# Teacher Support Materials

# Maths GCE

# Paper Reference MPC4

### *Question 1*

1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. *(2 marks)*

(b) Simplify the algebraic fraction $\dfrac{2x^2 + x - 3}{x^2 - 1}$. *(3 marks)*

### *Student Response*

Question number

Leave blank

1 (a) $2x^2 + x - 3 = f(x)$

$(2x+1) \quad x = -\frac{1}{2}$

$f(-\frac{1}{2}) = 2(-\frac{1}{2})^2 + (-\frac{1}{2}) - 3$ M1

$= \underline{\underline{-4}}$ A0 1

b) $\cancel{\dfrac{x(2x+1)-3}{x^2-1}}$

$x(2x+1) \overline{\smash{)}\, 2x^2 + x - 3}$ $x$

$2x^2 + x$

$-3$

$= \cancel{(2x-1)}\cancel{(x+3)}$

Leave blank

b) $\dfrac{2x^2 + x - 3}{(x^2 - 1)}$

$= \dfrac{2x^2 - 2x + 3x - 3}{x^2 - 1}$

$= \dfrac{2x(x-1) + 3(x-1)}{x^2 - 1}$

$= \dfrac{(x-1)(2x+3)}{x^2 - 1}$ B1 A0 1

$= \underline{\underline{\dfrac{2x+3}{x}}}$ A0 (2)

### *Commentary*

*This question was intended as a straightforward opening question. However, many candidates made numerical and algebraic errors which were unexpected at this level (A2)*

*The errors made by Nicola are typical.*

*In part (a) she has a correct expression for the right answer of –3, but has taken the square of a negative number as negative, rather than positive and so gets –4.*

*In part (b) she changes her mind about which method to use, from long division to using factors. Although the latter was expected either method is acceptable. Nicola correctly factorises the* *numerator but does not factorise the denominator, and makes an algebraic error, which costs her two marks. This is an elementary error, but was typical of what many candidates did in this question, indicating a lack of the expected algebraic skills of A2 candidates.*

### *Mark scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) | $2\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-3=-3$ | M1A1 | 2 | use of $\pm\frac{1}{2}$<br>SC NMS –3 1/2<br>No ISW, so subsequent answer "3" AO |
| | **Alt**<br>algebraic division: | | | |
| | $\begin{array}{r} x \\ 2x+1\overline{)2x^2+x-3} \\ 2x^2+x \\ \hline -3 \end{array}$ | (M1) | | complete division with integer remainder |
| | | (A1) | (2) | remainder = – 3 stated, or –3 highlighted |
| | **Alt**<br>$\frac{x(2x+1)-3}{2x+1}$ | (M1) | | attempt to rearrange numerator with $(2x+1)$ as a factor |
| | | (A1) | (2) | remainder $=-3$ stated, or –3 highlighted |
| (b) | $\frac{(2x+3)(x-1)}{(x+1)(x-1)}$ | B1<br>B1 | | $\left.\begin{array}{l}\text{numerator}\\ \text{denominator}\end{array}\right\}$ not necessarily in fraction |
| | $=\frac{2x+3}{x+1}$ | B1 | 3 | CAO in this form. Not $\frac{2x+3}{x+1}\frac{\cancel{x-1}}{\cancel{x-1}}$ |
| (b) | **Alternative**<br>$\frac{2x^2-2+x-1}{x^2-1}$ | | | |
| | $=2+\frac{x-1}{x^2-1}$ | (M1) | | |
| | $=2+\frac{x-1}{(x-1)(x+1)}$ | (B1) | | |
| | $=2+\frac{1}{x+1}$ | (A1) | (3) | |
| | **Total** | | **5** | |

***Question 2***

**2** (a) (i) Find the binomial expansion of $(1+x)^{-1}$ up to the term in $x^3$. *(2 marks)*

(ii) Hence, or otherwise, obtain the binomial expansion of $\dfrac{1}{1+3x}$ up to the term in $x^3$. *(2 marks)*

(b) Express $\dfrac{1+4x}{(1+x)(1+3x)}$ in partial fractions. *(3 marks)*

(c) (i) Find the binomial expansion of $\dfrac{1+4x}{(1+x)(1+3x)}$ up to the term in $x^3$. *(3 marks)*

(ii) Find the range of values of $x$ for which the binomial expansion of $\dfrac{1+4x}{(1+x)(1+3x)}$ is valid. *(2 marks)*

***Student response***

2 $(1+x)^{-1} = \cancel{1+x+x^2+x^3}$ $\quad 1+nx+\dfrac{n(n-1)}{2!}x^2$

$1 + -1.x + \dfrac{-1(-2)}{2!}x^2 + \dfrac{-1(-2)(-3)}{3!}.x^3$

$= 1 - x + x^2 - x^3 \ldots$ ✓

2

ii) $(1+3x)^{-1} = 1 + (3x)(-1) + \dfrac{-1(-2)}{2!}3x^2 + \dfrac{-1(-2)(-3)}{3!}.3x^3$

$1 - 3x + 3x^2 - 3x^3 \quad = 1 - 3x + 9x^2 - 27x^3 \ldots$ ✓

2

b) $\dfrac{1+4x}{(1+x)(1+3x)} = \dfrac{A}{1+x} + \dfrac{B}{1+3x} = \dfrac{A(1+3x)}{(1+x)(1+3x)} + \dfrac{B(1+x)}{(1+3x)(1+x)}$

$1+4x = A(1+3x) + B(1+x)$ when $x = -1, -1/3$.

