



General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2008 examination - January series

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	Rotation about the y -axis through $\cos^{-1} 0.8$	M1 A1 A1	3	Ignore direction or $\sin^{-1} 0.6$ or 36.87° or 0.644^c
(b)	Reflection in $y = x$	M1A1	2	Ignore if it is called a line
	Total		5	
2(a)(i)	$\mathbf{a} \cdot \mathbf{b} = 0$	B1	1	
(ii)	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 1 & 1 & -5 \end{vmatrix} = \begin{bmatrix} -16 \\ 11 \\ -1 \end{bmatrix}$	M1 A1	2	
(iii)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & -5 \\ 1 & 4 & 28 \end{vmatrix} = 0$	M1 A1	2	or via $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ ft in this case Do not allow = 0 via (a)(i)
(b)(i)	$\mathbf{a}, \mathbf{b}, \mathbf{a} \times \mathbf{b}$ mutually perpendicular	B1	1	
(ii)	$\mathbf{a}, \mathbf{b}, \mathbf{c}$ co-planar	B1	1	
	Total		7	
3(a)	Area invariant $\Rightarrow \text{Determinant} = 1 \Rightarrow pr + q^2 = 1$	M1 A1	2	MUST mention area Given answer justified
(b)(i)	$\begin{bmatrix} 4 & q \\ -q & r \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\Rightarrow 2q - 4 = 2$ and $q + 2r = -1$ $\Rightarrow q = 3$ and $r = -2$	M1 A1 A1	3	Either correct
(ii)	$x' = 4x + 3y$ and $y' = -3x - 2y$ Setting $x' = x, y' = y$ $y = -x$	B1 M1 A1	3	
	Alternative for (b)(ii): Setting $\lambda = 1$ $\Rightarrow 3x + 3y = 0$ (etc) ie $y = -x$	(M2) (A1)	(3)	
	Total		8	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{D} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}, \mathbf{U} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix},$ $\mathbf{U}^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$	B1B1		
		B1	3	ft \mathbf{U}^{-1}
(b)	$\mathbf{T}^n = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2^n & 0 \\ 0 & 2^n \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ $= \begin{bmatrix} 2^n & 2 \times 2^n \\ 2^n & 3 \times 2^n \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ <p>or</p> $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 \times 2^n & -2 \times 2^n \\ -2^n & 2^n \end{bmatrix}$ $= 2^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1 M1		For \mathbf{D}^n with n even For use of $\mathbf{U}^{-1} \mathbf{D}^n \mathbf{U}$ form
		m1 A1		
		A1	5	Shown legitimately
	<p>Alternative for (b):</p> $\mathbf{D}^n = \begin{bmatrix} 2^n & 0 \\ 0 & 2^n \end{bmatrix}$ $\mathbf{T}^n = \mathbf{U} (2^n \mathbf{I}) \mathbf{U}^{-1}$ $= 2^n (\mathbf{U} \mathbf{I} \mathbf{U}^{-1})$ $= 2^n \mathbf{I}$	(B1)		For \mathbf{D}^n with n even
		(M1) (m2) (A1)	(5)	Allow \equiv forms such as $3 \cdot 2^n - 2^{n+1}$
	Total		8	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	eg $3 \times (1) - (2) \Rightarrow 13y + 13z = -13$ $(3) - (2) \Rightarrow 15y + 11z = -5$ $x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$	M1 A1A1 M1 A1	5	Eliminating first variable Solving 2×2 system
	Alt I (Cramer's Rule): $\Delta = \begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ 3 & 11 & 13 \end{vmatrix}, \Delta_x = \begin{vmatrix} -2 & 3 & 5 \\ 7 & -4 & 2 \\ 2 & 11 & 13 \end{vmatrix},$ $\Delta_y = \begin{vmatrix} 1 & -2 & 5 \\ 3 & 7 & 2 \\ 3 & 2 & 13 \end{vmatrix}, \Delta_z = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -4 & 7 \\ 3 & 11 & 2 \end{vmatrix}$ $= 52, 312, 78 \text{ and } -130 \text{ respectively}$ $x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$ $x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$	(M1) (A1 A1) (M1) (A1)		Attempt at any two Δ correct; ≥ 1 other determinant correct At least one attempted numerically
	Alt II (Augmented matrix method): $\left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 3 & -4 & 2 & 7 \\ 3 & 11 & 13 & 2 \end{array} \right] \rightarrow$ $\left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & -13 & -13 & 13 \\ 0 & 2 & -2 & 8 \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 4 \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 5 \end{array} \right]$	(M1) (A1) (A1)	(5)	$R_2 \rightarrow R_2 - 3R_1$ $R_3 \rightarrow R_3 - 3R_1$ $R_3 \rightarrow R_3 - R_2$
