Teacher Support Materials 2008

Maths GCE Mechanics 4

MM04

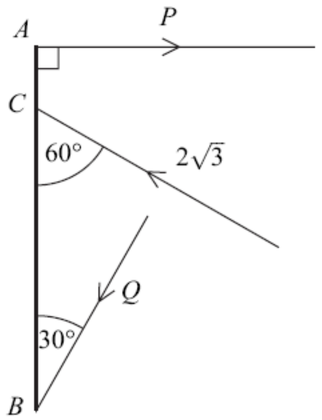
Question 1

1 A light rod AB has length 5 metres and the point C on the rod is 1 metre from A. The rod is on a smooth horizontal table and is acted upon by three horizontal forces of magnitude P, Q and $2\sqrt{3}$ newtons.

The force of magnitude P acts at A, at right angles to the rod.

The force of magnitude $2\sqrt{3}$ acts at C, at an angle of 60° to the rod.

The force of magnitude Q acts at B, at an angle of 30° to the rod, as shown in the diagram.



The three forces are equivalent to a couple.

(a) Show that $Q = 2$ and find the value of P. *(5 marks)*

(b) Determine the magnitude of the couple. *(3 marks)*

(c) State the sense of the couple. *(1 mark)*

Student Response

Question number

1. As couple : $F = 0$

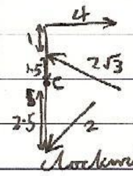
$\Rightarrow P = \sin 60 \times 2\sqrt{3} + Q \times \sin 30$ ✓ ✓

$\updownarrow$ $Q \cos 30 = 2\sqrt{3} \cos 60$ ✓

$\Rightarrow Q = \frac{2\sqrt{3} \cos 60}{\cos 30} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = 2 = Q$ ✓

$\Rightarrow P = \sin 60 \times 2\sqrt{3} + 2 \sin 30 = 4 = P$ ✓ 5

b)



C = C.O.M. $M(C)$: $1.5 \sin 60 \times 2\sqrt{3} - 2.5 \times 4 - 2.5 \times 2 \sin 30$ 3

(Mid pt) $= -8 \Rightarrow$ Couple $= 8$ Nm clockwise

c) clockwise 1

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Commentary

A very good starter question with candidates scoring an average of 82% of the total mark. Marks were rarely dropped, although some sign errors did occur. There was a great deal of variance in which point to take moments about. Candidates who chose point C tended to do better. A small number of candidates chose half way down the rod which meant that three individual moments were combined - meaning extra work.

The candidate's answer above illustrates the mid point method for part b). Note the clarity and efficiency of the solution throughout.

Mark scheme

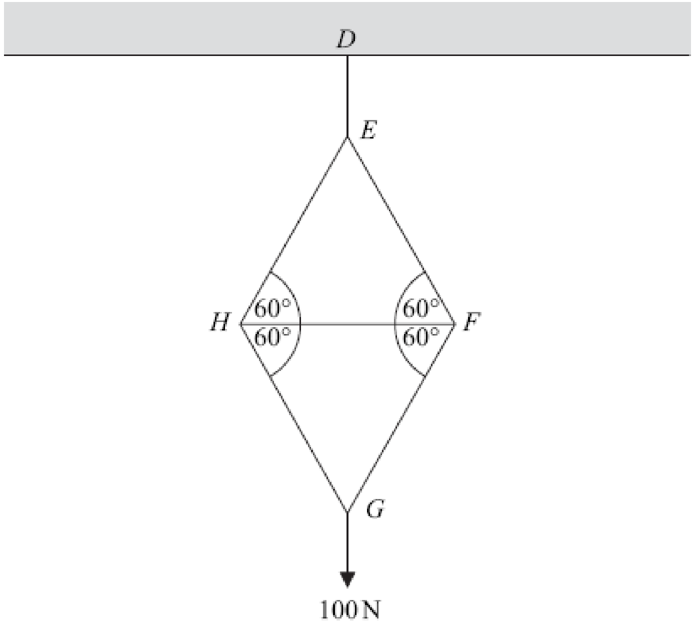
Q	Solution	Marks	Total	Comments
1(a)	Couple $\Rightarrow \Sigma$ horizontal component $=0$ Σ vertical component $=0$			
	Vertically: $2\sqrt{3}\cos 60° - Q\cos 30° = 0$	M1		Σ vertical component $=0$
	$\therefore Q = 2$	A1		AG
	Horizontally: $P - 2\sqrt{3}\sin 60° - Q\sin 30° = 0$	M1 A1		Σ horizontal component $=0$ one component correct (condone $\pm$)
	$\therefore P = 4$	A1	5	
(b)	Moments about B:			(N.B clockwise $-$ve/ anticlockwise +ve in solution below)
	$2\sqrt{3}\sin 60°(4) - 4(5)$	M1 A1√		Evidence of force × perp distance One term correct; ft error with P
	$= -8$ Magnitude $= 8$	A1√	3	
	Or Moments about A: $-2\sqrt{3}\sin 60°(1) - 2\sin 30°(5)$	(M1A1)		Evidence of force × perp distance One term correct
	$= -8$ Magnitude $= 8$	(A1)		No ft for Q
	Or Moments about C: $-4(1) - 2\sin 30°(4)$	(M1A1√)		Evidence of force × perp distance One term correct, ft error with P
	$= -8$ Magnitude $= 8$	(A1√)		
	Or Moments about centre of rod $-P(2.5) - Q(2.5\sin 30°) + 2\sqrt{3}(1.5\sin 60°)$	M1A1√		Evidence of force × perp distance One term correct, ft error with P
	$= -8$ Magnitude $= 8$	A1√		
	Or $\begin{bmatrix}0\\0\end{bmatrix}\times\begin{bmatrix}4\\0\end{bmatrix}+\begin{bmatrix}0\\-1\end{bmatrix}\times\begin{bmatrix}-3\\\sqrt{3}\end{bmatrix}+\begin{bmatrix}0\\-5\end{bmatrix}\times\begin{bmatrix}-1\\-\sqrt{3}\end{bmatrix}$	M1		Evidence of $\mathbf{r}\times\mathbf{F}$
	$= (0\ \ -3\ \ -5)\ \mathbf{k}$	A1√		one value correct
	$= -8\mathbf{k}$ Magnitude $= 8$	A1√		ft P value

