General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MS04**
Unit Statistics 4

Wednesday 18 June 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The volume of fuel consumed by an aircraft making an east–west transatlantic flight was recorded on 10 occasions with the following results, correct to the nearest litre.

68 860	71 266	69 476	68 973	69 318
70 467	71 231	68 977	70 956	69 465

These volumes of fuel may be assumed to be a random sample from a normal distribution with standard deviation σ .

(a) Construct a 99% confidence interval for σ . *(6 marks)*

(b) State one factor that may cause the volume of fuel consumed to vary. *(1 mark)*

2 (a) The discrete random variable X follows a geometric distribution with parameter p.

Prove that $\mathrm{E}(X) = \dfrac{1}{p}$. *(3 marks)*

(b) A fair six-sided die is thrown repeatedly until a six occurs.

(i) State the expected number of throws required to obtain a six. *(1 mark)*

(ii) Calculate the probability that the number of throws required to obtain a six is greater than the expected value. *(3 marks)*

(iii) Find the least value of r such that, when the die is thrown repeatedly, there is more than a 90% chance of obtaining a six on or before the rth throw. *(4 marks)*

3 A geologist is studying the effect of exposure to weather on the radioactivity of granite. He collects, at random, 9 samples of freshly exposed granite and 8 samples of weathered granite. For each sample, he measures the radioactivity, in counts per minute. The results are shown in the table.

	Counts per minute								
Freshly exposed granite	226	189	166	212	179	172	200	203	181
Weathered granite	178	171	141	133	169	173	171	160	

(a) Assuming that these measurements come from two independent normal distributions with a common variance, construct a 95% confidence interval for the difference between the mean radioactivity of freshly exposed granite and that of weathered granite. *(9 marks)*

(b) Comment on a claim that the difference between the mean radioactivity of freshly exposed granite and that of weathered granite is 10 counts per minute. *(2 marks)*

4 The lifetimes of electrical components follow an exponential distribution with mean 200 hours.

(a) Calculate the probability that the lifetime of a randomly selected component is:

(i) less than 120 hours; *(2 marks)*

(ii) more than 160 hours; *(2 marks)*

(iii) less than 160 hours, given that it has lasted more than 120 hours. *(3 marks)*

(b) Determine the median lifetime of these electrical components. *(3 marks)*

5 It is thought that the marks in an examination may be modelled by a triangular distribution with probability density function

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{1875}x & 0 \leqslant x < 50 \\ \dfrac{6}{75} - \dfrac{2}{1875}x & 50 \leqslant x \leqslant 75 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(2 marks)*

(b) A school enters 60 candidates for the examination. The results are summarised in the table.

Marks	0–	25–	50–75
Number of candidates	7	28	25

(i) Investigate, at the 5% level of significance, whether the triangular distribution in part (a) is an appropriate model for these data. *(9 marks)*

(ii) Describe, with a reason, how the test procedure in part (b)(i) would differ for a school entering 15 candidates, assuming that its results are summarised using the same mark ranges as in the table above. *(2 marks)*

Turn over for the next question

6 (a) The IQs of a random sample of 15 students have a standard deviation of 9.1 .

Test, at the 5% level of significance, whether this sample may be regarded as coming from a population with a variance of 225 . Assume that the population is normally distributed. *(6 marks)*

(b) The weights, in kilograms, of 6 boys and 4 girls were found to be as follows.

Boys	53	37	41	50	57	57
Girls	40	46	37	40		

Assume that these data are independent random samples from normal populations.

Show that, at the 5% level of significance, the hypothesis that the population variances are equal is accepted. *(7 marks)*

7 (a) The random variable X has a distribution with unknown mean μ and unknown variance σ^2 .

A random sample of size n, denoted by $X_1, X_2, X_3, \ldots, X_n$, has mean $\overline{X}$ and variance V, where

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad V = \left(\frac{1}{n}\sum_{i=1}^{n} X_i^2\right) - \overline{X}^2$$

(i) Show that

$$\mathrm{E}(X_i^2) = \sigma^2 + \mu^2 \quad \text{and} \quad \mathrm{E}(\overline{X}^2) = \frac{\sigma^2}{n} + \mu^2$$ *(3 marks)*

(ii) Hence show that $\dfrac{nV}{n-1}$ is an unbiased estimator for σ^2 . *(3 marks)*

(b) A random sample of size 2, denoted by X_1 and X_2 , is taken from the distribution in part (a).

Show that $\frac{1}{2}(X_1 - X_2)^2$ is an unbiased estimator for σ^2 . *(4 marks)*

END OF QUESTIONS