$-3 = A(-2) + 0.$

$3/2 = A.$ ✓ M1

$-1/3 = 0 + B\,4/3$ m1

$-1 = 4B \Rightarrow B = -1/4.$ ✗ A0

$\dfrac{3}{2(1+x)} - \dfrac{1}{4(1+3x)}$

2

c) $\dfrac{3}{2(1+x)} - \dfrac{1}{4(1+3x)}$ ✓ $= 3(2[1+x]^{-1}) - 4(1+3x)^{-1}$ ✗ M1

Leave blank

$3[2(1-x+x^2-x^3)] - 4(1-3x+9x^2-27x^3)$

M0

$3(2-2x+2x^2-2x^3) - (4-12x+36x^2-108x^3)$

$(6-6x+6x^2-6x^3) - (4-12x+36x^2-108x^3)$

$2+6x-30x^2+102x^3$ A0 1

ii) valid for when $|x| < 1$, $|x| < -1/3$ x, $|3x| < 1$

$1 < x \leq -1/3$ x M0 A0 0

### *Commentary*

Most candidates showed themselves to be familiar with binomial series expansions and the use of partial fractions in binomial expansions. Although there were many correct answers to most parts of this question, the response from Aneetha illustrates some typical errors.

Part (a) Aneetha has parts (i) and (ii) correct, but her answer to part (ii) shows bad practice. She should have brackets around the 3x; ie (3x). She isn't penalised as her answer is correct, but she is algebraically incorrect.

Part (b) Most candidates got the correct values for A and B here; Aneetha however has made an error in $-\frac{1}{3}+1$ and so gets the value of B wrong. Such a simple error could cost 2 marks as she now cannot get the correct answer to part (c)(i)

Part(c)(i) Here Aneetha makes the error of including the denominators of her fraction values for A and B in the negative index. By the time she expands the brackets, moving towards the answer, her coefficients of $\frac{3}{2}$ and $-\frac{1}{4}$ have become 6 and –4. Candidates should know that having found the partial fraction coefficients, they then do not change.

Part (c)(ii) Most candidates either didn't attempt this part of the question or gave an incorrect answer. Aneetha's answer shows some awareness, but also lack of understanding of the modulus notation. Had she not written $|x| < -\frac{1}{3}$ she would have scored 1 mark for two correct statements. Her conclusion is nonsense as binomial expansions are only valid for values of x near to nought.

### *Mark Scheme (next page)*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 2(a)(i) | $(1+x)^{-1}=1+(-1)x+px^2+qx^3$ | M1 | | $p\neq 0,\ q\neq 0$ |
| | $=1-x+x^2-x^3$ | A1 | 2 | SC 1/2 for $=1-x+px^2$ |
| (ii) | $(1+3x)^{-1}=1-3x+(3x)^2-(3x)^3$ | M1 | | $x$ replaced by $3x$ in candidate's (a)(i);condone missing brackets |
| | $=1-3x+9x^2-27x^3$ | A1 | 2 | CAO SC $x^3$-term : $1-3x+\frac{3}{9}x^2$ 1/2 |
| | **Alt (starting again)** | | | |
| | $(1+3x)^{-1}=1-(3x)+$ | | | |
| | $\frac{(-1)(-2)(3x)^2}{2!}+\frac{(-1)(-2)(-3)(3x)^3}{3!}$ | (M1) | | condone missing brackets accept 2 for 2!, 3.2 for 3! |
| | $=1-3x+9x^2-27x^3$ | (A1) | (2) | CAO |
| (b) | $\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ | M1 | | correct partial fractions form, and multiplication by denominator |
| | $1+4x=A(1+3x)+B(1+x)$ | | | |
| | $x=-1,\ x=-\frac{1}{3}$ | m1 | | Use (any) two values of $x$ to find $A$ and $B$ |
| | $A=\frac{3}{2},\ B=-\frac{1}{2}$ | A1 | 3 | $A$ and $B$ both correct |
| | **Alt:** | | | |
| | $\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$ | (M1) | | correct partial fractions form, and multiplication by denominator |
| | $1+4x=A(1+3x)+B(1+x)$ | | | |
| | $A+B=1,\ 3A+B=4$ | (m1) | | Set up and solve |
| | $A=\frac{3}{2},\ B=-\frac{1}{2}$ | (A1) | (3) | $A$ and $B$ both correct |
| (c)(i) | $\frac{1+4x}{(1+x)(1+3x)}=\frac{3}{2(1+x)}-\frac{1}{2(1+3x)}$ | M1 | | |
| | $=\frac{3}{2}(1-x+x^2-x^3)-\frac{1}{2}(1-3x+9x^2-27x^3)$ | m1 | | multiply candidate's expansions by $A$ and $B$, and expand and simplify |
| | $=1-3x^2+12x^3$ | A1 | 3 | CAO<br>SC $A$ and $B$ interchanged, treat as miscopy. $(1-4x+13x^2-40x^3)$ |
| | **Alt:** | | | |
| | $=\frac{1+4x}{(1+x)(1+3x)}=(1+4x)(1+x)^{-1}(1+3x)^{-1}$ | | | |
| | $=(1+4x)(1-x+x^2-x^3)(1-3x+9x^2-27x^3)$ | (M1) | | write as product, using expansions condone missing brackets on $(1+4x)$ only |
| | $=1-4x+13x^2-40x^3+4x-16x^2+52x^3$ | (m1) | | attempt to multiply the three expansions up to terms in $x^3$ |
| | $=1-3x^2+12x^3$ | (A1) | (3) | CAO |
| (ii) | $\lvert x\rvert<1$ and $\lvert 3x\rvert<1$ | M1 | | OE and nothing else incorrect |
| | $\lvert x\rvert<\frac{1}{3}$ (0.33) | A1 | 2 | OE Condone ≤ |
| | **Total** | | **12** | |