SC Max M1A0A0 for candidates who form an equation in part (b) without using a variable for couple
i.e. $4(2.5)+2\sqrt{3}(1.5\sin 60°)=2(2.5\sin 30°)$

1(c)	Clockwise	B1√		ft answer (b) if directions all clear

Question 2

2 A framework $EFGH$ consists of five identical light rods, EF, EH, FG, GH and FH, which are smoothly jointed at E, F, G and H. Each of the rods EF, EH, FG and GH makes an angle of 60° with the rod FH. The framework is suspended from a fixed point D by a string DE. The rod FH is horizontal, and G is vertically below D. A force of 100 N is applied vertically at G. The system, as shown in the diagram, is in equilibrium.



(a) State the magnitude of the force in the string DE, giving a reason for your answer. *(2 marks)*

(b) Explain why the forces in the rods EF, EH, FG and GH must be of equal magnitude. *(2 marks)*

(c) Find the magnitude of the forces in each of the rods EF, EH, FG and GH. *(2 marks)*

(d) Find the magnitude of the force in the rod FH. *(3 marks)*

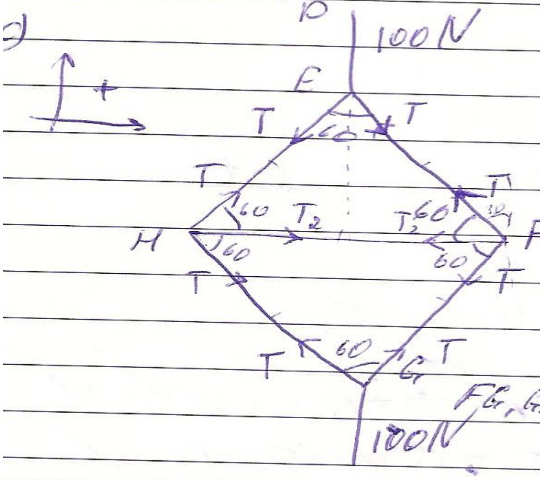
(e) State which of the five rods could be replaced by ropes, giving reasons for your answers. *(2 marks)*

Student response

a) 100 N, because if the system is in equillibrium the resultant force should be 0, so there should be a force of equal magnitude to the force at G acting in opposite direction

B) The framework is symmetrical about vertical and horizontal axis. The resultant must be 0.

c)



Joint E:

V: $-T\cos 30 \times 2 + 100 = 0$

H: $-T\sin 30 + T\sin 30 = 0$

$-2T\frac{\sqrt{3}}{2} = -100$

$\sqrt{3}\,T = 100$

$T = \frac{100}{\sqrt{3}}$

FG, GH, EF, EH $= \frac{100}{\sqrt{3}}$

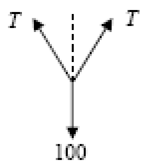
Commentary

Another good question with candidates scoring an average of 80% of the total mark. It did differentiate more than usual, largely due to the requests in a) and b) which required explanation. In a) a mark was lost if candidates did not clearly explain the idea of resolving the whole system and therefore balancing the 100N at G. In b) for both marks to be awarded candidates had to clearly refer to two axes of symmetry for the system. However both marks were awarded if a candidate noted that forces had to balance at each joint and then formed several equations to show that angles cancelled. A small number of candidates made serious errors in c) and d) when they resolved all forces within a rod, not at a joint, effectively double counting everything. The best solutions consisted of a clear labelled diagram with tensions marked correctly and which only used two letters due to the symmetry of the situation. Part e) was almost answered correctly by all candidates. The idea of replacing tensile rods with strings is well understood.

