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MS2B Statistics 2B

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General

It was again very pleasing to see the many fully correct solutions to each of the questions on the paper. However, it was still very noticeable that some candidates appeared to have neither prepared well enough for this paper nor covered the whole specification in sufficient depth to be able to gain reasonable marks.

The general standard of algebraic skills and integration techniques shown by most candidates was good. Hypotheses were usually stated clearly, but more emphasis needed to be placed on the interpretation of statistical results, a skill which remained quite weak.

Candidates should have access to a copy of the blue AQA booklet of formulae and statistical tables throughout the teaching of the unit, as familiarity with the content of the booklet can be essential to ultimate success. Relevant information therein is an integral part of this unit and should be treated as such. It sometimes appeared that a candidate had not seen the booklet's relevant content prior to taking the examination.

Question 1

This question, where many fully correct solutions were seen, proved to be a very good source of marks. The vast majority of candidates realised that this question required the use of a one-tailed test, correctly quoting the hypotheses as $\mathrm{H}_0{:}\,\mu = 5$ and $\mathrm{H}_1{:}\,\mu > 5$. However, there were still some candidates who quoted these hypotheses incorrectly, with $\mathrm{H}_0{:}\,\bar{x} = 5$ and $\mathrm{H}_1{:}\,\bar{x} > 5$, $\mathrm{H}_0{:} = 5$ and $\mathrm{H}_1{:} > 5$ or $\mathrm{H}_0{:}\,\mathrm{mean} = 5$ and $\mathrm{H}_1{:}\,\mathrm{mean} > 5$ being the most prevalent wrong answers. Candidates were expected to be aware that s^2 denoted an unbiased estimate of σ^2, as indicated in the specification and on page 12 of the blue formulae booklet. Those candidates who decided that $1.31 \times \frac{40}{39}$ was required only lost one mark on this occasion.

A few candidates, who decided to construct either a 98% or a 99% symmetric confidence interval instead of setting up the required hypothesis test, often did not state any hypotheses. This course of action inevitably resulted in a loss of marks. The final mark, for quoting a conclusion in context, was lost by many candidates. "Reject H_0" was often followed by disappointingly incorrect conclusions in context. Statements such as "Evidence to suggest customers have to queue for more than 5 minutes" or "Evidence to support David's claim that customers have to queue for more than 5 minutes" or "Evidence that the mean waiting time is not 5 minutes" were the most common incorrect attempts. It was expected that candidates would draw conclusions in context such as "Evidence to support David's claim at the 1% level" or "Evidence to support David's claim that customers have to queue for more than 5 minutes on average". Either of these statements gained full credit.

Question 2

In part (a)(i), there were surprisingly many candidates who did not know the relationship between the value of λ and the standard deviation of a Poisson distribution. In part (a)(ii), although the vast majority realised that the most efficient method of answering this part of the question required the use of the table on page 23 of the formulae booklet, very many failed to look up the appropriate values to enable them to show that $\mathrm{P}(6 < X < 12) = \mathrm{P}(X \le 11) - \mathrm{P}(X \le 6) = 0.596$. Those candidates who calculated the separate probabilities for $X = 7,\ 8,\ 9,\ 10 \text{ and } 11$ often reached the required answer.

In part (b)(i), it should be noted that $\lambda = 11.5$ and an indication that the distribution of T is Poisson were both required to gain the mark. In part (b)(ii), most candidates realised that "fewer than 2" implied that $\mathrm{P}(T \le 1) = \mathrm{P}(T = 0) + \mathrm{P}(T = 1)$. The correct answer of 0.000127 was then usually found by the use of $\lambda = 11.5$ and the formula $\mathrm{P}(X = x) = \mathrm{e}^{-\lambda} \dfrac{\lambda^x}{x!}$ (found on page 11 of the formulae booklet).

Although part (c) asked candidates to calculate appropriate numerical measures (plural), this was ignored by some candidates. Such candidates usually calculated $\bar{x}$ and, finding this to coincide with the given value of λ, stated inappropriately that $\mathrm{Po}(12)$ was a suitable model for the given data. Of those candidates who did calculate more than one numerical measure, most correctly calculated the mean and variance of the given data $(\bar{x} = 12;\ s^2 = 19.3 \text{ or } \sigma^2 = 17.4)$, and then indicated that, because the mean and variance were (very) different values, $\mathrm{Po}(12)$ was not a suitable model for the number of places booked each week. Some candidates incorrectly thought that the mean and the standard deviation had to be the same.

Question 3

Part (a) was usually very well done, with many correct answers of $k = \dfrac{1}{a+b}$ seen.

