



General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2007 examination - June series

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Key to mark scheme and abbreviations used in marking

| | | | |
|--------------|--|-----|----------------------------|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| ✓ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| -x EE | deduct x marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

| Q | Solution | Marks | Total | Comments |
|---|---|--------------|--|--|
| 1(a) | Samples are independent or random | B1 | | |
| | 98% $\Rightarrow z = 2.3263$ | B1 | | AWFW 2.32 to 2.33 |
| | CI for $\mu_1 - \mu_2$ is: | M1 | | Form |
| | $(\bar{x}_S - \bar{x}_A) \pm z \times \sqrt{\frac{s_S^2}{n_S} + \frac{s_A^2}{n_A}}$ | A1 | | Allow: sigmas, A&B or 1&2 and $n - 1$ Correct |
| | $(19268 - 17896)$ $\pm 2.3263 \times \sqrt{\frac{7321^2}{175} + \frac{8205^2}{225}}$ | A1✓ | | ✓ on z only $s_p = 7830$ to 7850 |
| ie $1372 \pm (1805 \text{ to } 1820)$ or $(-450 \text{ to } -430, 3170 \text{ to } 3200)$ | A1 | 6 | $1372 \pm (1830 \text{ to } 1845)$ AWFW | |
| (b) | Confidence interval includes zero so (at 5% level) | B1✓ ↑dep↑ | | ✓ on CI; OE |
| | Mean starting salaries may be equal | B1✓ | 2 | ✓ on CI; OE |
| | Total | | 8 | |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|------|---|----------------------------|-----------|--|
| 2(a) | $P(\geq 18 \text{Road}) = 0.85$ | B1 | 1 | CAO; OE; not 85 |
| (b) | $P(18 \text{ to } 64) =$ $P(\text{Route}) \times P(18 \text{ to } 64 \text{Route}) =$ $(0.25 \times 0.80) + (0.60 \times 0.35) + (0.55 \times 0.40)$ $= 0.20 + 0.21 + 0.22 = 0.63$ | M1 A1 A1 | 3 | Use of 3 possibilities, each the product of 2 probabilities At least 1 term correct CAO; OE |
| (c) | $P(\text{FR} \cap >64) = P(\text{FR}) \times P(>64 \text{FR})$ $= 0.35 \times 0.15$ $= 0.052 \text{ to } 0.053$ | B1 B1 | 2 | Correct expression AWFW (0.0525) |
| (d) | $P(\text{FR} >64) = \frac{(c)}{P(>64)} =$ $\frac{0.0525}{(0.25 \times 0.05) + (0.35 \times 0.15) + (0.40 \times 0.35)}$ $= \frac{0.0525}{0.0125 + 0.0525 + 0.1400} = \frac{0.0525}{0.205}$ $= 0.256 \text{ or } \frac{21}{82}$ | M1 M1 A1 A1 A1 | 5 | $\frac{\text{answer}(c)}{\sum(3 \times 2) \text{ probabilities}}$ At least 2 terms correct CAO AWRT/CAO; OE |
| | Total | | 11 | |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|---|---|-------|-----------|--|
| 3(a) | $H_0: p_K = p_S$ $H_1: p_K \neq p_S$ | B1 | | Both; OE; allow A&B or 1&2 |
| | SL $\alpha = 0.05$ CV $ z = 1.96$ | B1 | | CAO |
| | $\hat{p} = \frac{(150 \times 0.28) + (250 \times 0.34)}{400}$ | M1 | | Used |
| | $= \frac{127}{400}$ or 0.317 to 0.318 | A1 | | CAO/AFWW (0.3175) |
| | $z = \frac{(\hat{p}_K - \hat{p}_S) - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_K} + \frac{1}{n_S}\right)}}$ | M1 | | Used; accept unpooled denominator |
| | $ z = \frac{ 0.28 - 0.34 }{\sqrt{0.3175 \times 0.6825 \left(\frac{1}{150} + \frac{1}{250}\right)}}$ | A1✓ | | ✓ on \hat{p} ; accept no pooling |
| | $= 1.24 $ to $ 1.25 $ | A1 | | AWFW; $ 1.26 $ to $ 1.27 $ |
| | Thus accept H_0 as $ z < 1.96$ | A1✓ | | ✓ on z and CV with same sign |
| | Thus no evidence, at 5% level, of a difference between two proportions of male customers in two salons | E1✓ | 9 | ✓ on z and CV with same sign In context and qualified |
| | (b) | Zero | B1 | |
| since Cannot make a Type I error when H_0 is false | | B1 | 2 | OE |
| | Total | | 11 | |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|---|--|----------|----------|--|
| 4 | 98% $\Rightarrow z = 2.5758$ | B1 | | AWFW 2.57 to 2.58 |
| | CI width is $2 \times \frac{z\sigma}{\sqrt{n}}$ | M1 | | Used; allow $\frac{z\sigma}{\sqrt{n}}$ |
| | Thus $2 \times \frac{2.5758 \times 0.08}{\sqrt{n}} = 0.05$ | A1✓ | | OE; ✓ on z; allow no '2 ×' |
| | Thus $\sqrt{n} = 8.24256$ | m1 | | Solving for \sqrt{n} or n |
| | Thus $n = 67.9 \Rightarrow 68$ | A1✓ | | AWRT; ✓ on z |
| | Thus, to nearest 5, $n = 70$ | A1 | 6 | CAO |
| | Total | | 6 | |
| 5 | $D = \sum^3 X_i - \sum^2 Y_i$ or $D' = \sum^2 Y_i - \sum^3 X_i$ | M1 | | Used or implied |
| | have means $\mu = 162 - 166 = -4$ $\mu = 166 - 162 = +4$ | B1 | | CAO either |
| | and variance $\sigma^2 = (3 \times 2^2) + (2 \times 3^2) = 12 + 18$ $= 30$ | M1 A1 | | Use of $[a \times \text{Var}(Z)]$; implied CAO |
| | $P\left(\sum^3 X_i < \sum^2 Y_i\right) =$ $P(D < 0)$ or $P(D' > 0) =$ | M1 | | Used or implied |
| | $P\left(Z > \frac{0 - (-4)}{\sqrt{30}}\right)$ or $P\left(Z > \frac{0 - (+4)}{\sqrt{30}}\right) =$ | m1 | | Standardising 0 using μ and $\sqrt{\sigma^2}$ |
| | $P(Z < +0.73)$ or $P(Z > -0.73) =$ 0.767 to 0.768 | A1 | 7 | AWFW |
| | Total | | 7 | |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|----------------|--|----------|-------|--|
| 6(a)(i) | $E(X) = \sum_{x=0}^n x \times \binom{n}{x} p^x (1-p)^{n-x}$ | M1 | | Use of $\sum x \times P(X=x)$ |
| | $= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$ | M1 | | Expansion of ${}^n C_x$; cancelling of x (Ignore limits) |
| | $= np \times \sum_{x=1}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$ | M1 | | Factors of n and p (Ignore limits) |
| | $= np \times \sum P(X=x) B(n-1, p) = np$ | M1 | 4 | AG; must be convincing |
| (ii) | $\text{Var}(X) = E(X^2) - (E(X))^2$ | M1 | | Used |
| | $= [E(X^2) - E(X)] + E(X) - (E(X))^2$ | | | |
| | $= n(n-1)p^2 + np - n^2 p^2$ | m1 | | Attempted |
| | $= np(1-p)$ | A1 | 3 | AG; must be convincing |
| (iii) | Thus $np(1-p) = 3(1-p) = 2.97$ | M1 | | Substituting μ in σ^2 |
| | Thus $1-p = \frac{2.97}{3} = 0.99$ | | | |
| | Thus $p = 0.01$ and $n = 300$ | A1 A1 | 3 | CAO CAO |
| (iv) | $B(300, 0.01) \sim \text{Po}(3)$ | B1 | | CAO; PI |
| | $P(X > 2) = 1 - P(X \leq 2)$ | M1 | | Must be applied to Poisson |
| | $= 1 - 0.4232 = 0.577$ | A1 | 3 | AWRT |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|------|---|--|-----------|--|
| 6(a) | | | 13 | |
| (b) | $Y \sim B(500, 0.45)$ or $Y \sim (\text{normal})$ with mean $\mu = 225$ and variance $\sigma^2 = 123.75$ or standard deviation $\sigma = 11.124$ (At least) half $\Rightarrow (\geq) 250$ $P(Y_B \geq 250) = P(Y_N > 249.5) =$ $P\left(Z > \frac{249.5 - 225}{\sqrt{123.75}}\right) =$ $P(Z > 2.20) = 1 - P(Z < 2.20)$ $= 0.0138$ to 0.014 Note: Use of $\frac{0.5 - 0.45}{\sqrt{0.000495}} \Rightarrow$ max of 5 marks Use of $\frac{0.499 - 0.45}{\sqrt{0.000495}} \Rightarrow$ max of 7 marks | B1 B1 B1 B1 M1 m1 A1 | 7 | PI AWFW 123 to 124 AWFW 11.05 to 11.15 CAO CAO Standardising 249.5, 250 or 250.5 with c's μ and $\sqrt{\sigma^2}$ Area change Use of distribution of \hat{p} Use of distribution of \hat{p} with continuity correction |
| | Total | | 20 | |