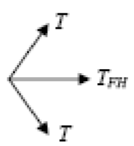
This candidate's answer has been chosen because of two reasons. The explanations in part a) and b) are succinct – no waffle. In part c) the candidate uses a clearly labeled diagram using the symmetry established in part b) to simplify notation. Equations are clearly stated.

Mark Scheme

2(a)	Magnitude $=100\text{ N}$	B1		
	Whole system must be in equilibrium and force in DE must balance the 100 N at G	E1	2	Reference to resolving whole system in equilibrium so $\sum F = 0$
(b)	Forces symmetrical about FH and $EG \Rightarrow$ equal magnitude	E(2,1,0)	2	E2 awarded for clear reference to **two** axes of symmetry
	Alternative			
	As any joint in the framework is in equilibrium, so resultant force is zero	E(2,1,0)		
	At F resolve vert $T_{EF}\sin 60° = T_{FG}\sin 60°$ $\therefore T_{EF} = T_{FG}$			
	At H resolve vert $T_{EH}\sin 60° = T_{HG}\sin 60°$ $\therefore T_{HG} = T_{EH}$			
	At G resolve horiz $T_{GH}\cos 60° = T_{GF}\cos 60°$ $\therefore T_{GH} = T_{GF}$			
	Hence $T_{GH} = T_{EF} = T_{EH} = T_{FG}$			
(c)	Consider forces at G, resolve vertically $T =$ Force in $FG =$ Force in GH			



	$2T\cos 30° = 100$	M1		Attempt to resolve at G or E Correct equation formed
	$T \simeq 57.7\text{ N}$	A1	2	$\frac{100}{\sqrt{3}}$ accepted
2(d)	Consider forces at H, resolve horizontally			



	$T_{FH} + 2T\cos 60° = 0$	M1		Attempt to resolve at H or F
		A1√		Correct equation formed. Follow through error for T
	$\Rightarrow \lvert T_{FH} \rvert = 57.7\text{ N}$	A1√	3	Solved; condone $\pm$ Follow through error for T
(e)	EH, EF, FG, HG can be replaced by ropes	B1		
	They are all in tension	B1	2	
	Or			
	FH can not be replaced by ropes	B1		
	It is the only one in thrust	B1		
	Total		**11**	

Question 3

3 A light rod has its ends at the points $A(2, 3, 5)$ and $B(4, 6, -1)$. A force $\mathbf{F}$ acts at B, where

$$\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$$

(a) Find $\overrightarrow{AB}$. *(1 mark)*

(b) Find the moment of $\mathbf{F}$ about the point A. *(3 marks)*

(c) Show that the magnitude of this moment is $10\sqrt{5}$. *(2 marks)*

(d) Hence, or otherwise, find the acute angle between $\mathbf{F}$ and the rod, giving your answer to the nearest degree. *(4 marks)*

Student Response

b) Moment about A $= \begin{vmatrix} i & 2 & 2 \\ j & 3 & -1 \\ k & -6 & 4 \end{vmatrix} = i\begin{vmatrix} 3 & -1 \\ -6 & 4 \end{vmatrix} - j\begin{vmatrix} 2 & 2 \\ -6 & 4 \end{vmatrix} + k\begin{vmatrix} 2 & 2 \\ 3 & -1 \end{vmatrix}$

$= i(12-6) - j(8+12) + k(-2-6)$

$= 6i - 20j - 8k$

c) ∴ Magnitude is $\sqrt{(6)^2+(20)^2+(8)^2}$

$= \sqrt{36+400+64}$

$= \sqrt{500}$

$= \sqrt{100 \times 5}$

$= 10\sqrt{5}$ N

Commentary

A more varied response to this question, with many candidates scoring full marks and others scoring very little. The average score was 72% of the total mark. Candiates often lost a mark in part b) through using Fxr or by making an error with the determinant. In part c0 candidates must appreciate that when answers are given full working must be shown. Very much mixed success with part d) with either the vector product or scalar product method being used. Sometimes these were mixed together. Other errors consisted of using the vectors from the wrong triangle eg OA and OB.

Two well organized solutions for part b) and c). Each show step by step methods – a good way to earn marks. Too often able candidates take short cuts, make errors and lose marks.

