the insert, is provided for use in this question.]

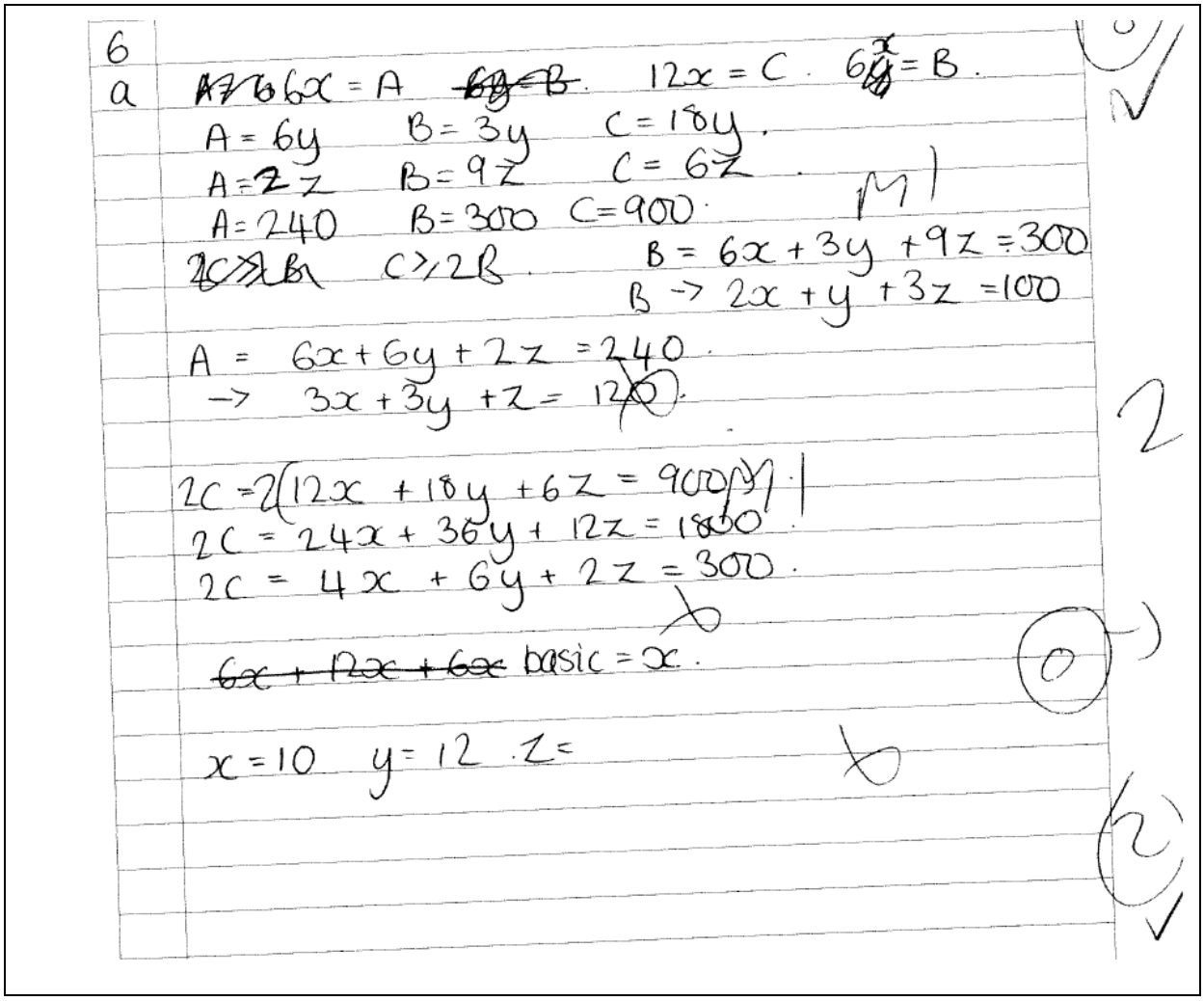
Each day, a factory makes three types of widget: basic, standard and luxury. The widgets produced need three different components: type A, type B and type C.

Basic widgets need 6 components of type A, 6 components of type B and 12 components of type C.
Standard widgets need 4 components of type A, 3 components of type B and 18 components of type C.
Luxury widgets need 2 components of type A, 9 components of type B and 6 components of type C.
Each day, there are 240 components of type A available, 300 of type B and 900 of type C.
Each day, the factory must use at least twice as many components of type C as type B.

Each day, the factory makes x basic widgets, y standard widgets and z luxury widgets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find four inequalities in x, y and z that model the above constraints, simplifying each inequality. *(8 marks)*

Student Response



Commentary

Candidates are expected to be able to translate a problem in words into a linear programming problem. This question was poorly answered and this script demonstrates a familiar incorrect response. This candidate was unable to separate the variables x, y and z from the given information. It is good practise for candidates to set out the information in a table as an interim step before transferring this information into a set of inequalities.

