General Certificate of Education January 2008 Advanced Level Examination



# MATHEMATICS Unit Pure Core 4

MPC4

Thursday 24 January 2008 9.00 am to 10.30 am

## For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

## **Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

#### **Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

## Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

P1029/Jan08/MPC4 6/6/6/ MPC4

## Answer all questions.

- 1 (a) Given that  $\frac{3}{9-x^2}$  can be expressed in the form  $k\left(\frac{1}{3+x} + \frac{1}{3-x}\right)$ , find the value of the rational number k.
  - (b) Show that  $\int_{1}^{2} \frac{3}{9 x^2} dx = \frac{1}{2} \ln \left( \frac{a}{b} \right)$ , where a and b are integers. (3 marks)
- 2 (a) The polynomial f(x) is defined by  $f(x) = 2x^3 + 3x^2 18x + 8$ .
  - (i) Use the Factor Theorem to show that (2x 1) is a factor of f(x). (2 marks)
  - (ii) Write f(x) in the form  $(2x-1)(x^2+px+q)$ , where p and q are integers. (2 marks)
  - (iii) Simplify the algebraic fraction  $\frac{4x^2 + 16x}{2x^3 + 3x^2 18x + 8}$ . (2 marks)
  - (b) Express the algebraic fraction  $\frac{2x^2}{(x+5)(x-3)}$  in the form  $A + \frac{B+Cx}{(x+5)(x-3)}$ , where A, B and C are integers. (4 marks)
- 3 (a) Obtain the binomial expansion of  $(1+x)^{\frac{1}{2}}$  up to and including the term in  $x^2$ .
  - (b) Hence obtain the binomial expansion of  $\sqrt{1+\frac{3}{2}x}$  up to and including the term in  $x^2$ .
  - (c) Hence show that  $\sqrt{\frac{2+3x}{8}} \approx a + bx + cx^2$  for small values of x, where a, b and c are constants to be found. (2 marks)

4 David is researching changes in the selling price of houses. One particular house was sold on 1 January 1885 for £20. Sixty years later, on 1 January 1945, it was sold for £2000. David proposes a model

$$P = Ak^t$$

for the selling price,  $\pounds P$ , of this house, where t is the time in years after 1 January 1885 and A and k are constants.

(a) (i) Write down the value of A.

(1 mark)

(ii) Show that, to six decimal places, k = 1.079775.

(2 marks)

- (iii) Use the model, with this value of k, to estimate the selling price of this house on 1 January 2008. Give your answer to the nearest £1000. (2 marks)
- (b) For another house, which was sold for £15 on 1 January 1885, David proposes the model

$$Q = 15 \times 1.082709^t$$

for the selling price,  $\pounds Q$ , of this house t years after 1 January 1885. Calculate the year in which, according to these models, these two houses would have had the same selling price.

(4 marks)

- 5 A curve is defined by the parametric equations  $x = 2t + \frac{1}{t^2}$ ,  $y = 2t \frac{1}{t^2}$ .
  - (a) At the point P on the curve,  $t = \frac{1}{2}$ .
    - (i) Find the coordinates of P.

(2 marks)

(ii) Find an equation of the tangent to the curve at P.

(5 marks)

(b) Show that the cartesian equation of the curve can be written as

$$(x - y)(x + y)^2 = k$$

where k is an integer.

(3 marks)

Turn over for the next question

6 A curve has equation  $3xy - 2y^2 = 4$ .

Find the gradient of the curve at the point (2, 1).

(5 marks)

- 7 (a) (i) Express  $6 \sin \theta + 8 \cos \theta$  in the form  $R \sin(\theta + \alpha)$ , where R > 0 and  $0^{\circ} < \alpha < 90^{\circ}$ . Give your value for  $\alpha$  to the nearest 0.1°. (2 marks)
  - (ii) Hence solve the equation  $6 \sin 2x + 8 \cos 2x = 7$ , giving all solutions to the nearest 0.1° in the interval  $0^{\circ} < x < 360^{\circ}$ . (4 marks)
  - (b) (i) Prove the identity  $\frac{\sin 2x}{1 \cos 2x} = \frac{1}{\tan x}$ . (4 marks)
    - (ii) Hence solve the equation

$$\frac{\sin 2x}{1 - \cos 2x} = \tan x$$

giving all solutions in the interval  $0^{\circ} < x < 360^{\circ}$ .

(4 marks)

8 Solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\cos 3x}{v}$$

given that y = 2 when  $x = \frac{\pi}{2}$ . Give your answer in the form  $y^2 = f(x)$ . (5 marks)

- **9** The points A and B lie on the line  $l_1$  and have coordinates (2, 5, 1) and (4, 1, -2) respectively.
  - (a) (i) Find the vector  $\overrightarrow{AB}$ .

(2 marks)

(ii) Find a vector equation of the line  $l_1$ , with parameter  $\lambda$ .

(1 mark)

- (b) The line  $l_2$  has equation  $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ .
  - (i) Show that the point P(-2, -3, 5) lies on  $l_2$ .

(2 marks)

(ii) The point Q lies on  $l_1$  and is such that PQ is perpendicular to  $l_2$ . Find the coordinates of Q. (6 marks)

# END OF QUESTIONS