3(a)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$	B1	1	
(b)	$\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & 2 & 2 \\ \mathbf{j} & 3 & -1 \\ \mathbf{k} & -6 & 4 \end{vmatrix}$	M1		Attempt at $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ M0 if no evidence of **i**, **j**, **k** components
	$= \begin{pmatrix} 6 \\ -20 \\ -8 \end{pmatrix}$	A2,1,0 √	3	One component correct = A1 Follow through $\overrightarrow{AB}$ [If $\mathbf{F} \times \mathbf{r}$ M1, A1, A0] max
(c)	$\sqrt{6^2 + 20^2 + 8^2} = \sqrt{500}$	M1		
	$= 10\sqrt{5}$ N	A1	2	AG must see $\sqrt{500}$ to award A1
(d)	$\sin\theta = \dfrac{10\sqrt{5}}{\left\lvert\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}\right\rvert \left\lvert\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right\rvert}$	M1		Use of $\sin\theta = \dfrac{\mathbf{a} \times \mathbf{b}}{\lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert}$ with correct vector pair
		B1		$\sqrt{49}$, 7 or $\sqrt{21}$ seen
	$= \dfrac{10\sqrt{5}}{7\sqrt{21}}$	A1√		Correct values ft their $\overline{AB}$
	$\theta \simeq 44°$	A1√	4	ft their $\overline{AB}$
	Total		**10**	

3(d)	SC if $90° - \theta$ found (wrong angle – correct triangle) ie $46°$ then award M1 B1 A1 A0 Max			
	Alternative			
3(d)	$\overrightarrow{AB}.\ \mathbf{F} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = -23$	B1√		Their $\overrightarrow{AB}.\ \mathbf{F}$
	$\cos\theta = \dfrac{-23}{\left\lvert\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}\right\rvert \left\lvert\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right\rvert} = \dfrac{-23}{7\sqrt{21}}$	M1A1 √		use of $\cos\theta = \left\lvert\dfrac{a.b}{\lvert a\rvert\lvert b\rvert}\right\rvert$ with correct vector pair ft their $\overline{AB}$. (May not be explicitly seen)
	$\theta = \cos^{-1}\left(\dfrac{-23}{7\sqrt{21}}\right) = 135.8°$ ...			
	$\therefore$ Required angle $= 180° - 135.8° = 44°$	A1√		ft their $\overrightarrow{AB}$

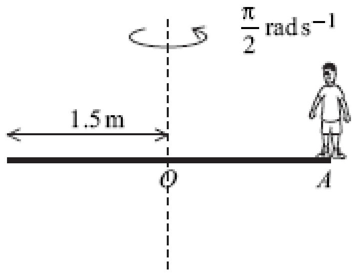
Question 4

4 (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}mr^2$. *(5 marks)*

(b) A roundabout in a playground can be modelled as a uniform circular disc of mass 200 kg and radius 1.5 m. The roundabout can rotate freely in a horizontal plane about a vertical axis through its centre O.

The roundabout is rotating at $\frac{\pi}{2}$ radians per second, with Dominic, a child of mass 25 kg, standing at a point A on the edge, as shown in **Figure 1**.

Figure 1

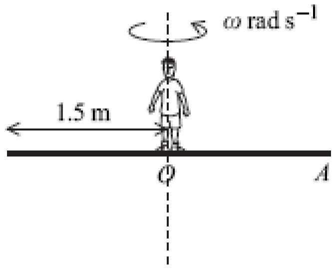


Assume that Dominic can be modelled as a particle.

(i) Show that the moment of inertia of the system about the vertical axis through O shown in **Figure 1** is 281.25 kg m^2. *(3 marks)*

(ii) Dominic then walks to the centre O, as shown in **Figure 2**. The angular speed of the roundabout changes from $\frac{\pi}{2}$ radians per second to ω radians per second.

Figure 2

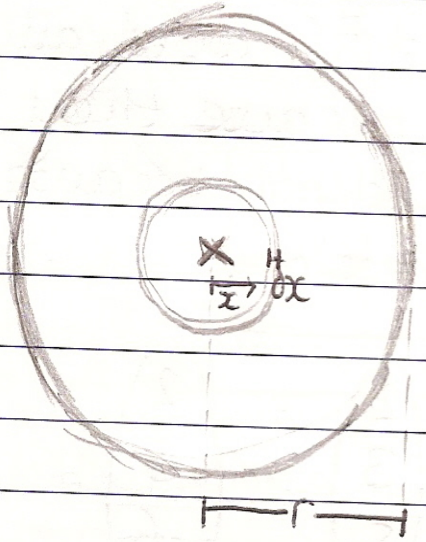


Explain why the total angular momentum of the system remains constant as Dominic walks from A to O. *(1 mark)*

(iii) Find the value of ω. *(4 marks)*

Student Response

a)



modelling the disc as a series of hoops:

mass of disc: $m = \pi r^2 \rho \Rightarrow \rho = \frac{m}{\pi r^2}$

mass of each hoop: $\delta m = \left[\pi(x+\delta x)^2 - \pi x^2\right]\rho$

$$\delta m = \frac{m\pi}{\pi r^2}\left(x^2 + 2x\,\delta x + (\delta x)^2 - x^2\right)$$

$$= \frac{m}{r^2}\left(2x\,\delta x + (\delta x)^2\right)$$

$$I_{hoop} = \delta m\, x^2 = \frac{mx^2}{r^2}\left(2x\,\delta x + (\delta x)^2\right)$$

$$I_{disc} = \lim_{\delta x \to 0} \sum_0^r I_{hoop}$$

$$= \frac{m}{r^2}\int_0^r 2x^3\,dx \qquad \left(\text{since } \delta x \to 0,\ (\delta x)^2 = 0\right)$$

$$= \frac{m}{r^2}\left[\frac{2x^4}{4}\right]_0^r = \frac{2mr^4}{4r^2} = \frac{1}{2}mr^2.$$

