Teacher Support Materials
2008

Maths GCE

Paper Reference MPC2

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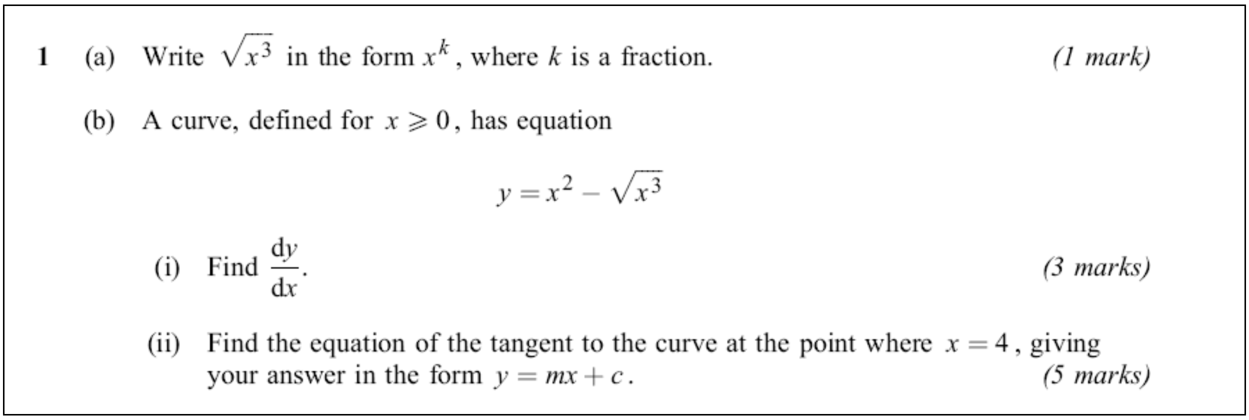
Question 1



1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

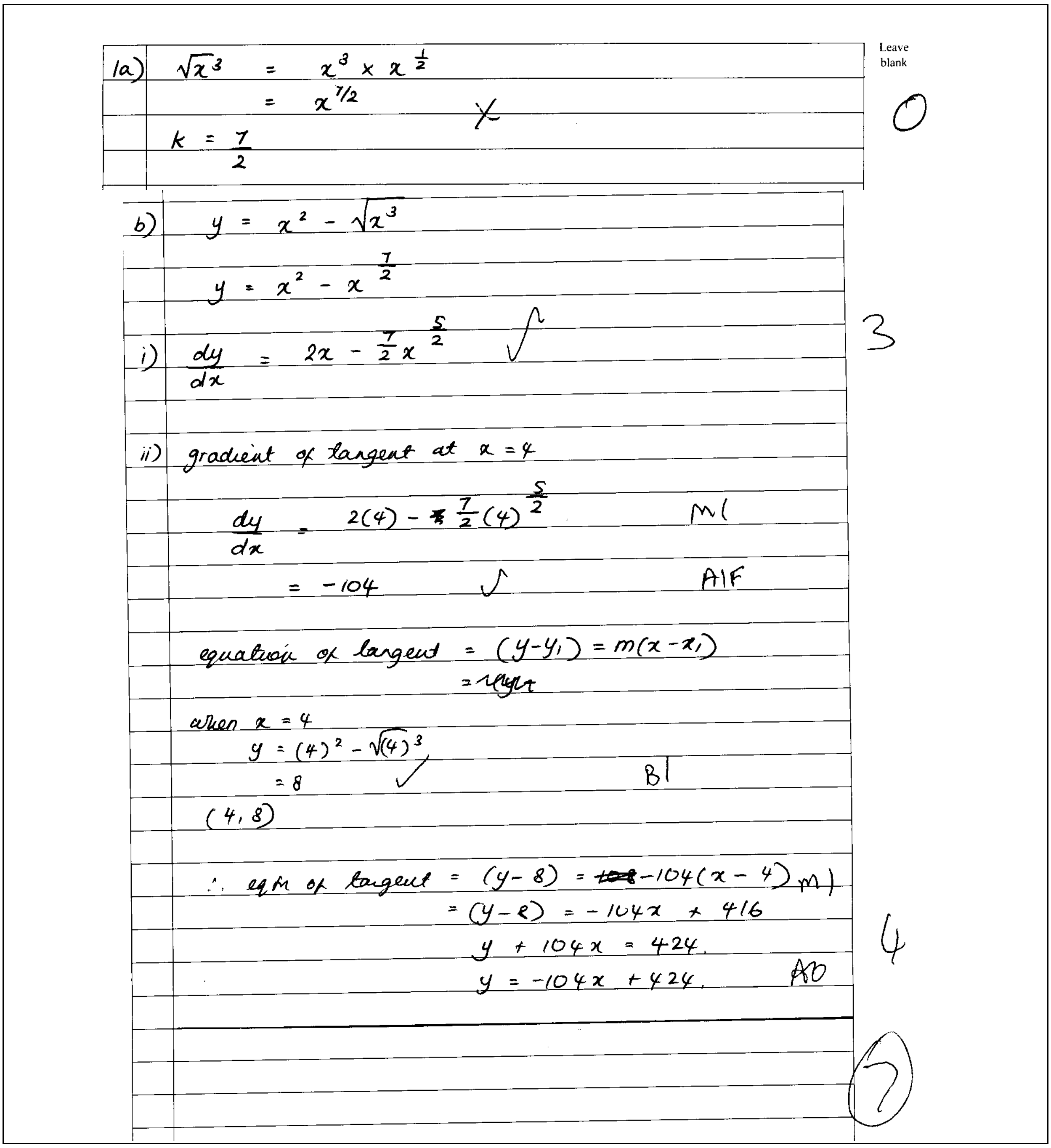
(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

Student Response



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1a) $\sqrt{x^3} = x^3 \times x^{\frac{1}{2}}$

$= x^{7/2}$ ✗

$k = \frac{7}{2}$

0

b) $y = x^2 - \sqrt{x^3}$

$y = x^2 - x^{\frac{7}{2}}$

i) $\frac{dy}{dx} = 2x - \frac{7}{2}x^{\frac{5}{2}}$ ✓

3

ii) gradient of tangent at $x = 4$

$\frac{dy}{dx} = 2(4) - \frac{7}{2}(4)^{\frac{5}{2}}$ M1

$= -104$ ✓ A1F

equation of tangent $= (y - y_1) = m(x - x_1)$

$=$ ~~[illegible]~~

when $x = 4$

$y = (4)^2 - \sqrt{(4)^3}$

$= 8$ ✓ B1

$(4, 8)$

$\therefore$ eqn of tangent $= (y - 8) =$ ~~[illegible]~~ $-104(x - 4)$ M1

$= (y - 8) = -104x + 416$

$y + 104x = 424.$

$y = -104x + 424.$ A0

4

7

Commentary

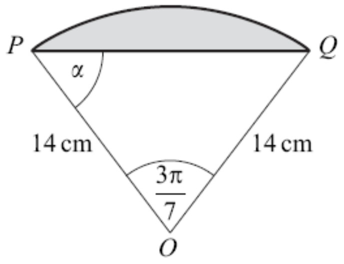
Writing $\sqrt{x^3}$ in the form x^k caused candidates more problems than anticipated. The exemplar illustrates a common wrong answer. The candidate has applied the index law $\left(x^m\right)^n = x^{m\times n}$ incorrectly as $\left(x^m\right)^n = x^m \times x^n$ to obtain the wrong value, $\frac{7}{2}$, for k. In part (b)(i) the candidate gains full marks for correctly differentiating the expression for y with their value of k obtained in part (a). A significant minority of candidates did not carry out this differentiation correctly. In part (b)(ii) many candidates realised that the gradient of the tangent, m, was given by the value of $\frac{dy}{dx}$ at $x = 4$ but fewer candidates applied a correct method to find the value of c. The exemplar illustrates a correct method to find the equation of the tangent, in particular the crucial step of finding the value for y when $x = 4$ and using the point (4, 8) to complete the solution. The candidate was awarded all the marks except for the final A1 which was only given to those who obtained the answer '$y = 5x-12$'. Candidates who found the equation of the normal to the curve instead of the tangent lost the final two marks.

