

# **General Certificate of Education**

# **Mathematics 6360**

# MFP4 Further Pure 4

# **Mark Scheme**

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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М	mark is for method				
m or dM	mark is dependent on one or more M marks and is for method				
А	mark is dependent on M or m marks and is for accuracy				
В	mark is independent of M or m marks and is for method and accuracy				
E	mark is for explanation				
$\sqrt{100}$ or ft or F	follow through from previous				
	incorrect result	MC	mis-copy		
CAO	correct answer only	MR	mis-read		
CSO	correct solution only	RA	required accuracy		
AWFW	anything which falls within	FW	further work		
AWRT	anything which rounds to	ISW	ignore subsequent work		
ACF	any correct form	FIW	from incorrect work		
AG	answer given	BOD	given benefit of doubt		
SC	special case	WR	work replaced by candidate		
OE	or equivalent	FB	formulae book		
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme		
–x EE	deduct <i>x</i> marks for each error	G	graph		
NMS	no method shown	c	candidate		
PI	possibly implied	sf	significant figure(s)		
SCA	substantially correct approach	dp	decimal place(s)		

## Key to mark scheme and abbreviations used in marking

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

### Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	Rotation	M1		
	about the y-axis	A1		Ignore direction
	through $\cos^{-1}0.8$	A1	3	or $\sin^{-1} 0.6$ or $36.87^{\circ}$ or $0.644^{\circ}$
(h)	Paflection in $y = r$	M1 A 1	r	Ignora if it is called a line
(0)	$\frac{1}{1} \frac{1}{1} \frac{1}$	MITAI	5	
2(a)(i)	$\mathbf{a} \mathbf{h} = 0$	B1	1	
2(a)(1)	<b>a.b</b> 0	DI	1	
(ii)	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 1 & 1 & -5 \end{vmatrix} = \begin{bmatrix} -16 \\ 11 \\ -1 \end{bmatrix}$	M1 A1	2	
(iii)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & -5 \\ 1 & 4 & 28 \end{vmatrix} = 0$	M1 A1	2	or via $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ ft in this case Do not allow = 0 via (a)(i)
(b)(i)	<b>a</b> , <b>b</b> , $\mathbf{a} \times \mathbf{b}$ mutually perpendicular	B1	1	
(ii)	<b>a</b> , <b>b</b> , <b>c</b> co-planar	B1	1	
()	Total		7	
<b>3(a)</b>	Area invariant	M1		MUST mention area
	$\Rightarrow$ Determinant = 1 $\Rightarrow pr + q^2 = 1$	A1	2	Given answer justified
(b)(i)	$\begin{bmatrix} 4 & q \\ -q & r \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\Rightarrow 2q - 4 = 2 \text{ and } q + 2r = -1$ $\Rightarrow q = 3 \text{ and } r = -2$	M1 A1 A1	3	Either correct
(ii)	x'=4x+3y and $y'=-3x-2ySetting x'=x, y'=yy=-x$	B1 M1 A1	3	
	Alternative for (b)(ii):			
	Setting $\lambda = 1$	(M2)		
	$\Rightarrow$ 3x + 3y = 0 (etc) ie y = -x	(A1)	(3)	
	Total		8	

### MFP4

MFP4 (cont)					
Q	Solution	Marks	Total	Comments	
4(a)	$\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \ \mathbf{U} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix},$	B1B1			
	$\mathbf{U}^{-1} = \begin{bmatrix} 3 & -2\\ -1 & 1 \end{bmatrix}$	B1	3	ft $U^{-1}$	
(b)	$\mathbf{T}^{n} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2^{n} & 0 \\ 0 & 2^{n} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	B1 M1		For $\mathbf{D}^n$ with <i>n</i> even For use of $\mathbf{U}^{-1}\mathbf{D}^n\mathbf{U}$ form	
	$= \begin{bmatrix} 2^n & 2 \times 2^n \\ 2^n & 3 \times 2^n \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	ml Al			
	or $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 \times 2^n & -2 \times 2^n \\ -2^n & 2^n \end{bmatrix}$				
	$=2^{n}\begin{bmatrix}1&0\\0&1\end{bmatrix}$	A1	5	Shown legitimately	
	Alternative for (b):				
	$\mathbf{D}^n = \begin{bmatrix} 2^n & 0\\ 0 & 2^n \end{bmatrix}$	(B1)		For $\mathbf{D}^n$ with <i>n</i> even	
	$\mathbf{T}^n = \mathbf{U} \left( 2^n \mathbf{I} \right) \mathbf{U}^{-1}$	(M1)			
	$=2^{n}\left(\mathbf{U}\mathbf{I}\mathbf{U}^{-1}\right)$	(m2)			
	$=2^{n}$ I	(A1)	(5)	Allow $\equiv$ forms such as $3 \cdot 2^n - 2^{n+1}$	
	Total		8		