Commentary

Proving the moment of inertia was challenging for some candidates who failed to identify an appropriate elemental piece. Some excellent answers were seen which correctly used appropriate notation to identify the elemental piece required. b)i) proved to be successful for almost every candidate and helped them to score an average of 65% of the total marks available for this question. Surprisingly few candidates answered ii) correctly by not realising that a comment about external forces was required. The last part was very successful although a few candidates tried to equate kinetic energy.

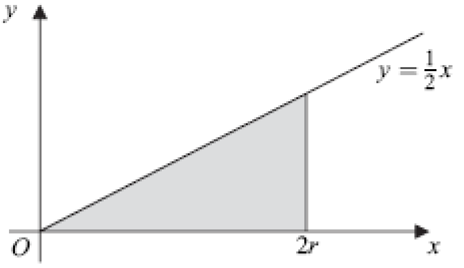
An excellent proof to establish the result is shown above. Elemental hoop identified. Shows how mass of hoop is obtained. Uses link between density, mass and area early on. Good, effective use of notation throughout. Agfain clear step by step explanation.

Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)	$m = \pi r^2 \rho \Rightarrow \rho = \frac{m}{\pi r^2}$	B1		ρ and m linked – used anywhere
	Mass of elemental 'hoop' $= 2\pi\rho\,\delta x\,x$	M1		Attempt to consider elemental 'hoop' – mass correct
	MI of each hoop $= 2\pi\rho\,\delta x\,x^3$	A1		Use of mr^2 with elemental 'hoop'
	MI disc $= \int_0^r 2\pi\rho\,\delta x\,x^3 = \int_0^r \frac{2m}{r^2}x^3\,dx$	m1		Attempt to integrate – dependant on first M1. Must be of form $\int kx^3 dx$
	$= \left[\frac{2mx^4}{4r^2}\right]_0^r = \frac{mr^2}{2}$	A1	5	AG
(b)(i)	$MI_{disc} = \frac{1}{2}mr^2 = \frac{1}{2}(200)(1.5)^2 = 225$	M1		Use of formula – either mr^2 or $\frac{1}{2}mr^2$
	$MI_{dom} = mr^2 = 25(1.5)^2 = 56.25$	A1		Both correct
	Total $= 225 + 56.25 = 281.25$	A1	3	AG Evidence of $MI_{disc} + MI_{dom}$
(ii)	No (resultant) external forces	E1	1	
(iii)	Momentum conserved Momentum at start $= I\omega$ $= 281.25\left(\frac{\pi}{2}\right)$	M1		Attempt at angular momentum (either)
	Momentum at end $= 225\omega$	A1		Both correct
	$\Rightarrow 225\omega = 281.25\left(\frac{\pi}{2}\right)$	M1		Equation formed – cons. of momentum
	$\omega = \frac{5\pi}{8} = 1.96\ \text{rad s}^{-1}$	A1	4	CAO
	Total		13	

Question 5

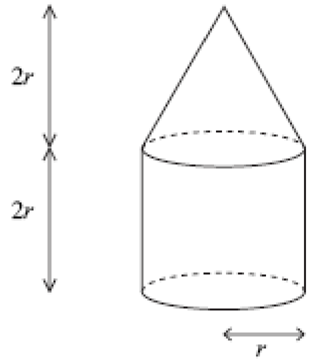
5 The region bounded by the line $y = \frac{1}{2}x$, the x-axis and the line $x = 2r$ is shown in the diagram.



This region is rotated about the x-axis to form a uniform solid cone of height $2r$ and radius r.

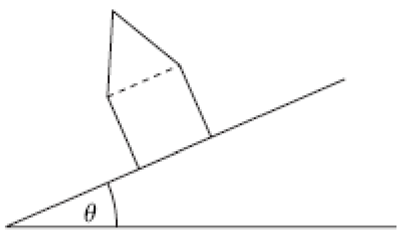
(a) Show, using integration, that the centre of mass of the cone is at a distance of $\frac{3r}{2}$ from the origin. *(5 marks)*

(b) A rocket consists of two parts. The lower part of the rocket may be modelled as a uniform solid cylinder with radius r, height $2r$ and density ρ. The upper part of the rocket may be modelled as a uniform solid cone of radius r, height $2r$ and density $k\rho$, as shown in the diagram.