***Question 3 (b)&(c)***

(b) Hence solve the equation $4\cos x + 3\sin x = 2$ in the interval $0° < x < 360°$, giving all solutions to the nearest $0.1°$. *(4 marks)*

(c) Write down the minimum value of $4\cos x + 3\sin x$ and find the value of $x$ in the interval $0° < x < 360°$ at which this minimum value occurs. *(3 marks)*

***Student Response***

b) $4\cos x + 3\sin x = 2$

$5\cos(x - 36.9) = 2.$

$\cos(x - 36.9) = \frac{2}{5}$ ✓

M1

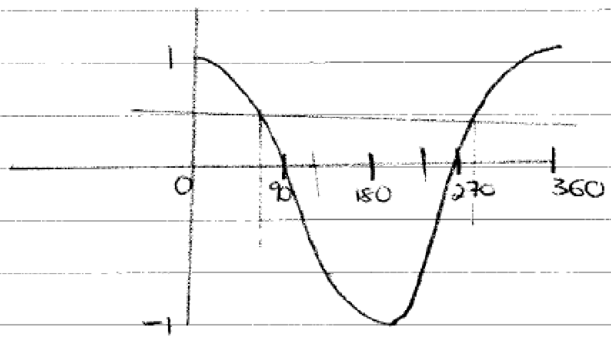
$x - 36.9 = \cos^{-1}\left(\frac{2}{5}\right)$

$x = \cos^{-1}\left(\frac{2}{5}\right) + 36.9$ A1

A1

$x = 103.3°$ ✓ d A0

3




$360 - 103.3° = 256.7°.$

$90 - 13.3° = 76.7°.$

$270 - 13.3° = 256°.$

$270 + 13.3° = 283.3°$

X

~~B0~~ ~~M0~~ 0

~~6~~

Question number

c) $4\cos x + 3\sin x = 36.9$

$36.9^{\circ}$ & $103.3^{\circ}$

$5\cos(x - 36.9) = 0$

$\cos(x - 36.9) = 0$

$x - 36.9 = \cos^{-1} 0$

$x = 90 + 36.9$ B0

$x = 126.9^{\circ}$

$360 - 126.9 = 233.1^{\circ}$ ✗ M0 0

Leave blank

### Commentary

Most candidates were successful in part (a) of this question and used it in their answer to part (b) as expected by the "hence" in the question.
Most candidates found one of the solutions of the equation correctly. Charlotte does that although she lives "dangerously" in not evaluating $\cos^{-1}\left(\frac{2}{5}\right)$ explicitly for had she not got 103.3° she would have lost 2 marks. Many candidates seemed uncertain over a second solution. Charlotte appears to be looking for a second solution but is unclear as to how many further solutions she thinks she is finding. She was given benefit of the doubt and lost 1 mark for a wrong second solution rather than being penalised further for finding too many solutions.

Many candidates either did not attempt part (c )or showed misunderstanding of what was required. Many confused maximum and minimum or didn't make it clear what they were trying to do; Charlotte's answer is like this. No minimum value is explicitly started, and her final answer is an angle. She has equated her expression to nought suggesting she thinks this is the minimum value and on the cosine curve it occurs at 90°. This error, and the similar error of equating to –1, instead of –5 were fairly common. Such errors might have been avoided by sketching the curve, particularly as candidates may use a graphics calculator.