Mark scheme

1(a)	$\sqrt{x^3} = x^{\frac{3}{2}}$	B1	1	OE; accept '$k = 1.5$'
(b)(i)	$\frac{dy}{dx} = 2x - \frac{3}{2}x^{\frac{1}{2}}$	M1		At least one index reduced by 1 and no term of the form $\sqrt{ax^2}$.
		B1		For $2x$
		A1F	3	For $-1.5\,x^{0.5}$ .Ft on ans (a) non-integer k
(ii)	When $x = 4$, $y = 8$	B1		
	$y'(4) =$;	M1		Attempt to find $\frac{dy}{dx}$ when $x = 4$
	$= 2(4) - 1.5(\sqrt{4}) = 5$	A1F		Ft on one earlier error provided non-integer powers in (a) and (b)(i)
	Tangent: $y - 8 = 5(x - 4)$	m1		$y - y(4) = y'(4)[x - 4]$ OE
	$y = 5x - 12$	A1	5	CSO; must be $y = 5x - 12$
	Total		**9**	

Question 2

2 The diagram shows a shaded segment of a circle with centre O and radius 14 cm, where PQ is a chord of the circle.



In triangle OPQ, angle $POQ = \frac{3\pi}{7}$ radians and angle $OPQ = \alpha$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) Find α in terms of π. *(2 marks)*

(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. *(2 marks)*

Student response

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2a) $r\theta$ = arc length ✓

$14 \times \left(\frac{3\pi}{7}\right) = \frac{42\pi}{7}$

$= 6\pi$ cm. ✓

2

b) $\frac{14}{\sin\frac{3\pi}{7}} = \frac{14}{\sin\alpha}$ ✗ MO

$14\sin\alpha = 14\sin\frac{3\pi}{7}$

$= \sin\alpha = \frac{14\sin\frac{3\pi}{7}}{14}$

$= \frac{196\sin 3\pi}{7}$

$= \frac{28\sin 3\pi}{7}$

$= \frac{1}{6}\pi.$ ✗

0

c) $14 + (\cancel{14}\ 14 \times (\frac{3\pi}{7}) = 32.8$ cm. ✗ MO

0

2

Commentary

In part (a) the exemplar illustrates good examination technique. The correct general formula for arc length has been quoted and substitution of 14 for *r* and $\frac{3\pi}{7}$ for θ has been clearly shown. The candidate evaluated the product correctly and, as requested, the answer has been left as a multiple of π. In parts (b) and (c) the exemplar illustrates a common error. The candidate has assumed incorrectly that triangle *OPQ* is equilateral and has used the length of chord *PQ* to be 14 in the sine rule. There are also further manipulation errors in part (b) but, even without these, writing *PQ* as 14 has resulted in no further marks being available.

Mark Scheme

2(a)	Arc $PQ = r\theta$ $= 6\pi$ (cm)	M1 A1	 2	$r\theta$ Condone missing units throughout the paper
(b)	$\alpha + \alpha + \frac{3\pi}{7} = \pi$	M1		OE
	$\alpha = \frac{2\pi}{7}$	A1	2	Accept equivalent fractions eg $\frac{4\pi}{14}$ and condone 0.286π or better
(c)	Chord $PQ = 2 \times 14 \times \cos\alpha$	M1		OE eg $2 \times 14 \times \sin\frac{3\pi}{14}$ or 17.45-17.5 inclusive or $\sqrt{14^2 + 14^2 - 2 \times 14^2 \times \cos\frac{3\pi}{7}}$
	Perimeter = 17.45… + 6π = 36.307… = 36.3 (cm)	A1	2	Condone > 3sf
	Total		**6**	

Question 3

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + \dots$$

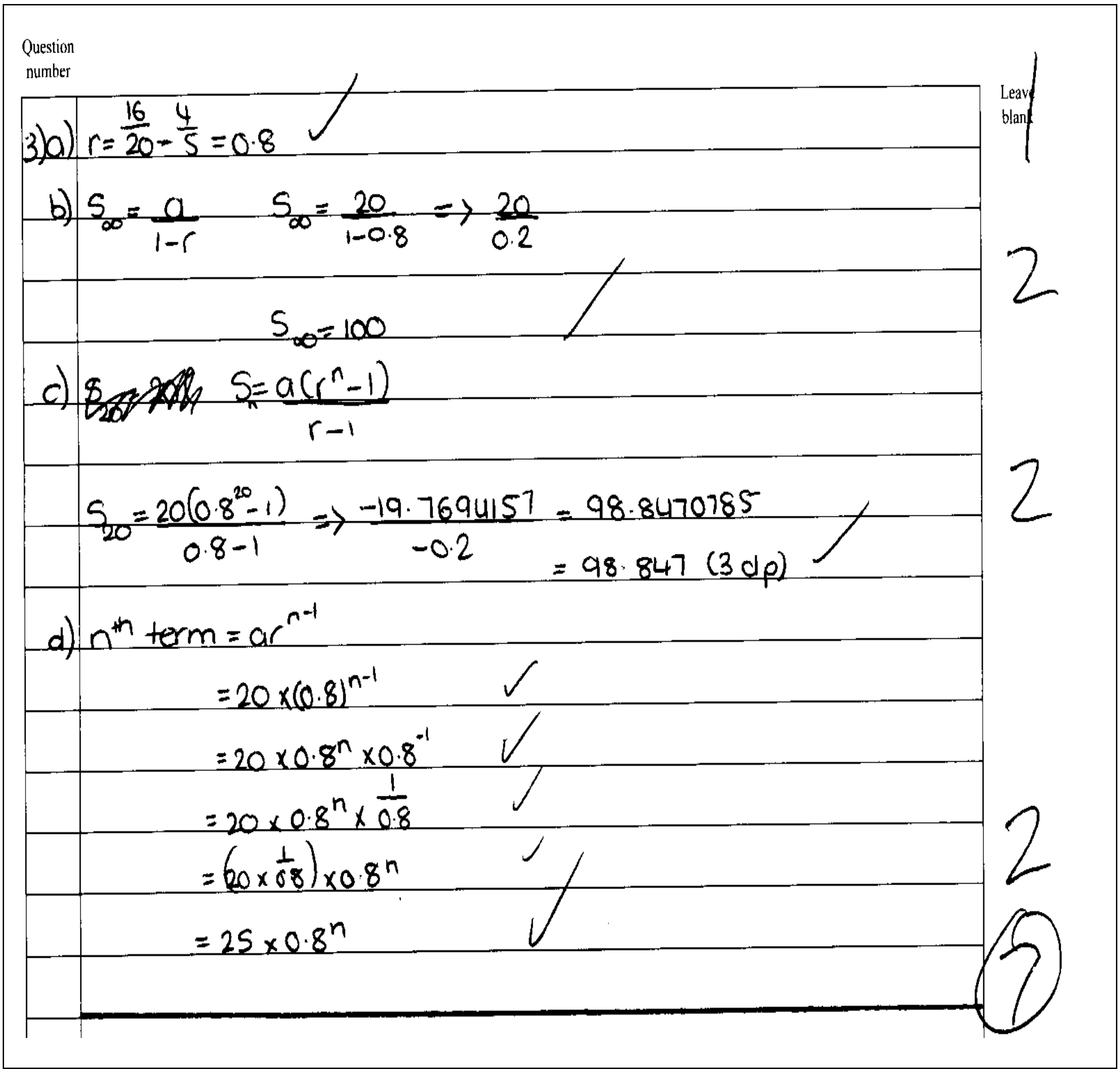
(a) Find the common ratio of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

