

General Certificate of Education

## Mathematics 6360

## MPC1 Pure Core 1

## Mark Scheme

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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## Key to mark scheme and abbreviations used in marking



## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) | Mid-point of $B C=(3,-2)$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ | 2 | Either coordinate correct Both cords correct. Accept $x=3, y=-2$ |
| (b)(i) | $\begin{aligned} & \frac{\Delta y}{\Delta x}=\frac{3-1}{-2-4} \\ & =-\frac{1}{3} \end{aligned}$ | M1 A1 | 2 | $\pm \frac{2}{6}$ OE implies M1 |
| (ii) | $\begin{aligned} & y-3=\text { "their } \operatorname{grad} "(x+2) \text { or } \\ & y-1=\text { "their } \operatorname{grad} "(x-4) \\ & \text { Hence } x+3 y=7 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | 2 | Or $y=m x+c$ and correct attempt to find $c$ |
| (iii) | $\begin{aligned} & y+5=\text { "their grad } A B "(x-2) \\ & y+5=-\frac{1}{3}(x-2) \text { or } x+3 y+13=0 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | 2 | Or "their $x+q y=c$ " and attempt to find $c$ OE |
| (c) | $\operatorname{Grad} B C=3\left(\right.$ from $\left.\frac{\Delta y}{\Delta x}=\frac{1+5}{4-2} \mathrm{OE}\right)$ $m_{1} m_{2}=-1$ stated or $\operatorname{grad} B C=3$ and $\operatorname{grad} A B=-\frac{1}{3}$ or $\operatorname{grad} B C \times \operatorname{grad} A B\left(=3 \times-\frac{1}{3}\right)$ | B1 <br> M1 |  | Or 2 lengths correct: $A B=\sqrt{40} ; B C=\sqrt{40} ; A C=\sqrt{80}$ <br> Or attempt at Pythagoras or Cosine Rule |
|  | Product of gradients $=-1$ <br> Hence $A B$ and $B C$ are perpendicular | $\begin{gathered} \text { A1 } \\ \text { CSO } \end{gathered}$ | 3 | $A C^{2}=A B^{2}+B C^{2} \Rightarrow \angle A B C=90^{\circ}$ <br> Completing proof and statement |
|  | Total |  | 11 |  |
| 2(a) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=4 x^{3}-32$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | 3 | Reduce one power by 1 One term correct All correct (no $+c$ etc) |
| (b) | Stationary point $\Rightarrow \frac{\mathrm{d} y}{\mathrm{~d} x}=0$ $\Rightarrow x^{3}=8$ | M1 <br> A1 $\checkmark$ |  | $x^{n}=k$ following from their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ |
|  | $\Rightarrow x=2$ | A1 | 3 | CSO |
| (c)(i) | $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=12 x^{2}$ | B1 $\checkmark$ | 1 | $\text { FT their } \frac{\mathrm{d} y}{\mathrm{~d} x}$ |
| (ii) | When $x=2, \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}$ considered $\Rightarrow$ minimum point | $\begin{gathered} \text { M1 } \\ \text { E1 } \checkmark \end{gathered}$ | 2 | Or complete test with $2 \pm \varepsilon$ using $\frac{\mathrm{d} y}{\mathrm{~d} x}$ |
| (d) | Putting $x=0$ into their $\frac{\mathrm{d} y}{\mathrm{~d} x}(=-32)$ $\frac{\mathrm{d} y}{\mathrm{~d} x}<0 \Rightarrow$ decreasing | M1 <br> A1 $\checkmark$ | 2 | Allow "increasing" if their $\frac{\mathrm{d} y}{\mathrm{~d} x}>0$ |
|  | Total |  | 11 |  |

MPC1 (cont)

\begin{tabular}{|c|c|c|c|c|}
\hline Q \& Solution \& Marks \& Total \& Comments <br>
\hline 3(a)

(b) \& \begin{tabular}{l}
$$
\begin{aligned}
& 5 \sqrt{8}=10 \sqrt{2} \\
& \frac{6}{\sqrt{2}}=\frac{6 \sqrt{2}}{2} \quad(=3 \sqrt{2})
\end{aligned}
$$ <br>
Answer $=13 \sqrt{2}$
$$
\frac{\sqrt{2}+2}{3 \sqrt{2}-4} \times \frac{3 \sqrt{2}+4}{3 \sqrt{2}+4}
$$ <br>
Numerator $=6+6 \sqrt{2}+4 \sqrt{2}+8$ <br>
Denominator $=18-16(=2)$ <br>
Final answer $=5 \sqrt{2}+7$

 \& 

B1 <br>
M1 <br>
A1 <br>
M1 <br>
m1 <br>
B1 <br>
A1
\end{tabular} \& 3

4 \& | Or $\frac{5 \sqrt{16}+6}{\sqrt{2}}$ gets B1 |
| :--- |
| then M1 for rationalising; and A1 answer $n=13$ |
| Multiplying top \& bottom by $\pm(3 \sqrt{2}+4)$ |
| Multiplying out (condone one slip) | <br>

\hline \& Total \& \& 7 \& <br>

\hline 4(a) \& $$
\begin{aligned}
& x^{2}+(y-5)^{2} \\
& \text { RHS }=5
\end{aligned}
$$ \& \[

$$
\begin{aligned}
& \text { B1 } \\
& \text { B1 }
\end{aligned}
$$

\] \& 2 \& \[

$$
\begin{aligned}
& b=5 \\
& k=5
\end{aligned}
$$
\] <br>

\hline (b)(i) \& Centre (0, 5) \& B1 $\checkmark$ \& 1 \& FT their $b$ from part (a) <br>
\hline (ii) \& Radius $=\sqrt{5}$ \& B1 $\checkmark$ \& 1 \& FT their $k$ from part (a); RHS must be $>0$ <br>