Mark Scheme

Q	Solution	Marks	Total	Comments
6				Working must be in x, y and z Equalities can only score M marks Strict inequalities: –1 first error only
(a)	$6x+4y+2z \le 240$	M1		
	$3x+2y+z \le 120$	A1		CAO
	$6x+3y+9z \le 300$	M1		
	$2x+y+3z \le 100$	A1		CAO
	$12x+18y+6z \le 900$	M1		
	$2x+3y+z \le 150$	A1		CAO
	$12x+18y+6z \ge 2(6x+3y+9z)$	M1		OE
	$y \ge z$	A1	8	**CSO**; OE in simplified form eg $y-z \ge 0$

Question 6b

(b) Each day, the factory makes the maximum possible number of widgets. On a particular day, the factory must make the same number of luxury widgets as basic widgets.

(i) Show that your answers in part (a) become

$$2x + y \leqslant 60, \quad 5x + y \leqslant 100, \quad x + y \leqslant 50, \quad y \geqslant x$$ *(3 marks)*

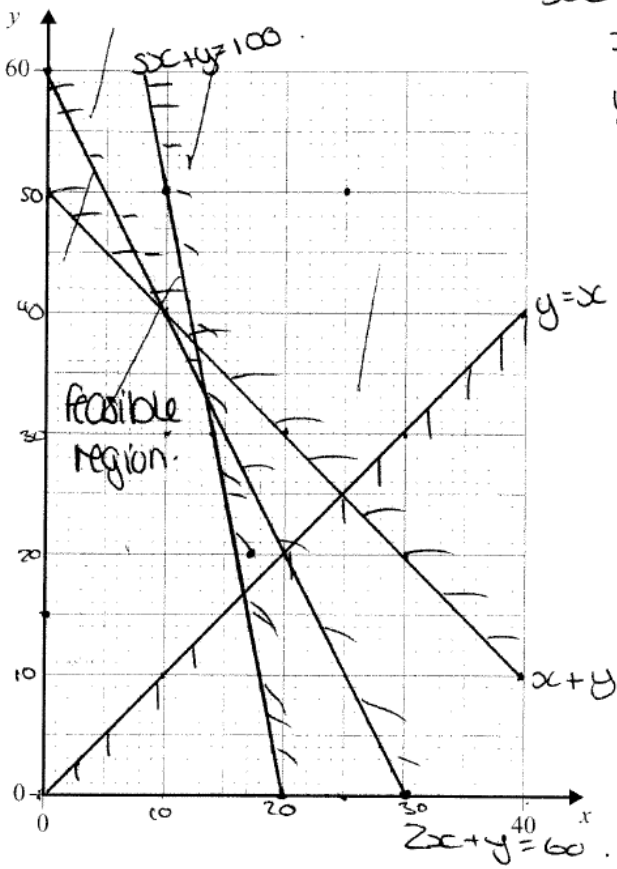
(ii) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region. *(5 marks)*

(iii) Find the total number of widgets made on that day. *(2 marks)*

(iv) Find all possible combinations of the number of each type of widget made that correspond to this maximum number. *(3 marks)*

Student Response

Figure 2 (for use in Question 6)



$y=0, x=30$
$x=0, y=60$

$2x+y \leq 60$
$5x+y \leq 100$
$x+y \leq 50$
$y \geqslant x$.

$x=10$
$y=50$

$x=20$
$y=$

$y=30$
$x=35$

$y=15$
$x=17$

$y=30$
$x=14$

5

20, 30
10, 40

$x=$ $x=$
$y=$
$z=$

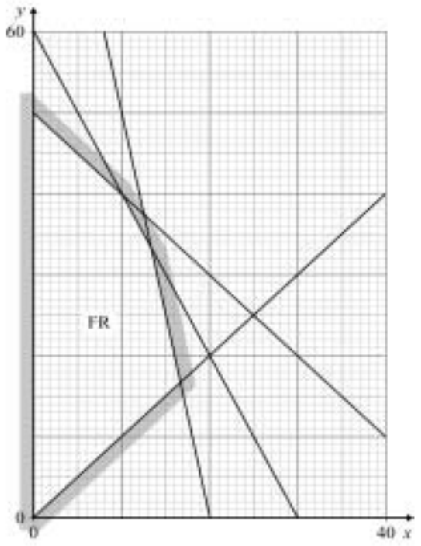
$x \times 5 = 5x = 30A + 30B + 60C$.
$5x = 120$
$y \times 5 = 5y = 20A + 15B + 90$
$5x + 5y = 210$

5

Commentary

Although candidates found the formulation of the inequalities in part (a) difficult, they were then given a simplified version so that they could then draw the graph. Student responses were poor, this solution showing many of the mistakes.
This candidate believes that the graph of y=x is a line drawn at 45 degrees **regardless** of scale. None of the other lines have been drawn correctly. This is work that we would expect a student in Year 10 to be able to do well. It is essential that students practise drawing graphs accurately. Although the line from (0, 60) to (40, 0) was an incorrect line it was still not drawn accurately at the point (0, 60), and if it had been a correct line to draw it would not have scored the marks due to the inaccuracy.