(i) Show that the centre of mass of the rocket is at a distance of $\left(\frac{6+5k}{6+2k}\right)r$ from the base of the rocket. *(5 marks)*

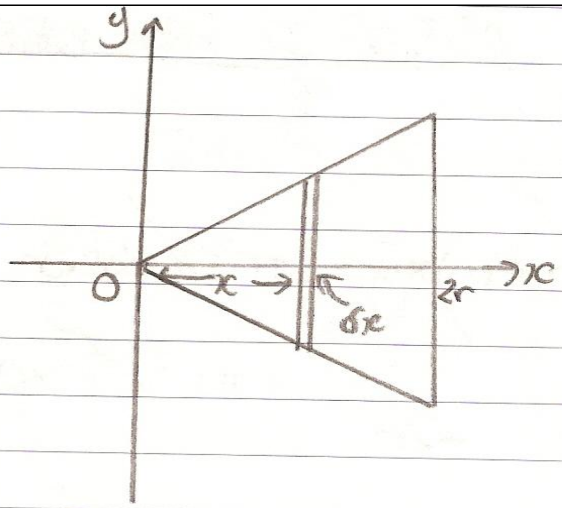
(ii) The rocket is now placed on a rough plane, which is inclined at an angle of θ to the horizontal, where $\tan\theta = \frac{2}{3}$.



Given that the rocket does **not** slide and is just on the point of toppling, find the value of k. *(5 marks)*

Student Response

a)



SLICE:

~~Area~~

Volume $= \pi y^2 \delta x$

$= \pi\left(\frac{1}{2}x\right)^2 \delta x$

$= \frac{1}{4}\pi x^2 \delta x$

CONE:

Volume $= \frac{1}{3}\pi r^2 \times 2r$

$= \frac{2}{3}\pi r^3$ ✓

Let w equal weight per unit volume:

$\Rightarrow \curvearrowright$: $\frac{2}{3}\pi r^3 \bar{x} w = \sum \frac{1}{4}\pi x^2 \delta x \, w \times x$

$\frac{2}{3}\pi r^3 \bar{x} = \int_0^{2r} \frac{1}{4}\pi x^3 \, dx$

$\frac{2}{3}\pi r^3 \bar{x} = \frac{1}{4}\pi \left[\frac{x^4}{4}\right]_0^{2r}$ ✓

$\frac{8}{3} r^3 \bar{x} = \frac{(2r)^4}{4}$

$= \frac{16r^4}{4}$ ✓

$\bar{x} = \frac{3}{8} \times 4 \times r$

$= \frac{3r}{2}$ ✓✓

∴ Centre of mass at $\frac{3r}{2}$ from origin.

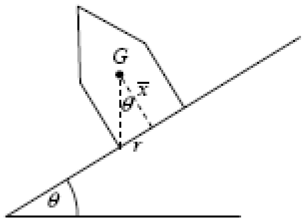
Commentary

In this question candidates scored an average of 73% of the marks available. A very pleasing response. All marks were lost in part a) if a 2 dimensional formula was used. The best solutions in part b) used a tabular approach before setting up the relevant equation. A common error was to have the distance of the centre of mass of the cone at 3.5r from the base not 2.5r. The last part was well understood with the correct principle applied. The best responses included a clear labelled diagram showing the principle concerned.

This candidate efficiently uses the standard result for the volume of a cone to simplify working. Good understanding shown, this candidate builds up the formula using knowledge of volumes of rotation. Good use of notation throughout.

Mark Scheme

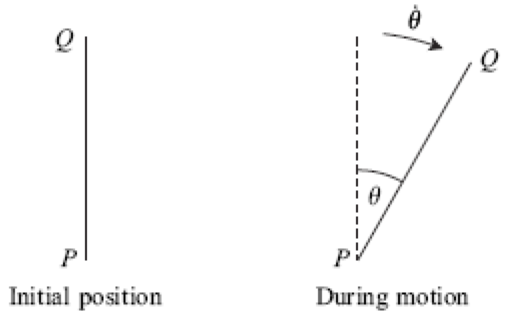
Q	Solution	Marks	Total	Comments
5(a)	$\int_0^{2r} xy^2\,dx = \int_0^{2r} \frac{x^3}{4}\,dx$	M1		Attempt to use formula $\int xy^2 dx$
	$= \left[\frac{x^4}{16}\right]_0^{2r}$	A1		Integration correct
	$= r^4$			
	$\int_0^{2r} y^2\,dx = \int_0^{2r} \frac{x^2}{4}\,dx$			Or use of $\frac{1}{3}\pi r^2 h$ to get $\frac{2}{3}\pi r^3$
	$= \left[\frac{x^3}{12}\right]_0^{2r}$			
	$= \frac{2r^3}{3}$	B1		
	$\Rightarrow \bar{x} = r^4 \div \frac{2r^3}{3} = \frac{3r}{2}$	M1A1	5	AG use of $\bar{x} = \frac{\pi\int_0^{2r} xy^2 dx}{\pi\int_0^{2r} y^2 dx}$ NB – consistent use of π throughout for M1A1 at end (or cancelled at start)
(b)(i)	(mass, distance) Lower: $\pi r^2(2r)\rho$, r Upper: $\frac{\pi r^2}{3}(2r)k\rho$, $2r+\frac{r}{2}$	B1		Any correct pairing seen anywhere (mass $\leftrightarrow$ distance)
	$\left(\pi 2r^3\rho + \frac{\pi 2r^3}{3}k\rho\right)\bar{x} = \pi 2r^3\rho(r)$	M1		Equation formed
	$+ \frac{\pi 2r^3}{3}k\rho\left(\frac{5r}{2}\right)$	A2,1,0		lose 1 each 'type' of error
	$\Rightarrow \left(1+\frac{k}{3}\right)\bar{x} = r + \frac{5rk}{6}$			
	$\Rightarrow (6+2k)\bar{x} = (6+5k)r$			
	$\bar{x} = \left(\frac{6+5k}{6+2k}\right)r$	A1	5	Rearrange to obtain printed answer