MFP4 (	MFP4 (cont)						
Q	Solution	Marks	Total	Comments			
5(a)	eg $3 \times (1) - (2) \implies 13y + 13z = -13$	M1		Eliminating first variable			
	$(3) - (2) \qquad \Rightarrow 15y + 11z = -5$	AlAl					
	$x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$	M1 A1	5	Solving $2 \times 2$ system			
	Alt I (Cramer's Rule):						
	$\Delta = \begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ 3 & 11 & 13 \end{vmatrix},  \Delta_x = \begin{vmatrix} -2 & 3 & 5 \\ 7 & -4 & 2 \\ 2 & 11 & 13 \end{vmatrix},$	(M1)		Attempt at any two			
	$\Delta_y = \begin{vmatrix} 1 & -2 & 3 \\ 3 & 7 & 2 \\ 3 & 2 & 13 \end{vmatrix}, \ \Delta_z = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -4 & 7 \\ 3 & 11 & 2 \end{vmatrix}$						
	= 52, 312, 78 and – 130 respectively	(A1 A1)		$\Delta$ correct; $\geq 1$ other determinant correct			
	$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$	(M1)		At least one attempted numerically			
	$x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$	(A1)	(5)				
	Alt II (Augmented matrix method):						
	$\begin{bmatrix} 1 & 3 & 5 &   & -2 \\ 3 & -4 & 2 &   & 7 \\ 3 & 11 & 13 &   & 2 \end{bmatrix} \rightarrow$	(M1)					
	$\begin{bmatrix} 1 & 3 & 5 &   & -2 \\ 0 & -13 & -13 &   & 13 \\ 0 & 2 & -2 &   & 8 \end{bmatrix}$	(A1)		$R_2 \rightarrow R_2 - 3R_1$ $R_3 \rightarrow R_3 - 3R_1$			
	$\rightarrow \begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 4 \end{bmatrix}$	(A1)					
	$\rightarrow \begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 5 \end{bmatrix}$			$R_3 \rightarrow R_3 - R_2$			
	Substituting back to get $x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$	(M1 A1)	(5)				
	Alt III (Inverse matrix method):						
	$C^{-1} = \frac{1}{52} \begin{bmatrix} -74 & 16 & 26 \\ -33 & -2 & 13 \\ 45 & -2 & -13 \end{bmatrix}$	(M1) (A1 A1)		M0 if no inverse matrix is given			
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1} \begin{bmatrix} -2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.5 \\ -2.5 \end{bmatrix}$	(M1) (A1)	(5)				