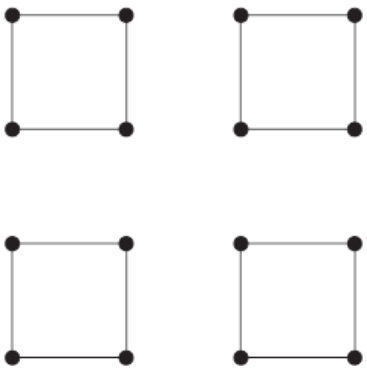
Mark Scheme

(b)(i)	$(z = x)$			
	$4x + 2y \le 120$ OE or $3x + 3y \le 150$ OE	M1		Correct unsimplified subst $x = z$ into either of these 2 correct inequs. (seen)
	$4x + 2y \le 120$ OE $\Rightarrow 2x + y \le 60$ **AG** $3x + 3y \le 150$ OE $\Rightarrow x + y \le 50$ **AG**	A1		Both correct and simplified
	$5x + y \le 100$, $y \ge x$ **AG**	A1	3	Correct subst $x = z$ into 4 correct inequs.
(ii)				
		B1		Line 1 correct at (0, 50) (25, 25)
		B1		Line 2 correct at (10, 50) (20, 0)
		B1		Line 3 correct at (0, 60) (30, 0)
		B1		Line 4 correct at (0, 0) (25, 25)
				Each line correct to $\frac{1}{2}$ square, horizontally or vertically
		B1	5	FR, must have all lines correct **and** labelled region (condone no shading)
(iii)	$N = x + y + z = 2x + y$	M1		Stated or PI
	Max = 60	A1	2	CSO; **SC** unsupported 60 scores 2/2
(iv)	10, 40, 10 11, 38, 11	B1		Any correct; may be earned in part (iii)
	12, 36, 12	B1		3 correct
	13, 34, 13	B1	3	4 correct **and** no extras
	Total		**21**	

Question 7

[**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

(a) The diagram shows a graph with 16 vertices and 16 edges.



(i) On **Figure 3**, add the minimum number of edges to make a connected graph. *(1 mark)*

(ii) On **Figure 4**, add the minimum number of edges to make the graph Hamiltonian. *(2 marks)*

(iii) On **Figure 5**, add the minimum number of edges to make the graph Eulerian. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) The number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(2 marks)*

Student Response

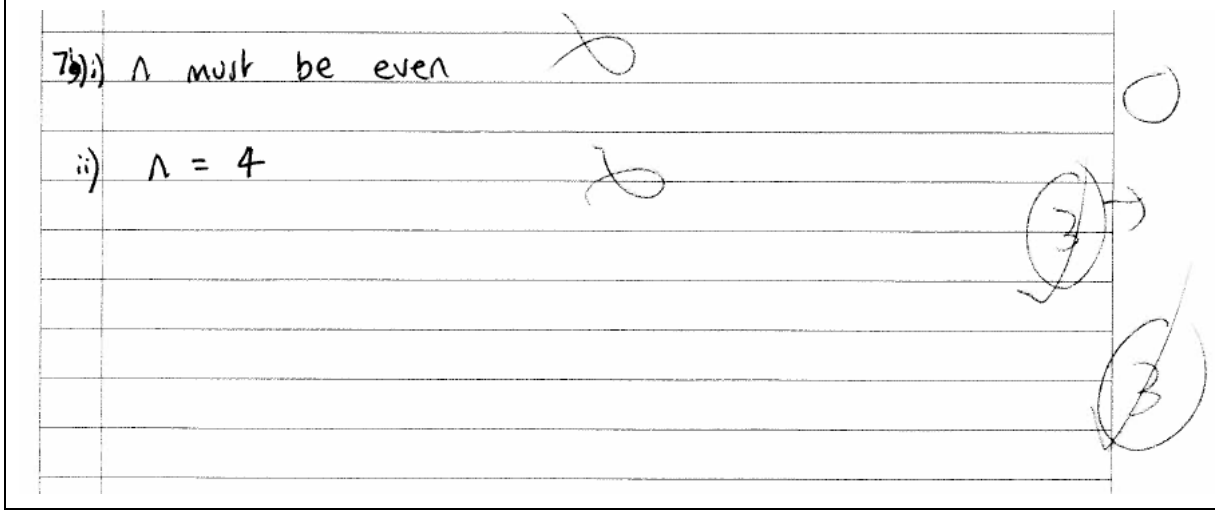


Figure 3 (for use in Question 7(a)(i))