Student Response



Question number

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3)a) $r = \frac{16}{20} = \frac{4}{5} = 0.8$ ✓ 1

b) $S_\infty = \frac{a}{1-r}$ $\quad S_\infty = \frac{20}{1-0.8} \Rightarrow \frac{20}{0.2}$

$S_\infty = 100$ ✓ 2

c) $S_n = \frac{a(r^n-1)}{r-1}$

$S_{20} = \frac{20(0.8^{20}-1)}{0.8-1} \Rightarrow \frac{-19.7694157}{-0.2} = 98.8470785$

$= 98.847$ (3 dp) ✓ 2

d) n^{th} term $= ar^{n-1}$

$= 20 \times (0.8)^{n-1}$ ✓

$= 20 \times 0.8^n \times 0.8^{-1}$ ✓

$= 20 \times 0.8^n \times \frac{1}{0.8}$ ✓

$= \left(20 \times \frac{1}{0.8}\right) \times 0.8^n$ ✓

$= 25 \times 0.8^n$ ✓ 2

7

Commentary

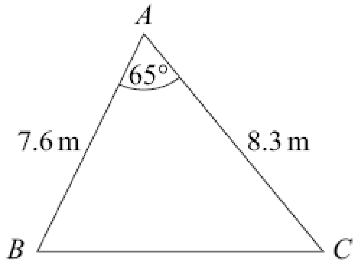
The exemplar illustrates good examination technique. In part (a) the correct value of the common ratio, r, is found to be 0.8. A common wrong answer was $r = 1.25$. Since part (b) asked for the sum to infinity, which only exists if $|r|<1$, candidates who obtained $r = 1.25$ in part (a) would have been well advised to consider the condition $|r|<1$ before proceeding. In parts (b) and (c) the exemplar again shows good practice, with relevant general formulae quoted correctly and substitution clearly shown. The candidate evaluated both the numerators and denominators before carrying out the divisions thus avoiding, for example, the common wrong evaluation of $\frac{20}{1 - 0.8}$ as 19.2. In part (c) the candidate has sensibly given the more accurate answer before rounding to the three decimal places requested. This practice frequently avoids marks being lost for rounding errors. The candidate presented an excellent step-by-step proof in part (d), by first quoting, from the formulae booklet, the general formula for the nth term of the geometric series. The critical step, replacing 0.8^{n-1} by $0.8^n \times 0.8^{-1}$, has been clearly shown in the candidate's proof.

Mark Scheme

3(a)	$r = 16 \div 20 = 0.8$	B1	1	OE
(b)	$\frac{a}{1-r} = \frac{20}{1-0.8}$	M1		OE Using a correct formula with $a = 20$ or $r =$ c's 0.8
	$= 100$	A1F	2	ft on c's value of r provided $\lvert r \rvert < 1$
(c)	$\{S_{20} =\}\frac{a(1-r^{20})}{1-r}$	M1		OE Using a correct formula with $n = 20$
	$= 100(1-0.8^{20})$ = 98.847{07..}	A1	2	Condone > 3dp
(d)	nth term = $20\ r^{n-1} = 20(0.8)^{n-1}$ $= 20 \times 0.8^{-1} \times 0.8^n$	M1		Ft on c's r . Award even if 16^{n-1} seen
	$= 25 \times 0.8^n$	A1	2	CSO; AG
	Total		**7**	

Question 4

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

Student Response

4a) $a^2 = b^2 + c^2 - 2bc\cos A$

$= 7.6^2 + 8.3^2 - 2 \times 7.6 \times 8.3 \times \cos 65$ ✓

~~$= 126.65 - 126.16\cos 65$~~

~~$= 126.65 - 53.31875$~~

~~$= 72.6824801$~~

~~$a = \sqrt{72.6824801} = 8.525$~~ ✓

$a^2 = 73.3324801$

$a = \sqrt{73.3324801}$ ✓

$= 8.5634\ldots m$

$= 8.56m$ to 3 SF.

3

4b $\frac{1}{2} \times a \times b \times \sin C$

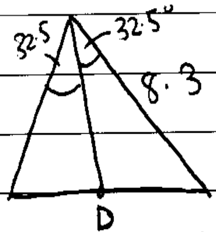
$= \frac{1}{2} \times 7.6 \times 8.3 \times \sin 65$ ✓

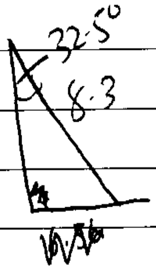
$= 28.584\ldots m^2$ ✓ sin

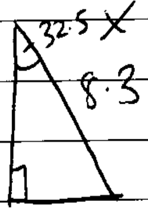
$= 28.6 m^2$

2

C







Mo

0

$(\cos 32.5) \times 8.3$

$= 7.00 m.$

x

5

Commentary

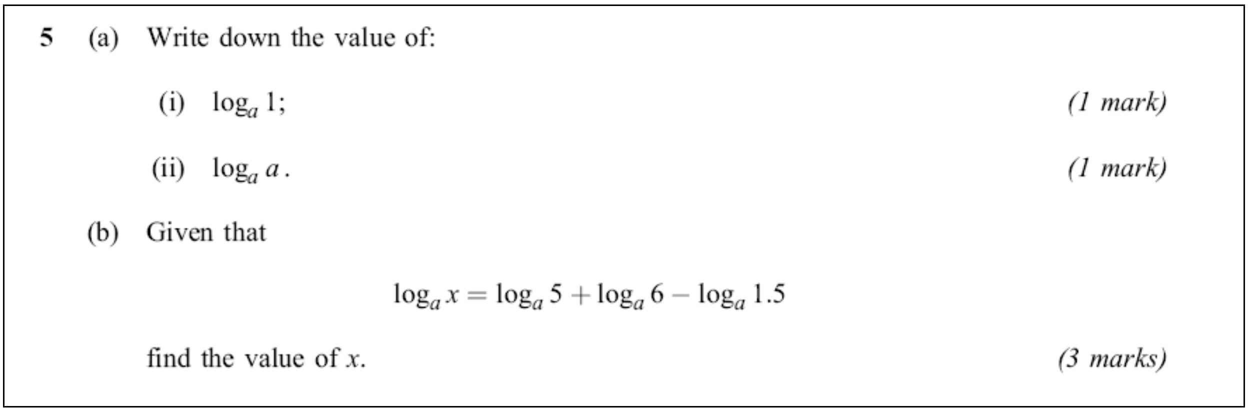
In part (a) the candidate has sensibly quoted the Cosine rule from the formulae booklet and substituted values correctly without performing any initial calculations. The next four lines have been crossed out even though the first error did not occur until the third of these lines; 126 instead of 126.65 has been used in the subtraction. The candidate has replaced these four lines by the correct 'unrounded' evaluation, has gone on to take the square root and has given a value for BC (8.5634) which was to a greater degree of accuracy than the printed three significant figure value.