Q	Solution	Marks	Total	Comments
5(b)(ii)				
	$\tan\theta = \dfrac{r}{\bar{x}}$	M1 A1		Use of $\tan\theta$ Correct structure
	$\Rightarrow \dfrac{2}{3} = \dfrac{r}{\left(\dfrac{6+5k}{6+2k}\right)r}$	B1		Substitution of $\bar{x}, \tan\theta$
	$\dfrac{2}{3} = \dfrac{6+2k}{6+5k}$			
	$12 + 10k = 18 + 6k$	M1		Attempt to solve
	$4k = 6$			
	$k = \dfrac{3}{2}$	A1	5	
	Total		**15**	

Question 6

6 A uniform rod PQ, of mass m and length $6a$, is free to rotate in a vertical plane about a fixed horizontal axis through P. Initially, the rod is at rest with Q vertically above P.

The rod is slightly disturbed from its initial position. In the subsequent motion, it makes an angle θ with the upward vertical at time t.



(a) (i) Show that the moment of inertia of the rod about the axis through P is $12ma^2$. *(1 mark)*

(ii) Show that $\dot{\theta}^2 = \frac{g}{2a}(1 - \cos\theta)$. *(4 marks)*

(iii) Hence, or otherwise, determine an expression for $\ddot{\theta}$ in terms of a, g and θ. *(2 marks)*

(b) Find, in terms of m, g and θ, the force at P which the axis exerts on the rod:

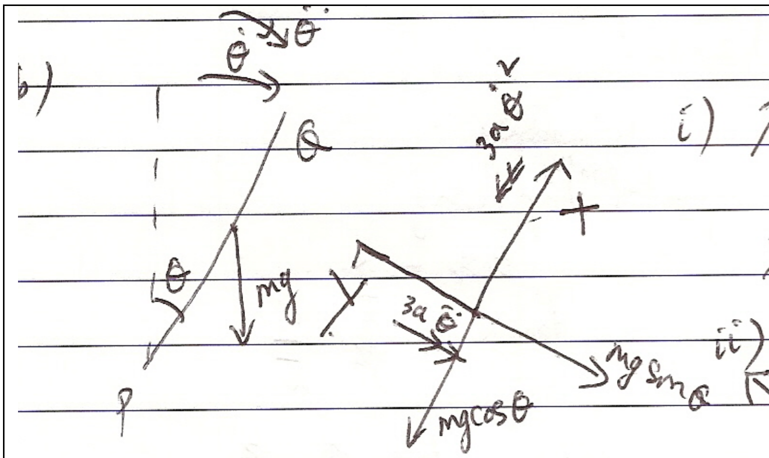
(i) in the direction PQ; *(4 marks)*

(ii) perpendicular to PQ. *(3 marks)*

(c) Determine the magnitude of the force exerted by the axis on the rod when Q is vertically below P. *(3 marks)*

Student Response

b)



i) $\nearrow$: $mg\cos\theta - X = m\,3a\,\dot{\theta}^2$

$mg\cos\theta - X = \frac{3}{2}g(1-\cos\theta)$

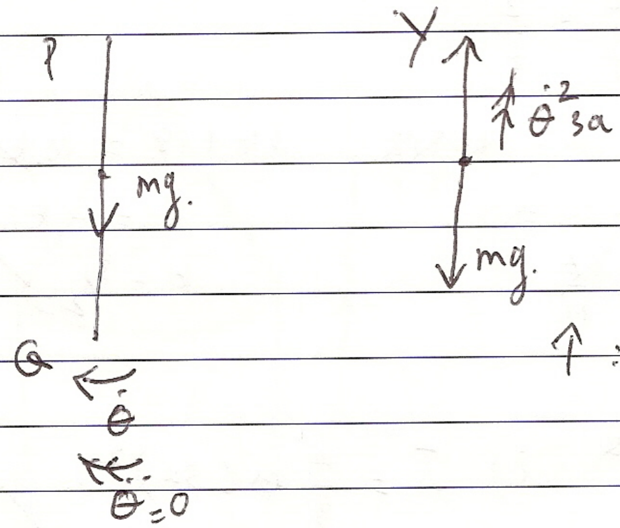
$X = mg\left(\frac{5}{2}\cos\theta - \frac{3}{2}\right)$

ii) $\nwarrow$: $mg\sin\theta - Y = m3a\ddot{\theta}$

$mg\sin\theta - Y = m\frac{3}{4}g\sin\theta$

$Y = mg\frac{1}{4}\sin\theta$

c)



conservation of energy

$E_k = E_p$

$\frac{1}{2}I\dot{\theta}^2 = mg\,6a$.