### Mark Scheme

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 3(a) | $R = 5$ | B1 | | |
| | $\tan\alpha = \frac{3}{4}$ (OE) $\alpha = 36.9°$ (ISW 216.9) | M1A1 | 3 | SC1 $\tan\alpha = \frac{4}{3}, \alpha = 53.1°$<br>$R, \alpha$ PI in (b) |
| (b) | $\cos(x-\alpha) = \frac{2}{R}$ | M1 | | |
| | $x - \alpha = 66.4°$ | A1 | | |
| | $x = 103.3°$ | A1F | | |
| | $x = 330.4°$ | A1F | 4 | accept 330.5°, –1 each extra<br>ft on acute $\alpha$ |
| (c) | minimum value $= -5$ | B1F | | ft on $R$ |
| | $\cos(x - 36.9) = -1$ | M1 | | SC $\cos(x+36.9)$ treat as miscopy |
| | $x = 216.9°$ | A1 | 3 | 216.9 or better accept graphics calculator solution to this accuracy<br>SC Find max:<br>max = 5 at $(x+36.9)$ stated 1/3<br><br>Max 8/10 for work in radians |
| | Total | | 10 | |

***Question 4***

**4** A biologist is researching the growth of a certain species of hamster. She proposes that the length, $x$ cm, of a hamster $t$ days after its birth is given by

$$x = 15 - 12e^{-\frac{t}{14}}$$

(a) Use this model to find:

(i) the length of a hamster when it is born; *(1 mark)*

(ii) the length of a hamster after 14 days, giving your answer to three significant figures. *(2 marks)*

(b) (i) Show that the time for a hamster to grow to 10 cm in length is given by $t = 14\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers. *(3 marks)*

(ii) Find this time to the nearest day. *(1 mark)*

(c) (i) Show that

$$\frac{dx}{dt} = \frac{1}{14}(15 - x)$$ *(3 marks)*

(ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. *(1 mark)*

***Student Response (below)***

4a) i) $x = 15 - 12e^{-\frac{t}{14}}$

when $t = 0$

$x = 15 - 12e^{0}$

$x = 15 - 12$

$= 3$ cm ✓

ii) when $t = 14$ days 1

$x = 15 - 12e^{\frac{14}{14}}$

$= 15 - 12e^{1}$

$= 15 - 4.414583294$

$x = 10.58544671$ cm

$x = 10.6$ cm ✓ 2

b) i) when $x = 10$cm.

$10 = 15 - 12e^{-t/14}$

$-5 = -12e^{-t/14}$ ✓ $= 5$ M1

$\frac{5}{12} = e^{-t/14}$

$\ln\frac{5}{12} = -\frac{t}{14}$ ✓ m1

$-14\ln\left(\frac{5}{12}\right) = -t$

$14\ln\left(\frac{5}{14}\right) = t$ × A0 2

ii) $t = 12$ days. ✓ 1

c) i) $x = 15 - 12e^{-t/14}$

$(x - 15) = -12e^{-t/14}$

$14\ln\frac{(x+15)}{12} = t$ M1

$t = 14\ln\frac{15-x}{12}$

Question number | Leave blank

c) i) $14\left[\ln(15-x) - \ln(12)\right] = t$.

$\frac{dt}{dx} = 14\frac{1}{(15-x)}$ negatives "cancelled." m1

$\frac{dx}{dt} = \frac{1}{\frac{dx}{dt}} = \frac{1}{14}(15-x)$. A0 2

ii) when $x = 8$

$\frac{dx}{dt} = \frac{1}{14}(15-8)$

$= \frac{1}{14}(7)$

$\frac{dx}{dt} = \frac{1}{2}$ ✓ 1

(9)

## *Commentary*

Most candidates evaluated the expressions required for part (a) correctly, as does Kathryn; however she wastes time by unnecessarily writing all the decimals off her calculator.

In part (b)(i) many candidates made a sign error. Kathryn starts her solution for t correctly but in her fifth line of working an extra minus sign appears. This is a better attempt than those of candidates who tried to take logs of negative numbers. Katherine's expression evaluates to give – 12, but like many others she apparently just ignores the minus sign in giving her answer to part (b)(ii). She might have reviewed her answer to (b)(i).

Part (c)(i) proved difficult for most candidates with few good quality responses seen. Kathryn doesn't take the expected approach of differentiating the given expression and finding $\frac{dx}{dt}$ , but decides to take the longer route of solving for t. In her third line of working, there are several alterations and she has lost a minus sign. She would have done better to start again at the top of the next page and kept it tidy. She continues to find $\frac{dt}{dx}$ but again drops a minus sign. Her expression is now "correct" and she thinks she has the result, albeit even if her chain rule expression is wrong. A lot of candidates got into a rather confused mess with this question, which a little more care and thought might have avoided.

Kathryn, like most others, gets part (c)(ii) correct, showing clearly how she gets her answer.