QSolutionMarksTotalComments5(b)(0) $\begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ a & 11 & 13 \end{vmatrix} = 26a - 26$ $a & 11 & 13 \end{vmatrix} = 26a - 26$ $a & 11 & 13 \end{vmatrix}$ M1Attempt at determinant; OESetting equal to zero and solving for a $a^{-1}$ m1 A13(ii) $x + 3y + 5z = -2$ $3x - 4y + 2z = 7$ $x + 11y + 13z = b$ m1 A13(iii) $x + 3y + 5z = -2$ $3x - 4y + 2z = 7$ $x + 11y + 13z = b$ B1 B1 B1 B1 $x^{-1} = x = -10$ Since, to be consistent, the 3 <sup>rd</sup> plane must contain the line of intersection of the first $x^{-1} = y = 10$ Substituting $x = 6$ $y = 1/4$ , $z = -2\frac{1}{6}$ M1 A12 $x^{-1} = y = 10$ 6(a)(i) $a = i + j + 2k$ and $b = 3i - 2j + 6k$ B11(ii)Equating for $\lambda$ : $\frac{x - 1}{3} = \frac{y - 1}{2} = \frac{z - 2}{6}$ M1 A12(iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ B1 Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ B1 A1f on 7These are the cosines of the angles between the line and the $x$ -, $y$ - and $z$ -axesB13(b)(i) $n = \begin{vmatrix} i & j & k \\ 1 & 1 & 3 \end{vmatrix} = 7i - 10j + k$ $\left( \frac{1}{2} - 10 \end{vmatrix} = 0$ M1 A14(iii) $d = 0 \Rightarrow$ plane through / contains the originB11(iii) $d = 0 \Rightarrow$ plane through / contains the originB11(iii) $d = 0 \Rightarrow$ plane through / contains the originB11(iii) $d = 0 \Rightarrow$ plane through / contains the originB11(iii) $d = 0 \Rightarrow$ plane through / contains the product of moduliM1 B1Mus	MFP4 (cont)						
$\begin{aligned} \mathbf{5(b)(i)} & \begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ 1 & 11 & 13 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ a = 1 \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ a = 1 \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ a = 1 \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ a = 1 \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ \text{Setting equal to zero and solving for a} \\ Setting equal to zero and solving for and for an and solving for a and for a must contain the line of intersection of the first contand the zero and zero and zero and zero and z$	Q	Solution	Marks	Total	Comments		
Setting equal to zero and solving for a a = 1 (ii) $x + 3y + 5z = -2$ 3x - 4y + 2z = 7 x + 11y + 13z = b NB $y + zz = -1$ (from before) B1 (3) - (1) = 8y + 8z = b + 2 $b + 2 = -8 \Rightarrow b = -10$ Alternative for (b)(ii): Substituting $x = 5$ , $y = 11/5$ , $z = -21/2$ into $x + 11y + 13z = b$ (M3) (x) = b = -10 (A1) (A1) (A1) (A1) (A2) (A1	5(b)(i)	$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ a & 11 & 13 \end{vmatrix} = 26a - 26$	M1		Attempt at determinant; OE		
(ii) $x + 3y + 5z = -2$ x + 11y + 13z = b NB $y + z = -1$ (from before) (3) - (1) $\Rightarrow 8y + 8z = b + 2$ B1 $b + 2 = -8 \Rightarrow b = -10$ Alternative for (b)(ii): Substituting $x = 6$ , $y = 11\%$ , $z = -21\%$ into $x + 11y + 13z = b$ $\Rightarrow b = -10$ (M3) $\Rightarrow b = -10$ (M3) (a = i + j + 2k and $b = 3i + 2j + 6k$ B1 (ii) Equating for $\lambda$ : $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$ (M1 (iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ These are the cosines of the angles between the line and the $x - , y -$ and $z$ -axes (b)(i) $\mathbf{n} = \begin{bmatrix} i & j & k \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix} = 7i - 10j + k$ (iii) $d = 0 \Rightarrow$ plane through / contains the origin B1 (c) $\sin\theta / \cos\theta = \frac{\operatorname{scalar product}}{\operatorname{product of moduli}}$ Numerator = 21 - 20 + 6 - 7 Denominator = $7,\sqrt{150}$ (b)(i) $n = -\frac{1}{7} - 102 + 6 - 7$ B1 Numerator = 27, $\sqrt{150}$ (b)(i) $a = -\frac{1}{7} - 102 + 6 - 7$ B1 Numerator = $7,\sqrt{150}$ (b)(i) $a = -\frac{1}{7} - 102 + 6 - 7$ (c) $\frac{\sin\theta}{\theta - \cos\theta} = \frac{\operatorname{scalar product}}{\operatorname{product of moduli}}$ (c) $\frac{\sin\theta}{\theta - 24, 7^{\circ}}$ (c) $\frac{1}{7}$ (c) $\frac{1}{7}$		Setting equal to zero and solving for $a = 1$	m1 A1	3			
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	(ii)	x + 3y + 5z = -23x - 4y + 2z = 7x + 11y + 13z = b					
Alternative for (b)(i): Substituting $x = 6$ , $y = 1/_2$ , $z = -2/_2$ into $x + 11y + 13z = b$ $\Rightarrow b = -10$ Since, to be consistent, the $3^{rd}$ plane must contain the line of intersection of the first $2$ planes, and therefore contains this pointTotal126(a)(i)a = i + j + 2k and $b = 3i + 2j + 6k$ B1IIGoal (A)(ii)Equating for $\lambda$ : $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$ M1 A1Q(iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ B1Direction cosines are $\frac{3}{7} \cdot \frac{2}{7}$ and $\frac{6}{7}$ B1These are the cosines of the angles between the line and the $x - , y - and z$ -axes (respectively)B13Allow just "angles" correctly described(iii) $d = 0$ M1 A1 $2$ (iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ B1Direction cosines are $\frac{3}{7} \cdot \frac{2}{7}$ and $\frac{6}{7}$ B1Allow just "angles" correctly described(b)(i) $n = \left[ \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} + $		NB $y + z = -1$ (from before) (3) - (1) $\Rightarrow 8y + 8z = b + 2$ $b + 2 = -8 \Rightarrow b = -10$	B1 B1 M1A1	4	Equating; CAO		
Total126(a)(i) $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ B11(ii)Equating for $\lambda$ : $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$ M1 A12(iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ B1ft on 7Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ B1ft on 7These are the cosines of the angles between the line and the $x - , y - $ and $z$ -axesB13(b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ M1A1 $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ M1 A14(ii) $d = 0 \Rightarrow$ plane through / contains the originB11(c) $\sin\theta/\cos\theta = \frac{scalar product}{product of moduli}$ M1 B1 Denominator $= 7, \sqrt{150}$ M1 B1Murerator $= 21 - 20 + 6 = 7$ Denominator $= 7, \sqrt{150}$ B1 A14CAO		Alternative for (b)(ii): Substituting $x = 6$ , $y = 1\frac{1}{2}$ , $z = -2\frac{1}{2}$ into $x + 11y + 13z = b$ $\Rightarrow b = -10$	(M3) (A1)	(4)	Since, to be consistent, the 3 <sup>rd</sup> plane must contain the line of intersection of the first 2 planes, and therefore contains this point		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		Total		12			