	Substituting back to get $x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$	(M1 A1)		M0 if no inverse matrix is given
	Alt III (Inverse matrix method): $C^{-1} = \frac{1}{52} \begin{bmatrix} -74 & 16 & 26 \\ -33 & -2 & 13 \\ 45 & -2 & -13 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1} \begin{bmatrix} -2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.5 \\ -2.5 \end{bmatrix}$	(M1) (A1 A1) (M1) (A1)	(5)	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
5(b)(i)	$\begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ a & 11 & 13 \end{vmatrix} = 26a - 26$ <p>Setting equal to zero and solving for a $a = 1$</p>	M1 m1 A1	3	Attempt at determinant; OE
(ii)	$\begin{aligned} x + 3y + 5z &= -2 \\ 3x - 4y + 2z &= 7 \\ x + 11y + 13z &= b \end{aligned}$ <p>NB $y + z = -1$ (from before) $(3) - (1) \Rightarrow 8y + 8z = b + 2$ $b + 2 = -8 \Rightarrow b = -10$</p> <p>Alternative for (b)(ii): Substituting $x = 6$, $y = 1\frac{1}{2}$, $z = -2\frac{1}{2}$ into $x + 11y + 13z = b$ $\Rightarrow b = -10$</p>	B1 B1 M1A1 (M3) (A1)	4 (4)	Equating; CAO Since, to be consistent, the 3 rd plane must contain the line of intersection of the first 2 planes, and therefore contains this point
	Total		12	
6(a)(i)	$\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$	B1	1	
(ii)	Equating for λ : $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{6}$	M1 A1	2	
(iii)	$\sqrt{3^2 + 2^2 + 6^2} = 7$ <p>Direction cosines are $\frac{3}{7}$, $\frac{2}{7}$ and $\frac{6}{7}$</p> <p>These are the cosines of the angles between the line and the x-, y- and z-axes (respectively)</p>	B1 B1 B1	3	ft on 7 Allow just "angles" correctly described
(b)(i)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ 1 & 1 & 3 \end{vmatrix} = 7\mathbf{i} - 10\mathbf{j} + \mathbf{k}$ $d = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -10 \\ 1 \end{bmatrix} = 0$	M1A1 M1 A1	4	ft \mathbf{n}
(ii)	$d = 0 \Rightarrow$ plane through / contains the origin	B1	1	
(c)	$\sin\theta / \cos\theta = \frac{\text{scalar product}}{\text{product of moduli}}$ <p>Numerator = $21 - 20 + 6 = 7$ Denominator = $7\sqrt{150}$ $\theta = 4.7^\circ$</p>	M1 B1 B1 A1	4	Must be $3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$ and their \mathbf{n} ft correct (unsimplified) ft both correct (unsimplified) CAO
	Total		15	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 4 & -3 & 3 \\ 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix}$ $\mathbf{M}^2 + 2\mathbf{I} = \begin{bmatrix} 4 & -3 & 3 \\ 3 & -2 & 3 \\ 3 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $= \begin{bmatrix} 6 & -3 & 3 \\ 3 & 0 & 3 \\ 3 & -3 & 6 \end{bmatrix} = 3\mathbf{M}$	M1 A1 A1	3	ie $k = 3$
(ii)	Multiplying by \mathbf{M}^{-1} to get $\mathbf{M} + 2\mathbf{M}^{-1} = 3\mathbf{I}$ so that $\mathbf{M}^{-1} = \frac{3}{2}\mathbf{I} - \frac{1}{2}\mathbf{M}$	M1 A1 A1	3	ft ie $a = -\frac{1}{2}$ and $b = \frac{3}{2}$
(b)(i)	Char. eqn. is $\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$ ie $(\lambda - 2)(\lambda - 1)^2 = 0$ giving $\lambda_1 = 1$ (twice) and $\lambda_2 = 2$	M1A1 A1A1 M1 A1	6	One A mark for each of the other coefficients Good factorisation attempt
(ii)	$\lambda = 1 \Rightarrow x - y + z = 0$ (thrice) Any two independent eigenvectors (eg) $\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\lambda = 2 \Rightarrow -y + z = 0$ $x - 2y + z = 0 \Rightarrow x = y = z$ $x - y = 0$ $\gamma \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	B1 M1 A1 M1 A1	5	Attempted
(iii)	For $\lambda = 1$, eigenvectors represent a plane of invariant points For $\lambda = 2$, eigenvectors represent an invariant line	M1 A1 B1	3	Plane
	Total		20	
	TOTAL		75	