## *Mark Scheme (next page)*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)(i)** | $t=0:\ x=3$ | B1 | 1 | |
| **(ii)** | $t=14:\ x=15-12e^{-1}$ | M1 | | or $15-12e^{\frac{-14}{14}}$ |
| | $=10.6$ | A1 | 2 | CAO |
| **(b)(i)** | $-5=-12e^{-\frac{t}{14}}$ | M1 | | substitute $x=10$; rearrange to form $p=qe^{-\frac{t}{14}}$ |
| | $\ln\left(\frac{5}{12}\right)=-\frac{t}{14}$ (OE) | m1 | | take lns correctly |
| | $t=14\ln\left(\frac{12}{5}\right)$ | A1 | 3 | must come from correct working |
| **(ii)** | $t=12.256...\approx 12$ days | B1F | 1 | ft on $a$, $b$ if $a>b$; accept $t=12$ NMS<br>Accept 12 from incorrect working in b(i)<br>Accept 13 if 12.2 or 12.3 seen |
| **(c)(i)** | $\frac{dx}{dt}=-\frac{1}{14}\times -12e^{-\frac{t}{14}}$ | M1 | | differentiate; allow sign error<br>condone $\frac{dy}{dx}$ used consistently |
| | $=-\frac{1}{14}(x-15)$ | m1 | | Or $\frac{1}{14}\left(12e^{-\frac{t}{14}}\right)$ and $12e^{-\frac{t}{14}}=15-x$ seen |
| | $=\frac{1}{14}(15-x)$ | A1 | 3 | AG – be convinced CSO |
| | **Alt:** $t=-14\ln\left(\frac{15-x}{12}\right)$ | (M1) | | attempt to solve given equation for $t$ |
| | $\frac{dt}{dx}=\frac{-14\left(-\frac{1}{12}\right)}{\left(\frac{15-x}{12}\right)}$ | (m1) | | differentiate wrt $x$, with $\frac{1}{\frac{15-x}{12}}$ seen; OE |
| | $\frac{dt}{dx}=\frac{14}{15-x}\Rightarrow\frac{dx}{dt}=\frac{1}{14}(15-x)$ | (A1) | (3) | AG – be convinced |
| | **Alt: (backwards)**<br>$\int\frac{dx}{15-x}=\int\frac{dt}{14}=\pm 14\ln(15-x)=t+c$ | (M1) | | |
| | Use $(0,3):-14\ln(15-x)+14\ln 12=t$ | (m1) | | |
| | Solve for $x$: $x=15-12e^{-\frac{t}{14}}$ | (A1) | (3) | All steps shown |
| **(ii)** | rate of growth = 0.5 (cm per day) | B1 | 1 | Accept $\frac{7}{14}$ |
| | **Total** | | **11** | |

### *Question 5*

**5** The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

(a) Show that $a = 1$. *(2 marks)*

(b) Find the gradient of the curve at $P$. *(7 marks)*

(c) Find an equation of the tangent to the curve at $P$. *(1 mark)*

### *Student Response*

5) $P(1,a)$ $a>0$ $y+4x = 5x^2y^2$

a) ~~y~~ when $x=1$

$y + 4 = 5(1)^2(y)^2$

$a + 4 = 5a^2$

~~a~~ $5a^2 - 4 - a$

$5a^2 - a - 4 = 0$ cardaic $a=5$ $b=-1$ $c=-4$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{+1 \pm \sqrt{1^2-(4 \times 5 \times -4)}}{10}$

$\frac{+1 \pm \sqrt{981}}{10}$

$= \frac{+1+9}{10}$ or $\frac{-1-9}{10}$ cardaic

$= \frac{10}{10}$ ✓ or $\frac{-10}{10}$ Since $a>0$ $\underline{a=1}$

2

5) b) Since $x = 1$

$y = 1$

$\cancel{1 + 4 = 5(1)^2(y)^2}$

$$y + 4x = 5x^2y^2$$

$$\frac{1}{y^2}(y + 4x) = 5x^2$$

$$\frac{1}{y^2} = \frac{5x^2}{y + 4x}$$

$$\frac{y}{y^2} = \frac{5x^2}{4x} \quad \times$$

$$\cancel{yy}(y)(y^{-2}) = 5x^2(4x^{-1})$$

M0
B0

by product rule

$u = y \quad v = y^{-2} \qquad u = 5x^2 \quad v = 4x^{-1}$

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = -2y^{-3} \qquad \frac{du}{dx} = 10x \quad \frac{dv}{dx} = -4x \;\times$

$uv' + vu' = uv' + vu'$

$$y(-2y^{-3}) + y^{-2}(1) = 5x^2(-4x) + 4x^{-1}(10x)$$

Leave blank

Leave blank

$$y(-2y^{-3}) + y^{-2}(1) \quad - 5x^2(-4x) + 4x^{-1}(10x)$$

$$-2y^{-2} + y^{-2} = \cancel{8} -20x^3 + \cancel{40x^2} \frac{10}{4}$$

$$-3y^{-2} = -20x^3 + \frac{10}{4}$$

$$3y^{-2} = 20x^3 - \frac{10}{4}$$

X

0

when $x=1$

$y=1$

$$3(1)^{-2} = 20(1)^3 - \frac{10}{4}$$

$$\frac{dy}{dx} = 20(1)^3 - \frac{10}{4} - 3(1)^{-2}$$

$$= \cancel{14.5} \quad \frac{29}{2}$$

X

1

c)

