General Certificate of Education January 2008 Advanced Subsidiary Examination

MATHEMATICS Unit Pure Core 1

MPC1

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

• an 8-page answer book

• the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The Examining Body for this paper is AQA. The Paper Reference is MPC1.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.



Answer all questions.

1	The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.			
	(a)	Find	the coordinates of the mid-point of BC.	(2 marks)
	(b)	(i)	Find the gradient of AB , in its simplest form.	(2 marks)
		(ii)	Hence find an equation of the line AB , giving your answer in the form $x + qy = r$, where q and r are integers.	(2 marks)
		(iii)	Find an equation of the line passing through C which is parallel to AB .	(2 marks)
	(c)	Prov	e that angle ABC is a right angle.	(3 marks)
2	2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.			
	(a)	Find	$\frac{\mathrm{d}y}{\mathrm{d}x}$.	(3 marks)
	(b)	Henc	the find the x-coordinate of M.	(3 marks)
	(c)	(i)	Find $\frac{d^2y}{d^2y}$	(1 mark)

(c) (i) Find
$$\frac{d^2 y}{dx^2}$$
. (1 mark)

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. (2 marks)

(d) Determine whether the curve is increasing or decreasing at the point on the curve where x = 0. (2 marks)

3 (a) Express
$$5\sqrt{8} + \frac{6}{\sqrt{2}}$$
 in the form $n\sqrt{2}$, where *n* is an integer. (3 marks)

(b) Express
$$\frac{\sqrt{2}+2}{3\sqrt{2}-4}$$
 in the form $c\sqrt{2}+d$, where c and d are integers. (4 marks)

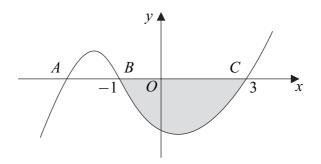
- 4 A circle with centre C has equation $x^2 + y^2 10y + 20 = 0$.
 - (a) By completing the square, express this equation in the form

$$x^{2} + (y - b)^{2} = k$$
 (2 marks)

- (b) Write down:
 - (i) the coordinates of C; (1 mark)
 - (ii) the radius of the circle, leaving your answer in surd form. (1 mark)
- (c) A line has equation y = 2x.
 - (i) Show that the *x*-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 4x + 4 = 0$. (2 marks)
 - (ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, *P*. (3 marks)
- (d) Prove that the point Q(-1, 4) lies inside the circle. (2 marks)
- 5 (a) Factorise $9 8x x^2$. (2 marks)
 - (b) Show that $25 (x + 4)^2$ can be written as $9 8x x^2$. (1 mark)
 - (c) A curve has equation $y = 9 8x x^2$.
 - (i) Write down the equation of its line of symmetry. (1 mark)
 - (ii) Find the coordinates of its vertex. (2 marks)
 - (iii) Sketch the curve, indicating the values of the intercepts on the *x*-axis and the *y*-axis. (3 marks)

Turn over for the next question

- 6 (a) The polynomial p(x) is given by $p(x) = x^3 7x 6$.
 - (i) Use the Factor Theorem to show that x + 1 is a factor of p(x). (2 marks)
 - (ii) Express $p(x) = x^3 7x 6$ as the product of three linear factors. (3 marks)
 - (b) The curve with equation $y = x^3 7x 6$ is sketched below.



The curve cuts the x-axis at the point A and the points B(-1, 0) and C(3, 0).

(i) State the coordinates of the point A. (1 mark)

(ii) Find
$$\int_{-1}^{3} (x^3 - 7x - 6) dx$$
. (5 marks)

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. (1 mark)

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point *B*. (3 marks)

- (v) Hence find an equation of the normal to the curve at the point *B*. (3 marks)
- 7 The curve C has equation $y = x^2 + 7$. The line L has equation y = k(3x + 1), where k is a constant.
 - (a) Show that the *x*-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0 (1 mark)$$

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$
 (3 marks)

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. (4 marks)

END OF QUESTIONS

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