Unfortunately, there were some candidates who simply quoted the formula $k = \dfrac{1}{b-a}$ from page 11 of the formulae booklet without realising that this was based on the interval $[a,\ b]$, whereas this question indicated an interval $[-a,\ b]$. Integration methods were also seen, which usually resulted in the correct expression for k. It was pleasing to see that the vast majority of candidates realised that $\mathrm{E}(T) = \int_{-a}^{b} kt\,\mathrm{d}t$, with most then integrating correctly.

Although the correct substitution of the given limits leading to $\frac{1}{2}k\left(b^2 - a^2\right)$ was often seen, many candidates then simply quoted the given answer without any further working. This resulted in a loss of marks. It was expected that candidates would factorise to get $b^2 - a^2 = (b-a)(b+a)$ and perform a cancellation before stating the final given result. When an answer is given in the question, it is essential for candidates to take all possible care to show all relevant working.

In part (b)(i), although the correct answer $\mathrm{E}(T) = 1$ was seen, many candidates thought that $\mathrm{E}(T) = 5$. This wrong answer usually resulted from candidates not realising that their answer to part (a)(ii), namely $\mathrm{E}(T) = \frac{1}{2}(b-a)$, was based on the given interval $[-a,\ b]$ and so $[-4,\ 6]$ meant that $a = 4$ and **not**, as they assumed, $a = -4$. Although the application of integration was also used here to obtain, in most cases, the correct result, it was not the most efficient method and should not be encouraged when dealing with rectangular distributions.

In part (b)(ii), those candidates who drew a diagram were then, more often than not, able to write down the correct answer of 0.4. However, although many candidates were able to calculate $\mathrm{P}(T < -3) = 0.1$ and $\mathrm{P}(T > 3) = 0.3$, they then seemed to be at a loss as to what to do next. They failed to realise that the word 'or' was asking them to add these two probabilities together. Some candidates calculated $\mathrm{P}(T < -3) = 0.1$ or $\mathrm{P}(T > 3) = 0.3$, obviously believing that they had a choice, whilst others incorrectly thought that $\mathrm{P}(T > 3) = \mathrm{P}(T \geq 4) = 0.2$. Candidates who drew a diagram usually then managed to write down the correct answers to all of part (b) with very little or no further working.

Question 4

In part (a), candidates were asked to state any assumption that they had made in constructing their confidence interval. Candidates should be aware that a meaningful interval cannot be constructed in this case unless they assume that they are dealing with a small sample from a normal population with unknown variance. It should also be noted that the assumption should be made with the context of the question in mind. Simply stating "Normal distribution" or "It is normal" or "the population is normal", without also stating what this population is, will not gain full credit. Also, many candidates incorrectly thought that the sample or the data had to be normally distributed.

Those candidates who did not state an assumption still calculated a confidence interval based upon the *t*-distribution. Most candidates calculated correctly that $\bar{v} = 117.9$ but were often less sure when it came to calculating an unbiased estimate, s^2 , for the variance of the population. Although a majority calculated correctly that $s^2 = \frac{1014.9}{9} = 112.8$ (or $s = 10.6$), there were many who thought wrongly that $s^2 = 101.49$ $(\text{or } s = 10.1)$ or $s = \sqrt{1014.9} = 31.9$. Those who stated that $\sigma = 10.1$, nearly always went on to wrongly state that $\frac{10.1}{\sqrt{10}}$ was the value for the standard error. Appropriate use of the formulae booklet provided could have eliminated many of these errors.

The majority of candidates who calculated the correct values for $\bar{v}$ and s then used $t = 3.250$ and went on to construct the required confidence interval as $(107, 129)$. Occasionally, the wrong value of t was extracted from the tables, and there were some candidates who thought that the reference distribution was $\mathrm{N}(0, 1)$, rather than t_{n-1} ; this lead them to use wrongly a variety of z-values in their calculations.

It seemed, from the responses to part (b), that very few candidates understand what a confidence interval shows. Many candidates firstly stated that John's claim was "wrong" and then indicated that this was because 99% of his serves were between 107 mph and 129 mph. Candidates should be aware that, when studying this section of the specification, emphasis should be given to both the construction and the interpretation of confidence intervals.

Question 5

Although it was stated clearly in the question that X was a discrete random variable, many candidates employed integration methods throughout this question, all to no avail. Those who did treat the variable as discrete usually did very well, with many gaining full marks. Although there were many fully correct answers of 0.5 seen to part (a), there were several candidates who, having lost both marks here, went on to gain all other available marks for the question.

In part (b), where the answers were given, it was very pleasing to see candidates usually supplying the required valid working to enable them to gain full credit.

Unfortunately, there were still a few candidates who calculated $\mathrm{E}(X)=\dfrac{17}{4}$ and then wrongly assumed that $\mathrm{E}\left(\dfrac{1}{X}\right)=\dfrac{1}{\mathrm{E}(X)}$. A similar incorrect argument followed when attempting to find the value of $\mathrm{Var}\left(\dfrac{1}{X}\right)$.