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 14.5(x-1)$$

$$y - 1 = \frac{29}{2}(x-1) \qquad \text{B1 F}$$

1

$$y - 1 = \frac{29x}{2} - \frac{29}{2}$$

$$y = \frac{29x}{2} - \frac{29}{2} - \frac{2}{2}$$

$$y = \frac{29x - 31}{2}$$

3

***Commentary***

This question was generally done well with most candidates demonstrating at least some knowledge and ability with implicit differentiation. However some candidates decided to rearrange the given equation so that they could attempt explicit differentiation, usually making algebraic and calculus errors. Daniel's response is like this.

In part (a) he shows he intends to set up a quadratic equation, although he doesn't equate it to zero. He makes an error in his solution, but is given benefit of the doubt because he has shown a=1 is the only positive solution. Many candidates just substituted (1,1) into the given equation and showed "it works" but didn't realise this fails to show a=1 is the only positive solution.

In part (b) Daniel decides he will rearrange the equation. He doesn't know, but teachers should be aware, that the presentation of the equation was intended to help candidates and implicit differentiation should be applied directly. Daniel is typical of candidates who took this approach; when he gets to his fourth line of working he makes a major algebraic error, but then doesn't simplify the expressions he now has. If he had attempted to use the product rule on the initially given equation, he might have got some marks for this part of the question. He appears to know what he is doing with the product rule, but there is no $\frac{dy}{dx}$ in his expression. However he is now so far from the intended question that he can gain no marks. He is allowed a compensatory mark in part (c) for correctly using what he believes the gradient to be.

### *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 5(a) | $x=1,\ 5a^2-a-4=0$ | M1 | | condone $y$ for $a$ |
| | $(5a+4)(a-1)=0,\quad a=1$ | A1 | 2 | AG – be convinced, both factors seen or $a=-\frac{4}{5}$ or $1 \Rightarrow a=1$<br>A0 for 2 positive roots<br>(substitute $(1,1) \Rightarrow 5=5$ no marks) |
| (b) | $\frac{dy}{dx}+4$ | B1B1 | | (Ignore '$\frac{dy}{dx}=$' if not used, otherwise loses final A1) |
| | $=10xy^2+10x^2y\frac{dy}{dx}$ | M1 | | attempt product rule, see two terms added |
| | | M1 | | chain rule, $\frac{dy}{dx}$ attached to one term only |
| | | A1 | | condone $5\times 2$ for 10 |
| | $x=1, y=1\quad \frac{dy}{dx}+4=10+10\frac{dy}{dx}$ | M1 | | two terms, or more, in $\frac{dy}{dx}$ |
| | $\frac{dy}{dx}=-\frac{6}{9}=\left(-\frac{2}{3}\right)$ | A1 | 7 | CSO |
| | **Alt (for last two marks)** | | | |
| | $\frac{dy}{dx}=\frac{10xy^2-4}{1-10x^2y}$ | (M1) | | find $\frac{dy}{dx}$ in terms of $x$, $y$ and substitute $x=1, y=1$ must be from expression with two terms or more in $\frac{dy}{dx}$ |
| | $(1,1) \Rightarrow \frac{10-4}{1-10}=-\frac{6}{9}$ | (A1) | | |
| (c) | $\frac{y-1}{x-1}=-\frac{2}{3}$ (OE) | B1F | 1 | ft on gradient<br>ISW after any correct form |
| | **Total** | | **10** | |

### *Question 6(b)*

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where $k$ is an integer. *(4 marks)*

### *Student Response*

(b) $x = \cos\theta$ $\quad y = 2\sin\theta\cos\theta$

$\theta = \frac{x}{\cos}$ x $\quad y = \sin\left(\frac{2x}{\cos}\right)$

$y = \frac{\sin 2x}{\cos}$ x $= \frac{2\sin x \cancel{\cos x}}{\cancel{\cos}}$ $\quad y = 2\sin x$

$y^2 = 2\sin x \times 2\sin x = 4\sin^2 x$ x

$\sin^2 x = 2(1 - \cos^2 x)$ $\quad 4(1 - \cos^2 x)$

4 $\quad$ x

$1 = \sin^2\theta + \cos^2\theta \times 4$

$= 4 = 4\sin^2\theta + 4\cos^2\theta$ $\quad \cos^2\theta = x^2$

$4 - 4\sin^2\theta = \cos^2\theta$ $\quad y^2 = 4x^2(1 - x^2)$ $\quad$ B0 M0

$1 - \sin^2\theta = \cos^2\theta$ $\quad y = (4\sin^2\theta)x^2(1 - x^2)$

$4(1 - \cos^2\theta) + 4\cos^2\theta$ $\quad y = 4\sin^2\theta + 4x^2(1 - x^2)$ $\quad$ 0

$4 - 4\cos^2\theta + 4\cos^2\theta = 4$

$\cos^2\left(\frac{1}{\cos} - \cos + 4\right)$ $\quad y = \quad y = (\tan^2 x + 4)x^2(1 - x^2)$

$\cos^2\theta\left(\frac{1}{\cos^2\theta} - 1 + 4\right)$ $\quad$ 1X $\quad$ 4

### *Commentary*

Most candidates completed part (a)(i) of this question successfully, with some making sign or coefficient errors. Part (a)(ii) was similarly answered well with most candidates demonstrating knowledge of the chain rule and using it correctly; relatively few had it upside down or attempted a product and relatively few had their calculators in degrees rather than radians.