(ii) Equating for $\lambda$ : $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$ (iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ These are the cosines of the angles between the line and the x-, y- and z-axes (respectively) (b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ (ii) $d = 0 \Rightarrow$ plane through / contains the origin (c) $\sin\theta/\cos\theta = \frac{\operatorname{scalar product}}{\operatorname{product of moduli}}$ Numerator = $21 - 20 + 6 = 7$ Denominator = $7, \sqrt{150}$ $\theta = 4, 7^{\circ}$ Numerator = $21, -20 + 6 = 7$ Denominator = $7, \sqrt{150}$ H H H H H H H H	6(a)(i)	$\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$	B1	1			
(iii) $\sqrt{3^2 + 2^2 + 6^2} = 7$ Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$ These are the cosines of the angles between the line and the x-, y- and z-axes (respectively) (b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ (ii) $d = 0 \Rightarrow$ plane through / contains the origin origin $B1$ (c) $\sin\theta/\cos\theta = \frac{\operatorname{scalar produt}}{\operatorname{product of moduli}}$ Numerator = $21 - 20 + 6 = 7$ Denominator = $7,\sqrt{150}$ $d = 4.7^\circ$ Total II II (c) $1 = \frac{1}{2} - \frac{1}{2$	(ii)	Equating for $\lambda : \frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$	M1 A1	2			
Direction cosines are $\frac{3}{7} \cdot \frac{2}{7}$ and $\frac{6}{7}$ These are the cosines of the angles between the line and the x-, y- and z-axes (respectively) (b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ (ii) $d = 0 \Rightarrow$ plane through / contains the origin (c) $\sin\theta / \cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$ Numerator = $21 - 20 + 6 = 7$ Denominator = $7 \cdot \sqrt{150}$ $d = 4 \cdot 7^{\circ}$ Total (b)(c) $\mathbf{n} = \begin{bmatrix} -7 \\ -10 \\ 1 \end{bmatrix} = 0$ $\mathbf{n} = \begin{bmatrix} M1 \\ A1 \\$	(iii)	$\sqrt{3^2 + 2^2 + 6^2} = 7$	B1				
These are the cosines of the angles between the line and the x-, y- and z-axes (respectively) (b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ (ii) $d = 0 \Rightarrow$ plane through / contains the origin (c) $\sin\theta / \cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$ Numerator = $21 - 20 + 6 = 7$ Denominator = $7 \cdot \sqrt{150}$ $\theta = 4.7^{\circ}$ M1 M1 M2 M1 M1 M1 M1 M1 M2 M1 M1 M1 M1 M1 M2 M1 M1 M2 M1 M1 M2 M1 M2 M1 M1 M2 M2 M1 M1 M2 M2 M1 M2 M2 M2 M1 M2 M2 M2 M1 M2 M2 M2 M2 M2 M2 M2 M2 M2 M2 M2 M2 M2		Direction cosines are $\frac{3}{7}, \frac{2}{7}$ and $\frac{6}{7}$	B1		ft on 7		
(b)(i) $\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ M1A1 $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$ M1 A1 4 ft <b>n</b> (ii) $d = 0 \Rightarrow$ plane through / contains the origin B1 1 (c) $\sin\theta / \cos\theta = \frac{\operatorname{scalar product}}{\operatorname{product of moduli}}$ M1 Numerator = $21 - 20 + 6 = 7$ B1 ft correct (unsimplified) Denominator = $7 \cdot \sqrt{150}$ B1 ft both correct (unsimplified) $\theta = 4.7^{\circ}$ A1 4 CAO		These are the cosines of the angles between the line and the <i>x</i> -, <i>y</i> - and <i>z</i> -axes (respectively)	B1	3	Allow just "angles" correctly described		
$d = \begin{bmatrix} 7\\5\\1 \end{bmatrix} \cdot \begin{bmatrix} 7\\-10\\1 \end{bmatrix} = 0 \qquad M1 \\ A1 \qquad 4 \qquad \text{ft } \mathbf{n}$ (ii) $d = 0 \Rightarrow \text{ plane through / contains the origin} \\ d = 0 \Rightarrow \text{ plane through / contains the origin} \\ (c) \qquad \sin\theta/\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}} \\ \text{Numerator} = 21 - 20 + 6 = 7 \\ \text{Denominator} = 7.\sqrt{150} \\ \theta = 4.7^{\circ} \qquad \text{Total} \qquad 15$	(b)(i)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$	M1A1				
(ii) $d = 0 \Rightarrow$ plane through / contains the originB11(c) $\sin\theta/\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$ M1Must be $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ and their $\mathbf{n}$ Numerator = $21 - 20 + 6 = 7$ B1ft correct (unsimplified)Denominator = $7.\sqrt{150}$ B1ft both correct (unsimplified) $\theta = 4.7^{\circ}$ Total15		$d = \begin{bmatrix} 7\\5\\1 \end{bmatrix} \cdot \begin{bmatrix} 7\\-10\\1 \end{bmatrix} = 0$	M1 A1	4	ft n		
(c) $\sin\theta/\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$ M1Must be $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ and their <b>n</b> Numerator = $21 - 20 + 6 = 7$ B1ft correct (unsimplified)Denominator = $7.\sqrt{150}$ B1ft both correct (unsimplified) $\theta = 4.7^{\circ}$ A14Total	(ii)	$d = 0 \implies$ plane through / contains the origin	B1	1			
Numerator = $21 - 20 + 6 = 7$ B1ft correct (unsimplified)Denominator = $7.\sqrt{150}$ B1ft both correct (unsimplified) $\theta = 4.7^{\circ}$ A14CAOTotal	(c)	$\sin\theta/\cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$	M1		Must be $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ and their <b>n</b>		
Denominator = $7.\sqrt{150}$ B1ft both correct (unsimplified) $\theta = 4.7^{\circ}$ A14CAOTotal		Numerator = $21 - 20 + 6 = 7$	B1		ft correct (unsimplified)		
$\theta = 4.7^{\circ} \qquad A1 \qquad 4 \qquad CAO$ $Total \qquad 15$		Denominator = $7\sqrt{150}$	B1		ft both correct (unsimplified)		
Total     15		$\theta = 4.7^{\circ}$	A1	4	CAO		
		Total		15			