Connected Graph

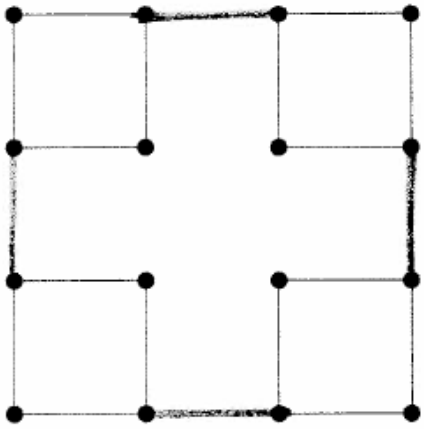


Figure 4 (for use in Question 7(a)(ii))

Hamiltonian Graph

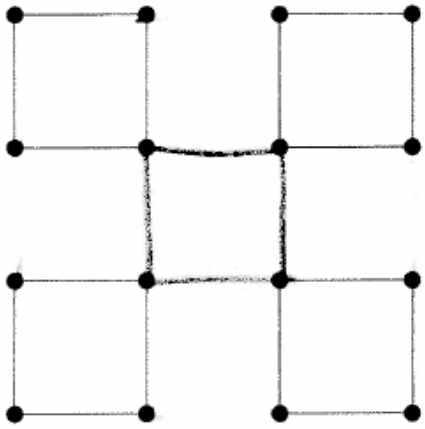
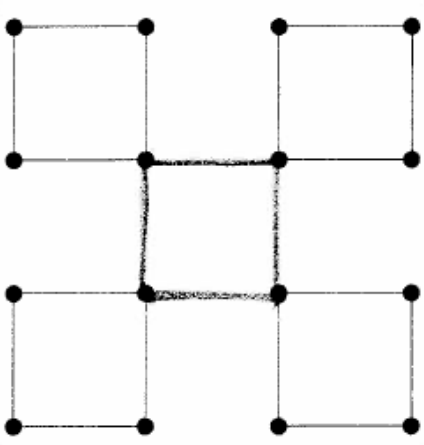


Figure 5 (for use in Question 7(a)(iii))

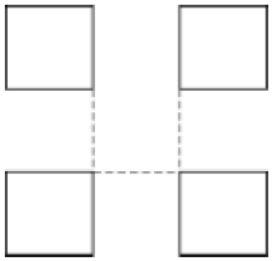
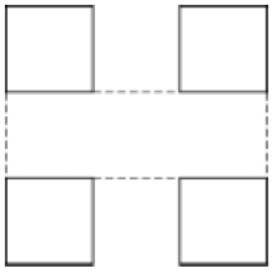
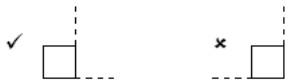
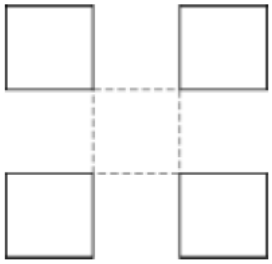
Eulerian Graph



Commentary

Although there were a number of correct responses to this question, this solution was the most common. Candidates do not like graph theory.
In part (a)(i) candidates must remember that a connected graph has to have all vertices connected, but it doesn't have to have cycles. As such this graph has one edge more than is necessary.
In part (a)(ii) the candidate has the correct number of edges, four, but it doesn't make the graph Hamiltonian. As to visit all vertices on this graph you must revisit some of the vertices.
In part (b), the candidate has realised that Eulerian graphs have something to do with even vertices, but the candidate hasn't a clear understanding of the concept. Although the order of the vertices must be even, this means that there must be an odd number of vertices. i.e. for a complete graph with nine vertices there are eight edges at each vertex.

Mark Scheme

Q	Solution	Marks	Total	Comments
7(a)(i)		B1	1	OE
(ii)		M1		4 edges
		A1	2	OE **Note**: new edges must meet each square at vertices on the opposite ends of a side of the square eg 
(iii)		M1		4 edges
		A1	2	Eulerian (all vertices are of even order)
(b)(i)	n odd	B1	1	$(n \pm 1)$ even
(ii)	(Triangle) $n = 3$	B2	2	Triangle, stated or drawn, scores B1
	Total		**8**	