If the candidate had only shown three significant figure values the final accuracy mark could not have been awarded. In part (b), correct substitution into a correct formula for the area of the triangle with correct evaluation has resulted in all marks being awarded. Again we see good examination technique in that the non-rounded answer is given before rounding has taken place. The exemplar illustrates a common error in part (c). The candidate has incorrectly assumed that the perpendicular, AD, bisects angle A and so no marks for (c) have been awarded.

Mark Scheme

4(a)	$\{BC^2 =\}\ 7.6^2 + 8.3^2 - 2 \times 7.6 \times 8.3 \cos 65$	M1		RHS of cosine rule used
	….. $= 57.76 + 68.89 - 53.3175...$	m1		Correct order of evaluation
	$BC = \sqrt{73.33..} = 8.563..$ (= 8.56 m)	A1	3	AG; must see $\sqrt{73.33....}$ or > 3sf value
(b)	Area triangle $= \frac{1}{2} \times 7.6 \times 8.3 \times \sin 65$	M1		Use of $\frac{1}{2} bc \sin A$ OE
	= 28.58… = 28.6 (m^2)	A1	2	Condone > 3sf
(c)	Area of triangle = 0.5× BC × AD	M1		Or valid method to find sin$B$ or sin$C$
	AD = [Ans (b)] ÷ [0.5×Ans (a)]	m1		Or AD = 7.6sin$B$; Or AD = 8.3sin$C$
	AD = 6.67.. = 6.7 (m)	A1	3	If not 6.7 accept 6.65 to 6.69 inclusive.
	Total		**8**	

Question 5



5 (a) Write down the value of:

(i) $\log_a 1$; *(1 mark)*

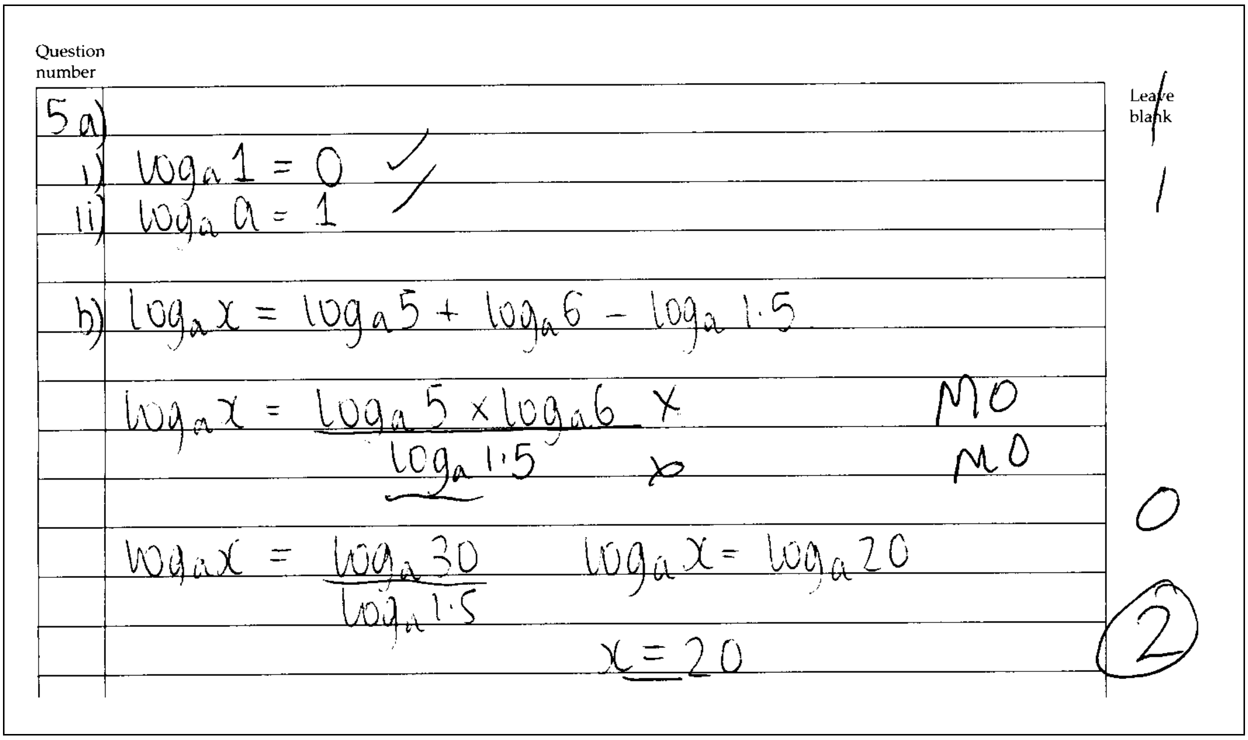
(ii) $\log_a a$. *(1 mark)*

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x. *(3 marks)*

Student Response



Question number

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5 a)