\hline (c)(i) \& $$
\begin{aligned}
& x^{2}+4 x^{2}-20 x+20=0 \\
& \Rightarrow x^{2}-4 x+4=0
\end{aligned}
$$ \& \[

$$
\begin{gathered}
\text { M1 } \\
\text { A1 }
\end{gathered}
$$
\] \& 2 \& May substitute into original or "their (a)" CSO; AG <br>

\hline (ii) \& | $(x-2)^{2}=0 \text { or } x=2$ |
| :--- |
| Repeated root implies tangent Point of contact is $P(2,4)$ | \& | M1 |
| :--- |
| E1 |
| A1 | \& 3 \& Or $b^{2}-4 a c$ shown $=0$ plus statement <br>

\hline (d) \& $$
\begin{aligned}
& \left(C Q^{2}=\right) 1^{2}+1^{2} \\
& \sqrt{2}<\sqrt{5} \Rightarrow Q \text { lies inside circle }
\end{aligned}
$$ \& \[

$$
\begin{gathered}
\text { M1 } \\
\text { A1 } \\
\text { CSO } \\
\hline
\end{gathered}
$$
\] \& 2 \& FT their $C$ $C Q$ or $C Q^{2}$ OE must appear for A1 <br>

\hline \& Total \& \& 11 \& <br>

\hline 5(a) \& $(9+x)(1-x)$ \& \[
$$
\begin{aligned}
& \text { M1 } \\
& \text { A1 }
\end{aligned}
$$

\] \& 2 \& | $\pm(9 \pm x)(1 \pm x)$ |
| :--- |
| Correct factors | <br>

\hline (b) \& $25-\left(x^{2}+8 x+16\right)=9-8 x-x^{2}$ \& B1 \& 1 \& AG <br>
\hline (c)(i) \& $x=-4$ is line of symmetry \& B1 \& 1 \& <br>
\hline (ii) \& Vertex is ( $-4,25$ ) \& B1,B1 \& 2 \& <br>

\hline \multirow[t]{2}{*}{(iii)} \& $$
y^{y}
$$ \& M1 \& \& General $\cap$ shape <br>

\hline \&  \& A1 \& 3 \& | 9 marked on $y$-axis and maximum to the left of $y$-axis |
| :--- |
| Must continue below $x$-axis at both ends | <br>

\hline \& Total \& \& 9 \& <br>
\hline
\end{tabular}

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 6(a)(i) | $\begin{aligned} \mathrm{p}(-1) & =-1+7-6 \\ & =0 \quad \text { therefore } x+1 \text { is a factor } \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ | 2 | Finding $\mathrm{p}(-1)$ <br> Shown to $=0$ plus statement |
| (ii) | $\mathrm{p}(x)=(x+1)\left(x^{2}-x-6\right)$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  | Long division/inspection (2 terms correct) Quadratic factor correct |
|  | $\mathrm{p}(x)=(x+1)(x+2)(x-3)$ | A1 | 3 | May earn M1,A1 for correct second factor then A1 for $(x+1)(x+2)(x-3)$ |
| (b)(i) | $A(-2,0)$ | B1 | 1 | Condone $x=-2$ |
| (ii) | $\frac{x^{4}}{}-\frac{7 x^{2}}{2}-6 x \quad(+c)$ | M1 |  | One term correct |
|  | $4{ }^{4} 220$ | A1 |  | Another term correct |
|  | (may have $+c$ or not) | A1 |  | All correct unsimplified |
|  | $\left[\frac{81}{4}-\frac{63}{2}-18\right]-\left[\frac{1}{4}-\frac{7}{2}+6\right]$ | m1 |  | $F(3)-F(-1)$ attempted in correct order |
|  | $=-32$ | A1 | 5 | CSO; OE |
| (iii) | Area of shaded region $=32$ | B1ヶ | 1 | FT their (b)(ii) but positive value needed |
| (iv) | $\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{2}-7$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ |  | One term correct <br> All correct (no $+c$ etc) |
|  | When $x=-1$, gradient $=-4$ | A1 | 3 | CSO |
| (v) | $\text { Gradient of normal }=\frac{1}{4}$ |  |  |  |
|  | $y=$ "their gradient" $(x \pm 1)$ | M1 |  | Must be finding normal, not tangent |
|  | $y=\frac{1}{4}(x+1)$ |  | 3 | CSO; any correct form eg $4 y-x=1$ |
|  | Total |  | 18 |  |
| 7(a) | $x^{2}+7=k(3 x+1) \Rightarrow x^{2}-3 k x+7-k=0$ | B1 | 1 | AG |
| (b) | $b^{2}-4 a c=(-3 k)^{2}-4(7-k)$ <br> (2 distinct roots when) $b^{2}-4 a c>0$ $9 k^{2}+4 k-28>0$ | $\begin{aligned} & \text { M1 } \\ & \text { B1 } \\ & \text { A1 } \end{aligned}$ | 3 | Clear attempt at $b^{2}-4 a c$ <br> Condone slip in one term of expression <br> Must involve $k$ <br> CSO; AG |
| (c) | $(9 k-14)(k+2)$ | M1 |  | Factors or formula correct unsimplified |
|  | Critical points -2 and $\frac{14}{9}$ | A1 |  |  |
|  |  |  |  | +ve -ve +ve |
|  | Sketch $\cup$ or sign diagram correct | M1 |  | -2 $\frac{14}{9}$ |
|  | $k<-2, k>\frac{14}{9}$ | A1 | 4 |  |
|  | Total |  | 8 |  |
|  | TOTAL |  | 75 |  |