The response to part (b) was very mixed. There were some clear demonstrations of the requested result, although some candidates had k=2 rather than 4, from squaring a correct expression for $\sin 2\theta$. However many candidates got themselves into difficulties through trying to recollect and manipulate trigonometric identities with little apparent thought and structure going into what they were attempting to do. The response from Mohammed is typical of the rather incoherent nature of such responses; it is difficult to follow his thinking through what is written down, and some of it makes no sense; for instance $\theta = \frac{x}{\cos}$ . His opening line of $y = 2\sin\theta\cos\theta$ is correct, but he then confuses himself over the roles of x and $\theta$ in this question and his fourth line of working doesn't relate to the opening line. He might well have done better had he just reviewed his work, had confidence in his opening line, and started again.

### *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6(a)(i) | $\frac{dx}{d\theta} = -\sin\theta \quad \frac{dy}{d\theta} = 2\cos 2\theta$ | B1 B1 | 2 | |
| (ii) | $\frac{dy}{dx} = -\frac{2\cos 2\theta}{\sin\theta}, \frac{dy}{dx} = -\frac{2\cos\frac{\pi}{3}}{\sin\frac{\pi}{6}} = -2$ | M1 | | use chain rule their $\frac{dy}{d\theta}$ / their $\frac{dx}{d\theta}$ and |
| | | | | substitute $\theta = \frac{\pi}{6}$ |
| | | A1 | 2 | |
| (b) | $y = 2\sin\theta\cos\theta = 2\sqrt{1-\cos^2\theta}\cos\theta$ | B1<br>B1 | | use $\sin 2\theta = 2\sin\theta\cos\theta$<br>use $\sin^2\theta = 1-\cos^2\theta$ |
| | $y = 2\sqrt{1-x^2}\ x$ | M1 | | $\sin\theta, \cos\theta$ in terms of $x$ |
| | $y^2 = 4x^2(1-x^2)$ | A1 | 4 | all correct CSO |
| | **Alt** | | | |
| | $y^2 = \sin^2 2\theta = (2\sin\theta\cos\theta)^2$ | (B1) | | use of double angle formula |
| | $= (4)\sin^2\theta\cos^2\theta = (4)(1-\cos^2\theta)\cos^2\theta$ | (B1) | | use of $s^2 + c^2 = 1$ to eliminate $\sin\theta$ |
| | $= (4)(1-x^2)x^2$ | (M1) | | Substitute $\cos\theta$ for $x$ |
| | $= 4(1-x^2)x^2$ | (A1) | (4) | CSO |
| | **Total** | | 8 | |

### *Question 7(b)&(c)*

(b) Show that $l_1$ and $l_2$ intersect and find the coordinates of the point of intersection, $P$. *(5 marks)*

(c) The point $A(-4, 0, 11)$ lies on $l_2$. The point $B$ on $l_1$ is such that $AP = BP$.

Find the length of $AB$. *(4 marks)*

### *Student Response*

b). $l_1$ and $l_2$ intersect.

$8 + \lambda 3 = -4 + \mu$ ①
$6 + -3\lambda = 2\mu$. ②
$-9 - \lambda = 11 - 3\mu$ ③. ✓

using equation ① and ②.
$8 + 3\lambda = -4 + \mu \cdot \times 2$.
$6 - 3\lambda = 2\mu$
$\cancel{8t}$ $16 + 6\lambda = -8 + 2\mu$. ①
$6 - 3\lambda = 2\mu$. ② ① − ② M1
$\cancel{9}$ $10 + 9\lambda = -8$
$\lambda = \dfrac{-8 - 10}{9}$ ✓ M1
$\underline{\lambda = -2}$.
substitute $\lambda = 2$ into equation ③ to find $\mu$.
$-9 + 2 = 11 - 3\mu$.
$\dfrac{-9 + 2 - 11}{-3} = \mu$.
$\underline{\mu = 6.}$ ✓ A1

Question number

Leave blank

the co-ordinates of the point of intersection are $P(2, 12, -7)$ M0 A0

3

c). $A(-4, 0, 11)$ lies on $l_2$. $\vec{AP} = \vec{BP}$.

$\vec{OP} - \vec{OA} = \vec{OP} - \vec{OB}$.

$\vec{AP} = \vec{OP} - \vec{OA}$.