i) $\log_a 1 = 0$ ✓ 1

ii) $\log_a a = 1$ ✓ 1

b) $\log_a x = \log_a 5 + \log_a 6 - \log_a 1{\cdot}5$

$\log_a x = \dfrac{\log_a 5 \times \log_a 6}{\log_a 1{\cdot}5}$ ✗ M0 ✗ M0

0

$\log_a x = \dfrac{\log_a 30}{\log_a 1{\cdot}5}$ $\log_a x = \log_a 20$

$\underline{\underline{x = 20}}$

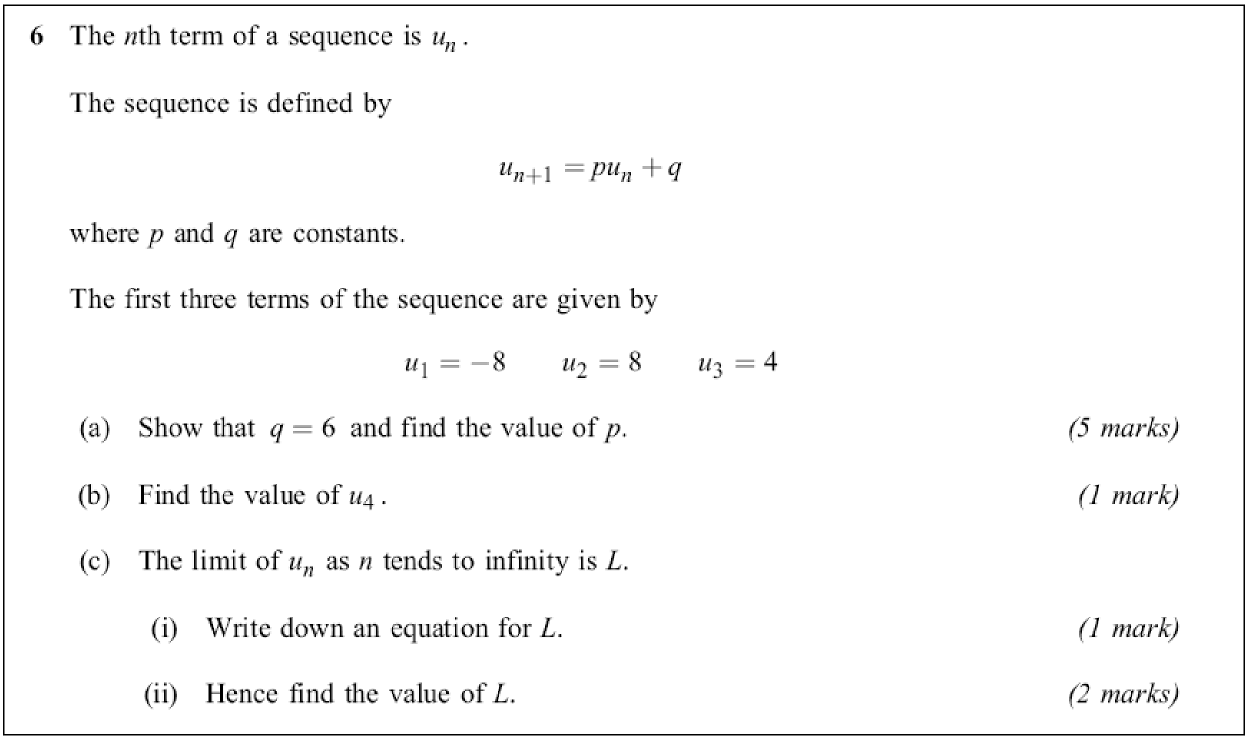
$\bigcirc\!\!\!\!2$

Commentary

Generally most candidates answered part (a) correctly but there was evidence of the use of wrong logarithmic laws in obtaining the ‘correct’ values of x in part (b). In the exemplar, the candidate gave the correct values for $\log_a 1$ and $\log_a a$, but in part (b) the candidate has used wrong laws of logarithms, namely, $\log m + \log n = \log m \times \log n$ and $\log m - \log n = \dfrac{\log m}{\log n}$ so no marks can be awarded even though the correct value of x has been stated.

5(a)(i)	$\log_a 1 = 0$	B1	1	
(ii)	$\log_a a = 1$	B1	1	
(b)	$\log_a x = \log_a (5 \times 6) - \log_a 1.5$ $\log_a x = \log_a \left(\frac{5 \times 6}{1.5}\right)$ $\log_a x = \log_a 20 \Rightarrow x = 20$	M1 M1 A1	 3	One law of logs used correctly A second law of logs used correctly
	Total		**5**	

Question 6



6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

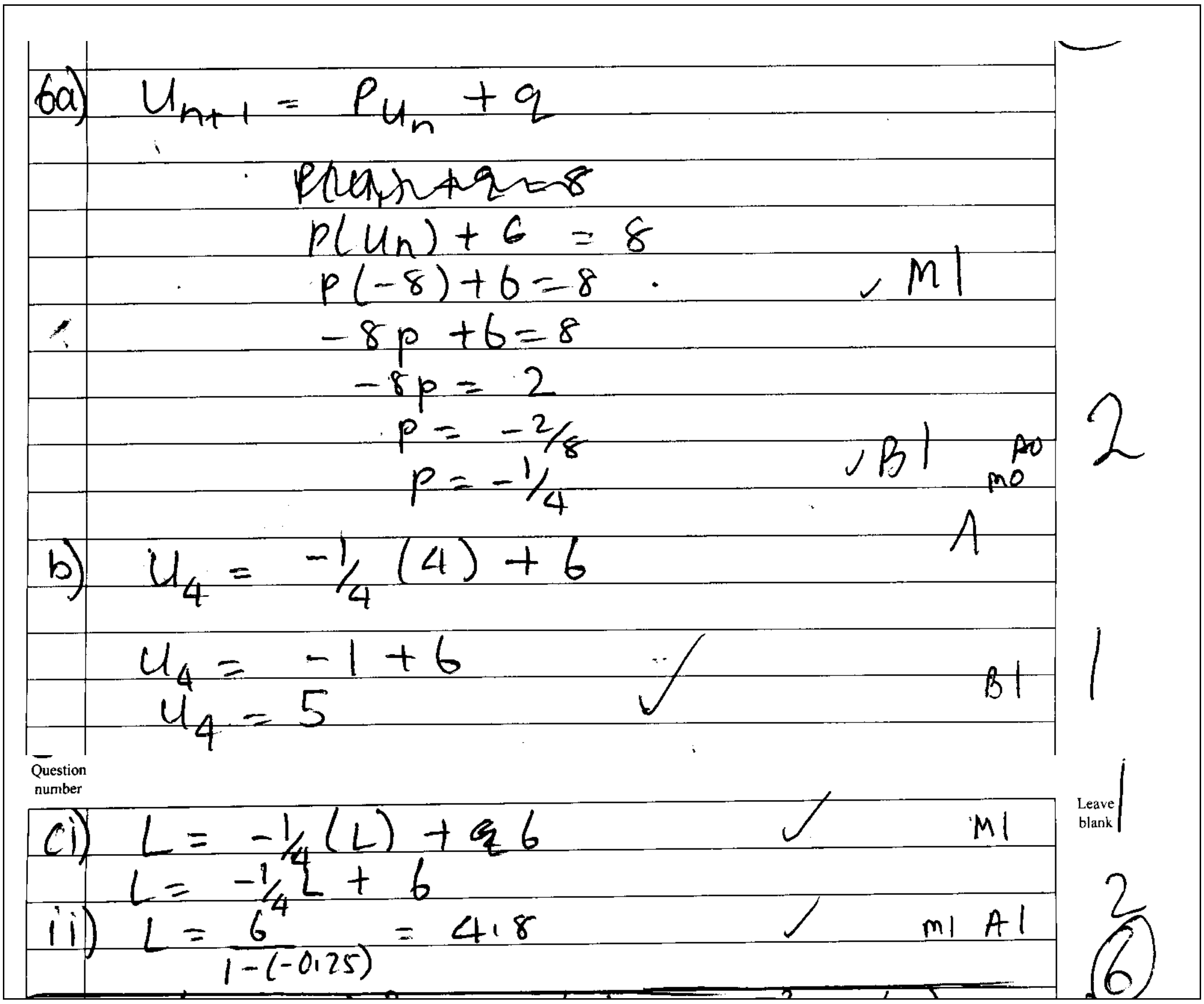
(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

Student Response



6a) $U_{n+1} = pu_n + q$

~~$p(u_n)+q=8$~~

$p(U_n) + 6 = 8$

$p(-8)+6=8$ ✓ M1

$-8p+6=8$

$-8p = 2$

$p = -2/8$

$p = -1/4$ ✓B1 A0 M0

A

2

b) $u_4 = -\frac{1}{4}(4) + 6$

$u_4 = -1 + 6$ ✓ B1

$u_4 = 5$

1

Question number

Leave blank

1

ci) $L = -\frac{1}{4}(L) + ~~q~~ 6$ ✓ M1

$L = -\frac{1}{4}L + 6$

ii) $L = \frac{6}{1-(-0.125)} = 4.8$ ✓ m1 A1

2

(6)

Commentary

In part (a) a significant number of candidates failed to form and solve two equations in p and q. This is illustrated in the exemplar where the candidate has used the printed value of q with the values of u_1 and u_2 to form and solve an equation in p only. No attempt was made to show that $q = 6$. The candidate has not used the other given information, $u_3 = 4$, in the solution for part (a). Those candidates who used u_3 with u_2 to form $4 = 8p + q$ and u_2 with u_1 to form $8 = -8p + q$ usually went on to correctly solve these equations simultaneously for five marks. Many candidates found the correct value of u_4 using the method illustrated in the exemplar. Part (c) defeated many candidates. The correct method is illustrated in the exemplar. The candidate's first equation displayed a thorough understanding of the topic as u_n and u_{n+1} were replaced by their limiting value, L. The candidate, having written the equation, then went on to rearrange and solve it to obtain the correct value, 4.8, for L. Some candidates just wrote down the answer 4.8 but this gained no credit as an equation for L had not been written down.