MFP4 (cont)					
Q	Solution	Marks	Total	Comments	
7(a)(i)	$\mathbf{M}^{2} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ $\begin{bmatrix} 4 & -3 & 3 \end{bmatrix}$	M1			
	$= \begin{bmatrix} 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix}$ $\mathbf{M}^{2} + 2\mathbf{I} = \begin{bmatrix} 4 & -3 & 3 \\ 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	A1			
	$ = \begin{bmatrix} 0 & -3 & 3 \\ 3 & 0 & 3 \\ 3 & -3 & 6 \end{bmatrix} = 3\mathbf{M} $	A1	3	ie $k = 3$	
(ii)	Multiplying by $\mathbf{M}^{-1}$ to get $\mathbf{M} + 2\mathbf{M}^{-1} = 3\mathbf{I}$ so that $\mathbf{M}^{-1} = \frac{3}{2}\mathbf{I} - \frac{1}{2}\mathbf{M}$	M1 A1 A1	3	ft ie $a = -\frac{1}{2}$ and $b = \frac{3}{2}$	
(b)(i)	Char. eqn. is $\lambda^3 - 4\lambda^2$ + $5\lambda - 2 = 0$ ie $(\lambda - 2)(\lambda - 1)^2 = 0$ giving $\lambda_1 = 1$ (twice) and $\lambda_2 = 2$	M1A1 A1A1 M1 A1	6	One A mark for each of the other coefficients Good factorisation attempt	
(ii)	$\lambda = 1 \implies x - y + z = 0$ (thrice) Any two independent eigenvectors	B1 M1		Attempted	
	(eg) $\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\lambda = 2 \implies -y + z = 0$ $x - 2y + z = 0 \implies x = y = z$	A1 M1			
	$\begin{array}{c} x - y = 0 \\ \gamma \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{array}$	A1	5		
(iii)	For $\lambda = 1$ , eigenvectors represent a plane of invariant points	M1 A1		Plane	
	For $\lambda = 2$ , eigenvectors represent an invariant line	B1	3		
	Total		20		
	TOTAL		75		