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Mathematics 6360

MFP4 Further Pure 4

Report on the Examination

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General

Just over 250 candidates sat this paper – a 25% increase on last January's entry – and, the overall standard was very high. Although no-one actually gained full marks on the paper, 7% of candidates scored 70 or more out of the 75 available. At the other end of the scale, only 14% of candidates failed to achieve a score of 30. Disappointingly, many of these candidates were clearly unprepared to sit the paper at this time; in many of these cases they did not seem to have studied much of the work on this module.

Many other candidates who scored in the 40 – 50 mark range were clearly competent mathematicians and produced some thorough responses. It was clear, however, that they were not yet fully prepared for the rigours of an MFP4 examination paper as there were questions whose demands they were obviously not quite au fait with. I expect many of these would do substantially better in the summer if they were to continue to prepare for and re-take the module paper then.

Only a very small proportion of candidates seemed to have difficulty completing attempts to all questions within the set time, but these generally were those who had difficulty getting through Question 7 in any case, and time did not appear to be a particular issue overall.

Question 1

This was a straightforward starter to the paper as might be expected, requiring candidates only to know their way round the Booklet of Formulae and interpret the standard results given therein. Centres should note that there is no intention to require candidates on this specification to specify what is meant by positive and negative directions or anti-clockwise and clockwise senses when describing rotational 3-d transformations, so no candidates were penalised for specifying such matters. Nor were candidates penalised in this case for describing $y = x$ as a line rather than a plane in part (b). Quite a few candidates did lose a mark, however, when they failed to distinguish between 'the plane $y = x$' and 'the x-y plane'.

Question 2

This was another fairly gentle starter to the paper, with most candidates scoring at least six of the seven marks. Where marks were lost, this was usually in failing to grasp what was required by the request for 'geometrical' interpretations in part (b) – as was also the case in questions 3, 6 and 7. Here, candidates were expected to point out that **a**, **b and a × b** are perpendicular to each other. Many simply noted that the vector product was perpendicular to both **a** and **b**. Those candidates who simply stated 'perpendicular' as a one-word answer were not given the benefit of the doubt, as there were many others who essentially said just this, but failed to be complete when going on to describe what was at right-angles to what. Many others missed the mark in part (b)(ii) by describing the algebraic relationship of linear dependence but failing to note the geometrical relationship of co-planarity.

Question 3

As in Question 2, many answers failed to point out in part (a) that the geometrical property of a shear required to explain the given result is the invariance of areas under the transformation. Otherwise, the question was very capably handled.

Question 4

This question was actually quite straightforward. However, apart from the final parts of Question 7, it managed to present the biggest obstacle on the paper. Surprisingly, the greatest difficulty arose when candidates were required to turn $(-2)^n$ into 2^n, in the case when n is even, when considering the matrix $\mathbf{D}^n$. Many failed to manage this at all. A disappointingly large number of candidates left it until well into their working and many such candidates then failed to cope with the complicated expressions they were trying to deal with. Those that did so at the outset found themselves with an alternative and simple route through the problem available to them, since it follows that $\mathbf{D}^n = 2^n\,\mathbf{I}$; it is now straightforward to re-write $\mathbf{T}^n$ and to note that $\mathrm{f}(n) = 2^n$.

Question 5

It was very pleasing to see a large numbers of candidates solving part (a)'s system of equations using some fairly high-powered techniques (see the end of the marking scheme for these: Cramer's Rule, the augmented matrix method, and the inverse matrix method) to find the unique solution, but most of these are actually more complicated than what was required. The "low-tech" algebraic approach does the job so easily that the others (whilst eminently worth teaching) often turn out to be longer and more prone to errors.

One approach that did turn up for part (b)(ii) is that one can simply substitute the values of x, y and z found in part (a) into the 3^{rd} equation – using the value of *a* from part (b)(i) – in order to find the value of b required for the final equation to be consistent. This is because the third plane must share the line of intersection of the first two planes, and hence the point on it found in part (a). Only a handful of students actually employed this approach.

Question 6

This question was handled very confidently by most candidates, and most were aware of the geometrical significance of the direction cosines of a line. Far fewer appreciated that the value $\mathrm{d} = 0$ meant that part (b)'s plane contained the origin; many had a stab at some perpendicularity arrangement instead. In part (c) a significant minority of candidates found the complementary angle to the one required.

Question 7

This question was definitely the most demanding one on the paper; partly for its length, and partly due to working with a 3 × 3 matrix. On top of this, there was a repeated eigenvalue to the matrix, and a lot of candidates clearly did not know quite what to make of it. Nonetheless, even the relatively weaker candidates were still able to gain up to 12 of the marks.

Part (a)(i) was very well handled by those who attempted it. Part (a)(ii) was less well handled. Many of those who fell down here did so because they then went off to find the inverse matrix $\mathbf{M}^{-1}$ itself, rather than follow the logic of the question. Many more who did multiply the matrix equation by $\mathbf{M}^{-1}$ got tangled up in some way – failing to spot that $\mathbf{M}\,\mathbf{M}^{-1} = \mathbf{I}$, for instance, or being unable to divide by 2 under exam conditions.

In part (b), the factorisation of the determinant $|\,\mathbf{M} - \lambda\,\mathbf{I}\,|$ was usually approached by row/column operations, which was very pleasing to see. However, it frequently (actually usually) led to the strange sight of candidates taking a partially or completely factorised expression, and then multiplying it out to get a standard cubic form, only to start to factorise it again to find the three eigenvalues.

Those who made a mistake in this working then shot themselves in the foot, losing the following method mark for the attempt to factorise, which they failed to pick up by presuming that their cubic actually *was* equal to the factorised form $(\lambda - 1)^2(\lambda - 2)$ rather than doing any of the necessary working or going back to check their previous, incorrect working.

The final two parts to the question were demanding, but actually not as difficult as candidates found them to be. Despite the fact that most candidates getting this far quite correctly deduced that $x - y + z = 0$ for the repeated eigenvalue $\lambda = 1$, almost all of them then believed this represented the equation of a line. This error then prevented them from finding a pair of independent representative eigenvectors (or an alternative parametric plane form) and also from describing the result as a plane of invariant points, rather than as a line.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the **Results statistics** page of the AQA Website.