Mark Scheme

6(a)	$8 = -8p + q$	M1		Either equation. PI eg by combined eqn.
	$4 = 8p + q$	A1		Both (condone embedded values for the M1A1)
		m1		Valid method to solve two simultaneous equations in p **and** q to find either p or q
	$q = 6$	A1		AG (condone if left as a fraction)
	$p = -0.25$	B1	5	OE
(b)	$u_4 = 5$	B1F	1	Ft on $(6 + 4p)$
(c)(i)	$L = pL + q$; $(L = -0.25L + 6)$	M1	1	OE
(ii)	$L = \frac{q}{1-p}$	m1		Rearranging
	$L = \frac{6}{1.25} = 4.8$	A1F	2	Ft on $\frac{6}{1-p}$ Dependent on previous two marks
	Total		**9**	

Question 7

7 (a) The expression $\left(1+\frac{4}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.
(3 marks)

(b) (i) Hence find $\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_1^2\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

Student Response

7a) $(1+ax)^n$

$1 + nax + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3$

$(1+\frac{4}{x^2})^3$

$n = 3$

$ax = \frac{4}{x^2}$

✓ ✓

$1 + (3\times\frac{4}{x^2}) + \frac{3(3-1)}{2!}(\frac{4}{x^2})^2 + \frac{3(3-1)(3-2)}{3!}(\frac{4}{x})^3$

$1 + \frac{12}{x^2} + 3\times(\frac{4}{x^2})^2 + 1\times(\frac{4}{x^2})^3$

✓ ✓

$1 + \frac{12}{x^2} + \frac{48}{x^4} + \frac{64}{x^6}$

3

$p = 12$ ✓

$q = 48$

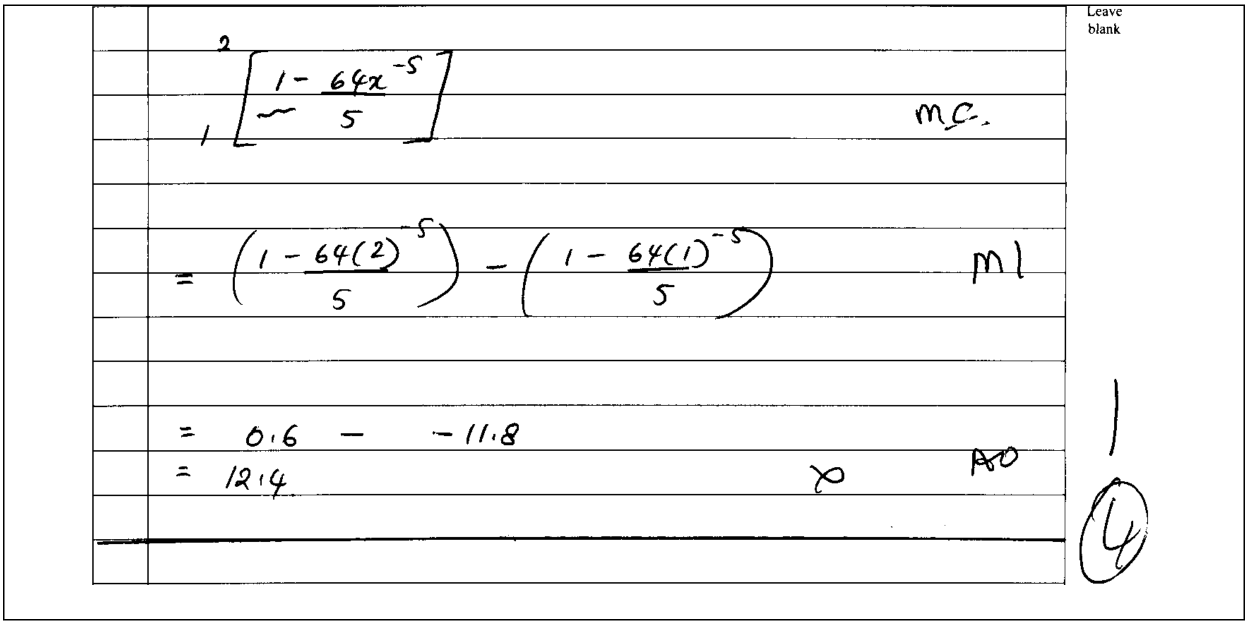
bi) $\int(1+\frac{4}{x^2})^3$

$1^3 + (\frac{4}{x^2})^3 \overset{\times}{=} 1 + \frac{64}{x^6} = \cancel{64}\ 1 + 64x^{-6}$

$\int(1+64x^{-6}) = x + \frac{64x^{-5}}{5}$

M0

0



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$$\left[1-\frac{64x^{-5}}{5}\right]_1^2$$ mc.

$$=\left(1-\frac{64(2)^{-5}}{5}\right)-\left(1-\frac{64(1)^{-5}}{5}\right)$$ M1

$= 0.6 - -11.8$

$= 12.4$ ✗ A0

1

④

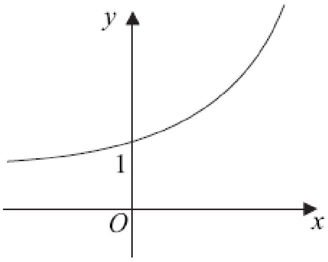
Commentary

In part (a) the candidate has quoted the general form of the binomial expansion with the correct expression, $\frac{4}{x^2}$, for ax and the correct value for n, simplified the terms to obtain the correct expansion and stated the correct values for p and q. Part (b)(i) of Question 7 starts with 'Hence', a word deliberately used by the examiner to give guidance to candidates that part (a) should be used. In the exemplar the candidate ignores this guidance and expands $\left(1+\frac{4}{x^2}\right)^3$ incorrectly this time, as a two term expression. The only mark available to the candidate, in the remaining parts of the question, following this error has been scored for showing how to deal correctly with the limits in the definite integral.

7(a)	$\left(1+\frac{4}{x^2}\right)^3 =$			
	$\left[1^3\right]+3(1^2)\left(\frac{4}{x^2}\right)+3(1)\left(\frac{4}{x^2}\right)^2+\left[\left(\frac{4}{x^2}\right)^3\right]$	M1		Any valid method as far as term(s) in $1/x^2$ and term(s) in $1/x^4$
	$= [1]+\frac{12}{x^2}+\frac{48}{x^4}+\left[\frac{64}{x^6}\right]$	A1		p = 12 Accept $\frac{12}{x^2}$ even within a series
		A1	3	q = 48 Accept $\frac{48}{x^4}$ even within a series
(b)(i)	$\int\left(1+\frac{4}{x^2}\right)^3\, dx$			
	$=\int\ (1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6})\ dx$	M1		Integral of an 'expansion', at least 3 terms PI by the next line
	$= x-px^{-1}-\frac{q}{3}x^{-3}-\frac{64}{5}x^{-5}\ (+c)$	m1 A2F,1	4	At least two powers correctly obtained Ft on c's non-zero integer values for p and q (A1F for two terms correct; can be unsimplified)
	$= x-12x^{-1}-16x^{-3}-\frac{64}{5}x^{-5}\ (+c)$			Condone missing c but check that signs have been simplified at some stage before the award of both A marks.
(ii)	$\left(2-\frac{p}{2}-\frac{q}{3(8)}-\frac{64}{5(32)}\right)-$			
	$\left(1-p-\frac{q}{3}-\frac{64}{5}\right)$	M1		F(2) − F(1), where F(x) is cand's answer or the correct answer to (b)(i).
	= 33.4	A1	2	CSO
	Total		**9**	

Question 8

8 The diagram shows a sketch of the curve with equation $y = 6^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, \mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, \mathrm{d}x$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(2 marks)*

Student Response

(8)

$\int_0^2 6^{x}\,dx$

$\frac{2-0}{4} = \frac{1}{2}$ B1

$n = \frac{1}{2}$

x	0	0.5	1	1.5	2
	1	2.449	6	14.697	36

when $x = 0$

$6^0 = 1$

when $x = 0.5$

$6^{0.5} = 2.449$

when $x = 1$

$6^1 = 6$

when $x = 1.5$

$6^{1.5} = 14.697$

when $x = 2$

$6^2 = 36$

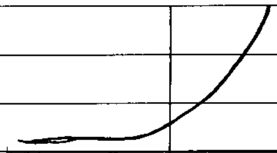
M1 A1

$\frac{1}{2} \times \frac{1}{2}\left(1 + 36 + 2\left(2.449 + 6 + 14.697\right)\right)$

$= 20.823$

~~= 20.823~~ $= 20.82$ A0

(ii)

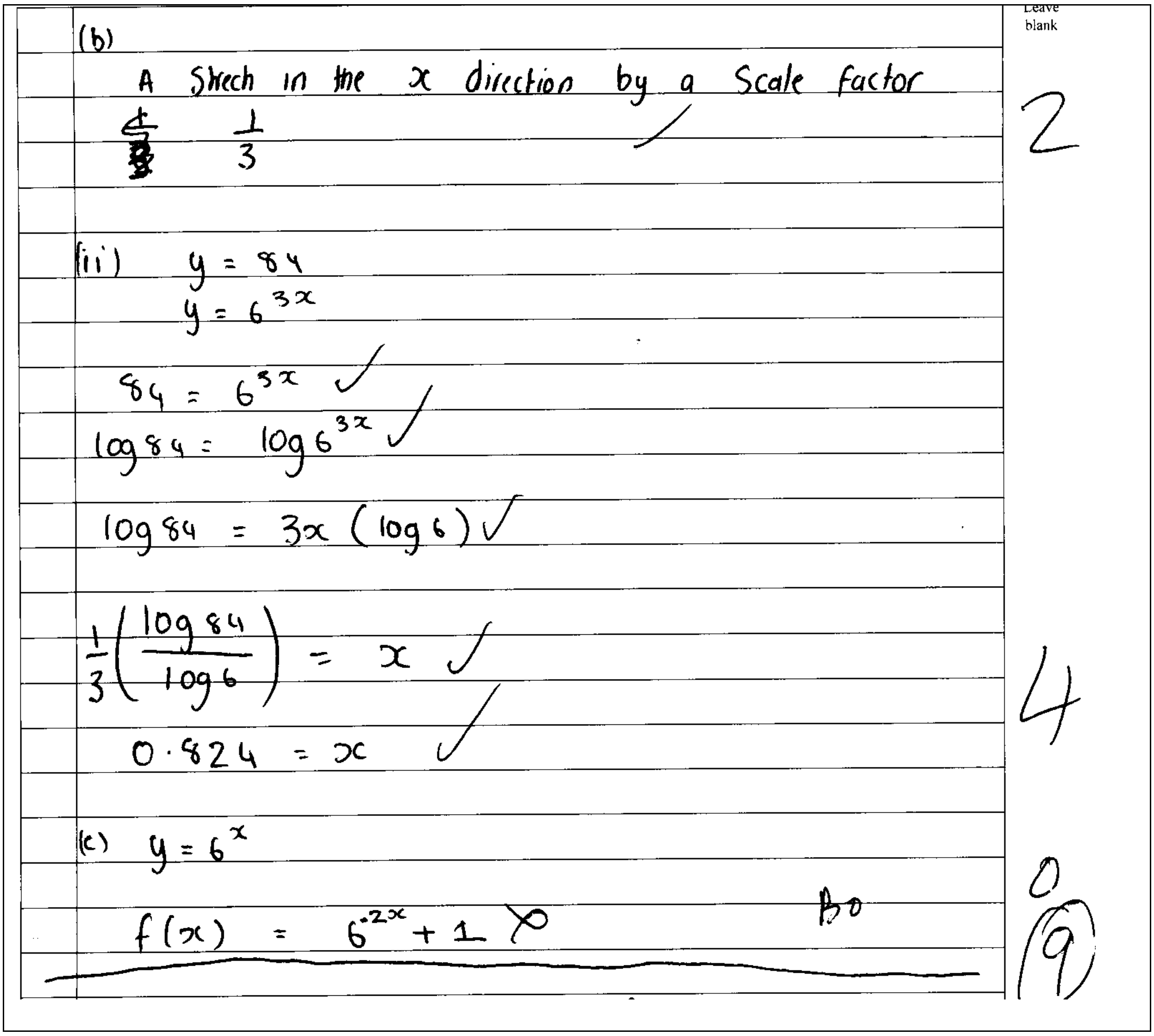


The graph is overestimate

M0

3

0



Leave blank

(b)

A Stech in the x direction by a Scale factor

~~$\frac{4}{3}$~~ $\frac{1}{3}$ ✓

2

(ii) $y = 84$

$y = 6^{3x}$

$84 = 6^{3x}$ ✓

$\log 84 = \log 6^{3x}$ ✓

$\log 84 = 3x(\log 6)$ ✓

$\frac{1}{3}\left(\frac{\log 84}{\log 6}\right) = x$ ✓

$0.824 = x$ ✓

4

(c) $y = 6^{x}$

$f(x) = 6^{-2x} + 1$ ✗ B0

0

(9)

Commentary

In the exemplar the candidate applied the trapezium rule correctly but failed to give the final answer to the required three significant figure accuracy and so did not score the final accuracy mark. The candidate did not draw any trapeziums on the diagram and did not explain or justify the statement ‘The graph is overestimate’. For full marks candidates needed to show four relevant trapeziums on a copy of the sketch of the curve to explain that the sum of the areas of these trapeziums was greater than the area of the region under the curve.

In the exemplar the candidate gave a full correct description (condoning the spelling mistake) of the geometrical transformation required in part (b)(i). In part (b)(ii) the candidate formed the correct equation, $6^{3x} = 84$, and solved it correctly using logarithms, showing clearly the steps involved including the use of the logarithmic law $\log a^{n} = n\log a$. The candidate’s answer, $f(x) = 6^{-2x} + 1$, was incorrect and did not match either component in the given translation vector so no marks were awarded.

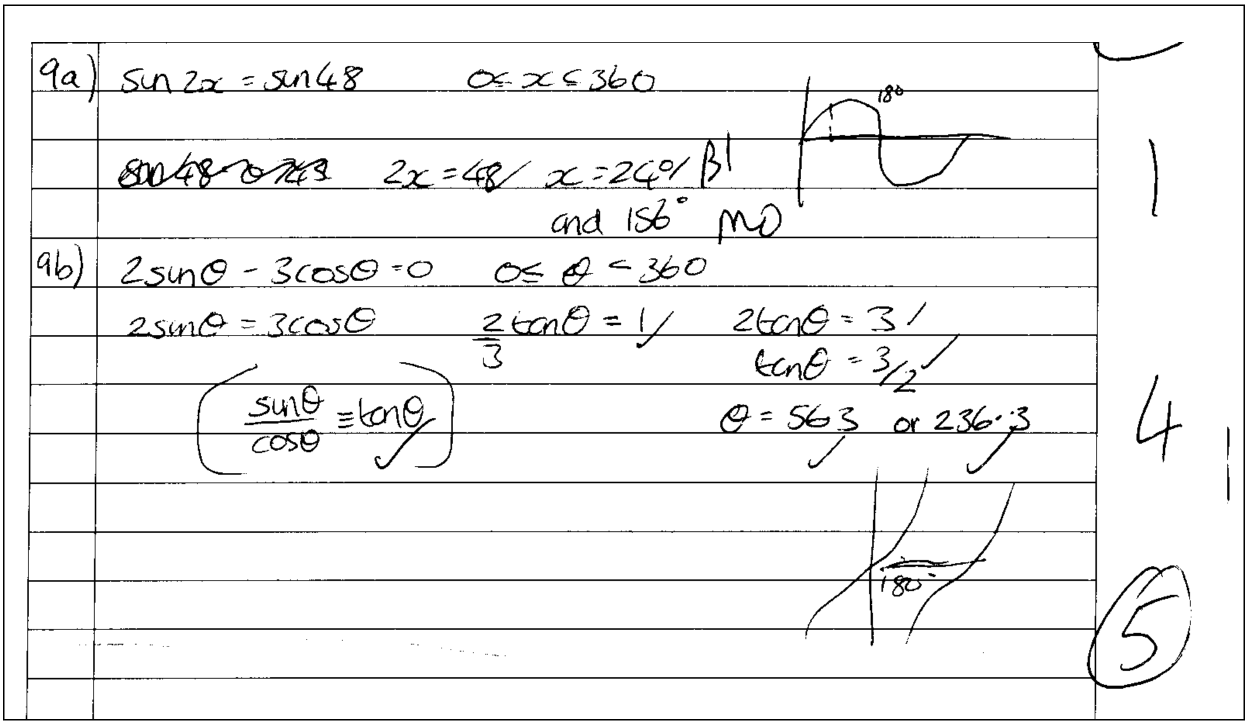
8(a)(i)	$h = 0.5$ Integral = $h/2$ {……}	B1		PI
	{..}=f(0)+2[f$\left(\frac{1}{2}\right)$+f(1)+ f$\left(\frac{3}{2}\right)$]+f(2)	M1		OE summing of areas of the four traps.
	{}= $1+2\left[\sqrt{6}+6+6\sqrt{6}\right]+36$ = 1+2[2.449..+ 6 + 14.6969..] + 36 = 37 + 2 × 23.146.. = 83.292…	A1		Condone 1 numerical slip. Accept 3sf values if not exact.
	Integral = 0.25 × 83.292.. = 20.8 (3sf)	A1	4	CAO; must be 20.8
(ii)	Relevant trapezia drawn on a copy of given graph	M1		Accept single trapezium with its sloping side above the curve
	{Approximation is an}overestimate	A1	2	Dep. on 4 trapezia with each of their upper vertices lying on the curve
(b)(i)	Stretch **(I)** in x-direction **(II)**	M1		Need **(I)** and one of **(II)**, (**III**) M0 if more than one transformation
	(scale factor) $\frac{1}{3}$ **(III)**	A1	2	
(ii)	$6^{3x}=84$	M1		PI
	$\log_{10}6^{3x}=\log_{10}84$	M1		Take logs of both sides of $a^x = b$, PI by 'correct' value(s) later or $3x=\log_6 84$
	$3x\log_{10}6=\log_{10}84$	m1		Use of $\log 6^{3x}=3x\log 6$ OE or $3x=\log_6 84$ seen
	$x=\frac{\lg 84}{3\lg 6}$			
	x = 0.82429…. = 0.824 (to 3dp)	A1	4	Must see that logs have been used before any of the last 3 marks are awarded in (b)(ii). Condone > 3dp
(c)	f(x) = $6^{x-1}-2$	B2,1	2	B1 for either 6^{x-1}+2 or for 6^{x+1}−2
	Total		**14**	

Question 9

9 (a) Solve the equation $\sin 2x = \sin 48°$, giving the values of x in the interval $0° \leqslant x < 360°$. *(4 marks)*

(b) Solve the equation $2\sin\theta - 3\cos\theta = 0$ in the interval $0° \leqslant \theta < 360°$, giving your answers to the nearest 0.1°. *(4 marks)*

Student Response



9a) $\sin 2x = \sin 48$ $\quad 0 \leq x \leq 360$

~~$\sin 48 = 0.743$~~ $\quad 2x = 48$ ✓ $\quad x = 24°$ ✓ B1

and 156° M0

9b) $2\sin\theta - 3\cos\theta = 0$ $\quad 0 \leq \theta < 360$

$2\sin\theta = 3\cos\theta$ $\quad \frac{2}{3}\tan\theta = 1$ ✓ $\quad 2\tan\theta = 3$ ✓

$\tan\theta = 3/2$ ✓

$\left(\frac{\sin\theta}{\cos\theta} \equiv \tan\theta\right)$ ✓

$\theta = 56.3$ ✓ or 236.3 ✓

1

4

5

Commentary

Part (a) of the exemplar illustrates the most common error. The candidate had started correctly by equating $2x$ to 48 but did not write down the other three values for $2x$ in the interval $0 \leq 2x \leq 720$. Instead the candidate found $x = 24$ and effectively went on to solve the equation $\sin x = \sin 24$ which does not have the same set of solutions as the equation $\sin 2x = \sin 48$. Those candidates who replaced $2x$ by u and solved the equation $\sin u = \sin 48$ to get $u = 48, 132, 408, 492$ then divided u by 2 to get the values for x were usually more successful. The candidate produced a fully correct solution to part (b) and used good examination technique, explicitly stating the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

9(a)	$2x = 48$	B1		PI by $x = 24^\circ$
	$2x = 180 - 48$	M1		Accept equivalents for x
	$2x = 360 + 48$ and $2x = 360+180-48$	M1		Accept equivalents for x
	$x = 24^\circ,\ 66^\circ,\ 204^\circ,\ 246^\circ$	A1	4	CAO; need all four, no extras in given interval
(b)	$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$	M1		Stated or used
	$2\sin\theta - 3\cos\theta = 0 \Rightarrow \tan\theta = 1.5$	A1		
	$\theta = 56.3^\circ$	A1		Condone > 1dp
	$\theta = 56.3^\circ + 180^\circ = 236.3^\circ$	A1F	4	Ft on c’s PV+180° dep only on the M1 provided no ‘extra’ solutions in the given interval.
	Total		**8**	