

General Certificate of Education
January 2008
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 It is given that $z_1 = 2 + i$ and that z_1^* is the complex conjugate of z_1 .

Find the real numbers x and y such that

$$x + 3iy = z_1 + 4iz_1^* \quad (4 \text{ marks})$$

- 2 A curve satisfies the differential equation

$$\frac{dy}{dx} = 2^x$$

Starting at the point $(1, 4)$ on the curve, use a step-by-step method with a step length of 0.01 to estimate the value of y at $x = 1.02$. Give your answer to six significant figures. (5 marks)

- 3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π .

(5 marks)

- 4 (a) Find

$$\sum_{r=1}^n (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers.

(5 marks)

- (b) It is given that

$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

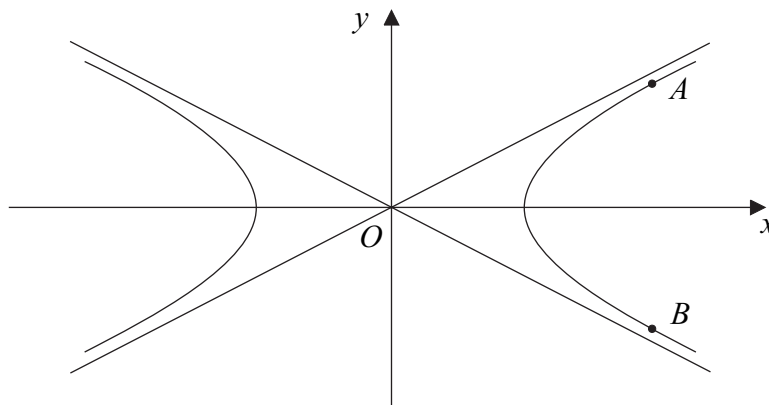
Without calculating the value of S , show that S is a multiple of 2008.

(2 marks)

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. (2 marks)

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B .

Find, in surd form, the y -coordinates of A and B . (2 marks)

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y -coordinates of the image points of A and B under this translation; (1 mark)

(ii) the equations of the hyperbola and the asymptotes after the translation. (3 marks)

Turn over for the next question

Turn over ►

6 The matrix \mathbf{M} is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and \mathbf{I} is the 2×2 identity matrix. (3 marks)

(ii) Show that the matrix \mathbf{M} can be written in the form

$$q \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ \sin 60^\circ & -\cos 60^\circ \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. (3 marks)

(b) The matrix \mathbf{M} represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; (1 mark)

(ii) the equation of the mirror line of the reflection. (1 mark)

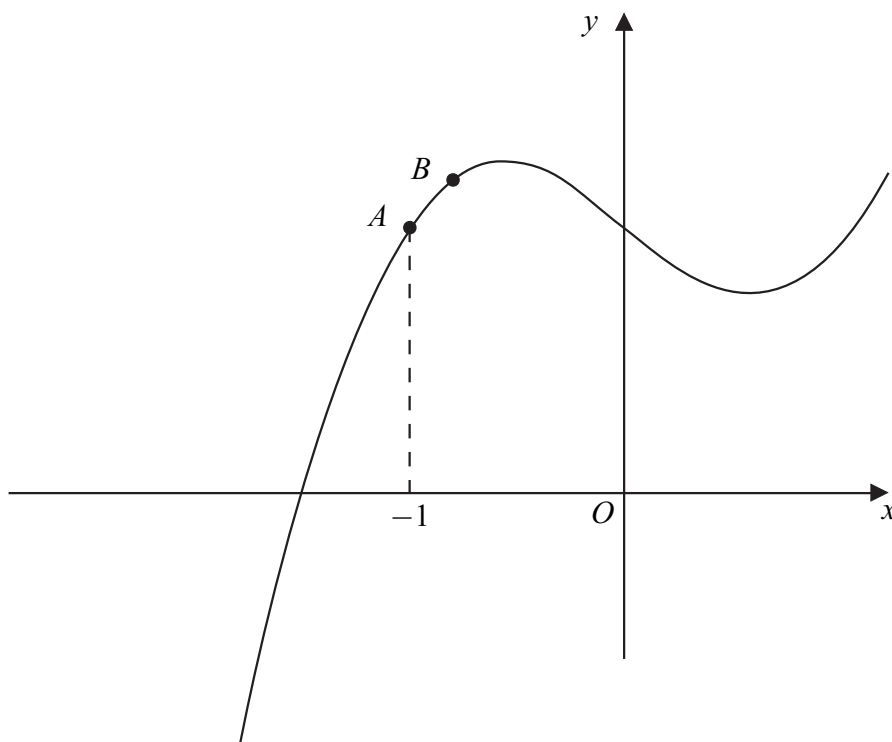
(c) Describe fully the geometrical transformation represented by \mathbf{M}^4 . (2 marks)

7 [Figure 1, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x -coordinates -1 and $-1 + h$ respectively.



- (a) (i) Show that the y -coordinate of the point B is

$$1 + 2h - 3h^2 + h^3 \quad (3 \text{ marks})$$

- (ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p , q and r are integers. (3 marks)

- (iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A . State the value of this gradient. (2 marks)

- (b) The equation $x^3 - x + 1 = 0$ has one real root, α .

- (i) Taking $x_1 = -1$ as a first approximation to α , use the Newton-Raphson method to find a second approximation, x_2 , to α . (2 marks)

- (ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram.

(2 marks)

Turn over ►

- 8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0 \quad (6 \text{ marks})$$

- (ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. (2 marks)

- (b) Solve the equation

$$x^2 - 2x + 4 = 0 \quad (2 \text{ marks})$$

- (c) Use your answers to parts (a) and (b) to show that

$$(1 + i\sqrt{3})^3 = (1 - i\sqrt{3})^3 \quad (2 \text{ marks})$$

- 9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

- (a) Write down the equations of the three asymptotes of C . (3 marks)

- (b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) (6 marks)

- (c) Sketch the curve C . (3 marks)

END OF QUESTIONS

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Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

Insert for use in **Question 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 7)