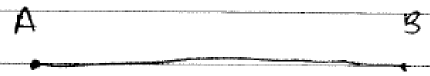
$$= \begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix} - \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ -18 \end{bmatrix}$$ M1

$\vec{BP} = \begin{bmatrix} 6 \\ 12 \\ -18 \end{bmatrix}$. $\vec{BP} = \vec{OP} - \vec{OB}$. A0

$$\begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 12 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 12 \\ -7 \end{bmatrix} - \begin{bmatrix} 6 \\ 12 \\ -18 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$ M0 A0 1

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix}$$




6

$\vec{AB} = \vec{OB} - \vec{OA}$

$$= \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} - \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \therefore \text{length} = 0.$$

## *Commentary*

Most candidates were successful in part (a) of this question, showing they both knew the result for two lines to be perpendicular and clearly demonstrating it in this case.

In part (b) most candidates knew they were to set up simultaneous equations and solve them for

$\lambda$ and $\mu$ which many did successfully. Charlotte has done that in her response but she has found $\lambda$ from her first two equations and $\mu$ from the third one. She doesn't check that her solutions satisfy all three equations so doesn't score the marks for finding the intersection point, although she has this correct. Charlotte could have shown that the intersection point lies on both lines by substituting her values of $\lambda$ and $\mu$ into the equations of the lines, but she just wrote the coordinates of the point down.

In part (c) Charlotte finds the vector **AP** correctly but them makes the common mistake of interpreting the question as meaning the vector **AP** and **BP** are equal rather than their length or moduli. Having made this assumption she proceeds sensibly to find the point B, but seems to just accept she has shown A and B are the same point, with a zero distance between them. She might have thought this odd and looked for an error or reread the question. The point B cannot easily be found from the information given, although many candidates were determined that it could. With a careful reading of the question many more might have been successful here.

## *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 7(a) | $\begin{bmatrix}3\\-3\\-1\end{bmatrix}\bullet\begin{bmatrix}1\\2\\-3\end{bmatrix}=3-6+3=0$ | M1 | | attempt at sp, 3 terms, added |
| | $=0 \Rightarrow$ perpendicular | A1 | 2 | $=0 \Rightarrow$ perpendicular seen $\left(\text{or } \cos\theta=0\Rightarrow\theta=90^\circ\right)$ |
| | | | | Allow $\begin{array}{c}3\\-6\\3\\\hline 0\end{array}$ but not $\begin{bmatrix}3\\-6\\3\end{bmatrix}=0$ |
| (b) | $8+3\lambda=-4+\mu$<br>$6-3\lambda=2\mu$<br>$-9-\lambda=11-3\mu$ | M1 | | set up any two equations |
| | $\lambda=-2, \mu=6$<br>verify third equation | m1 A1<br>m1 | | solve for $\lambda$ and $\mu$<br>substitute $\lambda,\mu$ in third equation |
| | intersect at $(2,12,-7)$ | A1 | 5 | CAO |
| | **Alt (for last two marks)**<br>substitute $\lambda$ into $l_1$ and $\mu$ into $l_2$ | (m1) | | |
| | intersect at $(2,12,-7)$, condone $\begin{pmatrix}2\\12\\-7\end{pmatrix}$ | (A1) | | $(2,12,-7)$ found from both lines<br>Note: working for (b) done in (a): award marks in (b) |
| 7(c) | $\overline{AP}=\begin{pmatrix}6\\12\\-18\end{pmatrix}$ | M1 | | $\overline{AP}=\pm\left\{\text{their } \overline{OP}-\begin{pmatrix}-4\\0\\11\end{pmatrix}\right\}$ |
| | $AP^2=504$ | A1F | | ft on $P$ |
| | $AB^2=2AP^2$ | M1 | | Calculate $AB^2$ |
| | $AB = 12\sqrt{7}$ | A1 | 4 | OE accept 31.7 or better |
| | **Total** | | **11** | |

### *Question 8*

**8** (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1+2y}}{x^2}$$

given that $y = 4$ when $x = 1$. *(6 marks)*

(b) Show that the solution can be written as $y = \frac{1}{2}\left(15 - \frac{8}{x} + \frac{1}{x^2}\right)$. *(2 marks)*

### *Student Response*

Leave blank

8 $\frac{dy}{dx} = \frac{\sqrt{1+2y}}{x^2}$ $y=4$ $x=1$

$dx = \frac{\sqrt{1+2y}}{dy \cdot x^2}$

$x^2 dx = \frac{\sqrt{1+2y}}{dy}$

$\frac{x^2}{dx} = \frac{\sqrt{1+2y}}{dy}$ $(1+2y)^{1/2}$

$\int_1^x \frac{x}{dx} = \int_4^y \frac{1^{1/2}}{dy} + 2y \int \frac{1}{dy}$

~~$\frac{dy}{dx} = \sqrt{1+2y} \times x^{-2}$~~

$\int_1^x x^{-2} \cdot dx = \int_4^y (1+2y^{-1/2}) dy$ ