

General Certificate of Education  
June 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 5**

**MM05**

Wednesday 18 June 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 A simple pendulum of length 2 metres is set in motion.

- (a) (i) Show that the period of the motion is 2.84 seconds, correct to three significant figures. (2 marks)
- (ii) Show that the frequency of the motion is 0.352 cycles per second, correct to three significant figures. (1 mark)
- (b) The length of the pendulum is adjusted so that the period of its motion is 2.5 seconds. Find the adjusted length of the pendulum. (2 marks)

2 A particle moves in a straight line with simple harmonic motion such that its displacement at time  $t$  seconds relative to a fixed origin on this line is  $x$  metres. The motion of the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 16x = 0$$

(a) Verify that

$$x = A \cos 4t + B \sin 4t$$

where  $A$  and  $B$  are constants, is a solution to this differential equation. (4 marks)

- (b) When  $t = 0$ , the particle is momentarily at rest. Show that  $B = 0$ . (1 mark)
- (c) Given that  $x = h$  ( $h > 0$ ) when  $t = \frac{\pi}{12}$ , find  $A$  in terms of  $h$ . (2 marks)
- (d) Find the maximum speed of the particle in terms of  $h$ . (1 mark)
- (e) The mass of the particle is  $m$  kg. Find the magnitude of the maximum force acting on the particle during the motion. Give your answer in terms of  $h$  and  $m$ . (2 marks)

3 A particle  $P$  moves in a plane so that, at time  $t$ , its polar coordinates  $(r, \theta)$  with respect to a fixed origin,  $O$ , are given by

$$r = t^2 \quad \theta = \frac{9}{\pi^2} \sin \frac{\pi t}{6}$$

- (a) Find the radial and transverse components of the velocity of  $P$  when  $t = 3$ . (4 marks)
- (b) Find the radial and transverse components of the acceleration of  $P$  when  $t = 3$ . (5 marks)
- (c) Determine the angle between the acceleration of  $P$  and  $OP$  when  $t = 3$ . (2 marks)

- 4 A rocket is launched from the ground so that it travels vertically upwards. The rocket ejects burnt fuel vertically downwards at a speed of  $1400 \text{ m s}^{-1}$  relative to the rocket at a constant rate of  $10 \text{ kg s}^{-1}$ .

The initial mass of the rocket and its fuel is  $1000 \text{ kg}$ .

The velocity of the rocket at time  $t$  seconds after it is launched is  $v \text{ m s}^{-1}$ .

It may be assumed that the only external force acting on the rocket is gravity. The acceleration due to gravity should be taken as constant.

- (a) Show that

$$\frac{dv}{dt} = -9.8 + \frac{1400}{100 - t} \quad (8 \text{ marks})$$

- (b) Given that  $v = 0$  when  $t = 0$ , show that

$$v = -9.8t + 1400 \ln\left(\frac{100}{100 - t}\right) \quad (3 \text{ marks})$$

- (c) When  $t = 80$ , the fuel in the rocket has all been burnt. Find the total time taken for the rocket to reach its maximum height. (4 marks)

- 5 A particle, of mass  $2 \text{ kg}$ , is suspended from a fixed point  $O$  by a light spring of natural length  $0.5 \text{ metres}$  and modulus of elasticity  $49 \text{ N}$ .

- (a) Initially, the particle hangs at rest in equilibrium below  $O$ . Find the extension of the spring in this position. (2 marks)

- (b) A force,  $F$  newtons, is then applied to the particle in a vertically downwards direction. The displacement of the particle below its equilibrium position at time  $t$  seconds later is  $x$  metres. Given that  $F = 12 \cos nt$ , where  $n$  is a positive constant, show that

$$\frac{d^2x}{dt^2} + 49x = 6 \cos nt \quad (5 \text{ marks})$$

- (c) In the case where  $n = 5$ , find an expression for  $x$  at time  $t$ . (10 marks)
- (d) State the value of  $n$  for which resonance occurs. (1 mark)

**Turn over for the next question**

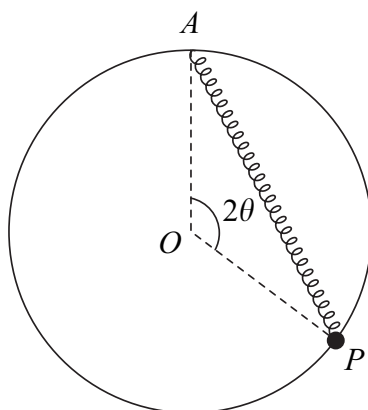
**Turn over ►**

6 A smooth circular wire, of radius  $a$  and centre  $O$ , is fixed in a vertical plane.

A small smooth bead,  $P$ , of mass  $m$ , can move freely on the wire.

The bead is attached to one end of a light spring, which has modulus of elasticity  $4mg$  and natural length  $a$ . The other end of the spring is attached to  $A$ , the highest point on the wire.

The angle subtended by the spring at  $O$  is  $2\theta$ , as shown in the diagram, where  $0 < \theta \leq \frac{\pi}{2}$ .



- (a) (i) Show that the elastic potential energy stored in the spring in this position is given by

$$2mga(2 \sin \theta - 1)^2 \quad (3 \text{ marks})$$

- (ii) The gravitational potential energy is taken to be zero at the level of the lowest point on the wire. Show that the total potential energy,  $V$ , is given by

$$V = 2mga(3 \sin^2 \theta - 4 \sin \theta + 2) \quad (5 \text{ marks})$$

- (b) Find the two values of  $\theta$  for which the bead is in equilibrium, giving your answers to two decimal places. (4 marks)
- (c) Determine, for each of these values, whether the bead is in stable or unstable equilibrium. (4 marks)

**END OF QUESTIONS**