MS03 (cont)

| Q | Solution | Marks | Total | Comments |
|------|---|-------|-----------|--|
| 7(a) | $H_0: \lambda = 13$ | B1 | | CAO; OE |
| | $H_1: \lambda < 13$ | B1 | | CAO; OE |
| | $P(R \leq 10 \mid \text{Po}(13))$ | M1 | | Used or implied |
| | $= 0.2517$ | A1 | | AWFW 0.251 to 0.252 |
| | Prob of $0.2517 > 0.10$ (10%) $z = -0.83$ to $-0.70 > -1.28$ | M1 | | Comparison of prob with 0.10 Comparison of z with -1.28 |
| | Thus no evidence, at 10% level, of a reduction in the mean value of R | A1✓ | 6 | ✓ on probability or z In 'context' and qualified |
| (b) | Require $P(R \leq r \mid \text{Po}(13)) \approx 0.10$ | M1 | | Stated or implied |
| | Critical Region is $R \leq 8$ or $R < 9$ | A1 | 2 | Accept $R = 8$ May be scored in (a) |
| (c) | Require $P(\text{accept } H_0 \mid H_0 \text{ false})$ | B1 | | OE; PI |
| | $= P(R > 8 \mid \text{Po}(6.5))$ | M1 | | Use of $\text{Po}(6.5)$ |
| | $= 1 - P(R \leq 8 \mid \text{Po}(6.5))$ | m1 | | |
| | $= 1 - 0.7916$ | | | |
| | $= 0.208$ to 0.209 | A1 | 4 | AWFW (0.2084) |
| | Total | | 12 | |
| | TOTAL | | 75 | |