$\frac{1}{2}(12ma^2)\dot{\theta}^2 = mg6a$.

$\dot{\theta}^2 = \frac{g}{a}$

$\uparrow$: $Y - mg = m\,3a\,\dot{\theta}^2$

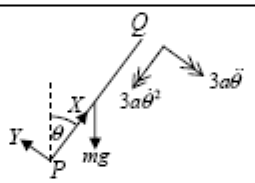
$Y - mg = 3mg$.

$Y = 4mg$.

Commentary

. A demanding question with many candidates scoring less than half marks (on average, 39% of the marks were scored). In a)i) several candidates used the incorrect radius 6a in the incorrect formula to get the correct answer (no marks). In part ii) several candidates tried to equate energy but again used 6a not 3a, clearly not realising that it was the location of the centre of mass that was required. Attempts to differentiate to obtain the angular acceleration varied, although the mark scheme awarded an easy mark if sine was seen.
Parts b) and c) were non existent for many candidates. The best solutions here used clear labelled diagrams indicating forces and accelerations.
It was disappointing to see elements of M2 done so badly here.

The work shown uses clearly labeled diagrams in part b) and as such makes no sign errors. In part c) the alternative method is used – clearly explained. Again another diagram is used to aid thinking. This solution was one of the best seen.

Q	Solution	Marks	Total	Comments
6(a)(i)	$\frac{4}{3}m(3a)^2 = 12ma^2$	B1	1	
(ii)	Use conservation of energy			
	PE lost = KE gained			
	$mg3a(1-\cos\theta) = \frac{1}{2}(12ma^2)\dot{\theta}^2$	M1 A1,A1		Equation formed A1 each side
	$\dot{\theta}^2 = \frac{g}{2a}(1-\cos\theta)$	A1	4	AG
(iii)	Differentiate			
	$2\dot{\theta}\ddot{\theta} = \frac{g}{2a}(\sin\theta)\dot{\theta}$	M1		Attempt to differentiate – $\sin\theta$ seen $\Rightarrow$ M1
	$\ddot{\theta} = \frac{g}{4a}\sin\theta$	A1	2	$\dot{\theta}$ cancelled – clear indication
	Alternative			
6(a)(iii)	using $C = I\ddot{\theta}$ $mg3a\sin\theta = 12ma^2\ddot{\theta}$	M1		
	$\therefore \ddot{\theta} = \frac{g\sin\theta}{4a}$	A1	2	
6(b)(i)				
	Along PQ			Use of F = mass × acc. along PQ
	$mg\cos\theta - X = 3ma\dot{\theta}^2$	M1		M1 for either $(\pm mg\cos\theta \pm X)$ or $m(3a)\dot{\theta}^2$ or $\frac{m(3a\dot{\theta})^2}{3a}$
		A1		A1 fully correct
	$mg\cos\theta - X = 3ma\left[\frac{g}{2a}(1-\cos\theta)\right]$	A1		Use of (a)(ii) to replace $\dot{\theta}^2$
	$X = mg\cos\theta - \frac{3mg}{2} + \frac{3mg}{2}\cos\theta$ or			
	$\frac{mg}{2}[5\cos\theta - 3]$	A1	4	Can be unsimplified
(ii)	Perpendicular to PQ $mg\sin\theta - Y = 3ma\ddot{\theta}$	M1		Use of F = mass × acc perp to PQ, must have attempted both sides
	$mg\sin\theta - Y = 3ma\left[\frac{g}{4a}\sin\theta\right]$	A1√		Use of (a)(iii) to replace "their" $\ddot{\theta}$
	$Y = mg\sin\theta - \frac{3mg}{4}\sin\theta$ or $\frac{mg}{4}\sin\theta$	A1√	3	Follow through (a)(iii) (condone ± for b (i)(ii))
(c)	When Q is vertically below P $\theta = \pi$ $\Rightarrow Y = 0$	B1		Stated or implied
	$X = \frac{mg}{2}[-5-3] = -4mg$	M1		Substituting $\theta = \pi$
	$\Rightarrow$ magnitude of total force $= 4mg$	A1	3	CAO
(c)	**Alternative** Conservation of energy (at top) $\frac{1}{2}I\dot{\theta}^2 = mg6a$	B1		
	$\therefore \dot{\theta}^2 = \frac{g}{a}$			
	vertically $Y - mg = m3a\dot{\theta}^2$	M1		
	$Y - mg = 3mg$ $Y = 4mg$	A1		
	Total		17	