Part (c) was very well done, even by some of those candidates who failed to gain any credit on the previous parts of this question.

Question 6

This, once again, proved to be a very popular topic and an excellent source of marks. Although there were still a few candidates who failed to state any hypotheses at all, those who did usually did so correctly. Most realised that $\nu=1$ and that Yates' correction was required. Unfortunately, there were some who could not apply this correction in the correct way. Although the majority of candidates correctly considered $|O_i-E_i|-0.5=7.3$, and then went on to calculate $X^2=\sum_i\left(7.3^2/E_i\right)$, there were some who thought wrongly that

$X^2=\sum_i\left\{(O_i-E_i-0.5)^2/E_i\right\}$ or even that $X^2=\sum_i\left\{\left((O_i-E_i)^2-0.5\right)/E_i\right\}$ were appropriate.

It was pleasing to see good conclusions in context.

In part (b), most candidates understood what a Type I error was, stating that this occurred when "H_0 was rejected when in fact H_0 was correct". Unfortunately, many could not then interpret this Type I error in the context of the question.

Question 7

In general, the attempts at this question were very poor. It should be emphasised, yet again, that when an answer is given in a question, candidates must take extra care to explain what they are doing. This was often not seen in answers to parts (a)(ii), (a)(iii) and (b)(i), where many candidates showed insufficient working to achieve full credit.

Part (a)(i) asked candidates to sketch the graph of F. Many interpreted this as allowing the drawing of a rather scruffy freehand diagram without a scale. This was not what was intended. It was expected that scaled (approximately) and labelled axes be drawn (using a ruler and preferably a pencil) and care be taken in drawing the curved part of the sketch.

Although most candidates attempted the straight line from the origin to $\left(1,\tfrac{1}{2}\right)$ correctly, there was much less success with the curve from $\left(1,\tfrac{1}{2}\right)$ to $(4,1)$. The part of the graph defined by $\mathrm{F}(x)=1$ was often not drawn, and the part defined by $\mathrm{F}(x)=0$ was rarely seen at all.

Part (a)(ii) was not explained well by many candidates. Many thought incorrectly that dividing 4 (the length of the interval over which most of them drew their sketch) by 4 would give them the value of the lower quartile, whilst others assumed that (the value of the lower quartile) = $\dfrac{1}{2}\times$(the value of the median) without any explanation as to why this was the case for this distribution.

It was, however, pleasing to see good candidates state correctly that $\mathrm{F}(q_1) = \frac{1}{2}q_1 = 0.25$, leading to the given answer of $q_1 = \frac{1}{2}$. A variety of methods, mostly correct, were employed in part (a)(iii). The vast majority of candidates realised that the upper quartile was in the interval $[1, 4]$ and used the fact that $\mathrm{F}(q_3) = 0.75$ to good effect. Most used different rearrangements of the resultant cubic equation, whilst others used their calculators to show that $q_3 = 1.62$ (3 sf), to achieve the required given result.

The attempts at part (b)(i) were varied and mostly disappointing, with many candidates using the result $\alpha = \frac{1}{2}$ to show that $\beta = \frac{1}{18}$ without firstly explaining why they thought that $\alpha = \frac{1}{2}$. Some found only one relationship between α and β (usually either $\alpha = 9\beta$ or $\alpha + 9\beta = 1$) and then simply substituted the given values of α and β to show that these values did work. This was not felt to be sufficient to gain full credit since each of these equations, taken in isolation, had infinitely many solutions, of which the given solution was just one such pair of values. However, there were a few excellent solutions, the most successful of these first showing that on the interval $[0, 1]$, $\mathrm{f}(x) = \alpha = \mathrm{F}'(x) = \frac{1}{2}$. This was then followed by use of the property that f was continuous at $x = 1$ to give $\mathrm{f}(1) = \alpha = 9\beta$, thus giving $\beta = \frac{1}{18}$ as required.

Part (b)(ii) was a good source of marks, even for those candidates who gained virtually no credit on earlier parts of the question. These candidates showed good algebraic and integration skills in finding correctly that $\mathrm{E}(X) = 1.125$. A small minority of candidates, instead of using any integration methods, used their calculators to find the correct numerical answer, usually after stating correctly that $\mathrm{E}(X) = \int_0^1 \frac{1}{2}x\,\mathrm{d}x + \int_1^4 \frac{1}{18}x(x-4)^2\,\mathrm{d}x$. This method gained the full 5 marks for those candidates who stated the correct numerical value but only 1 mark for those who did not. Whilst the use of calculators is encouraged in data analysis and in the evaluation of numerical expressions, candidates run the risk of losing marks if they acquire an incorrect numerical answer without any intermediate working.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the **Results statistics** page of the AQA Website.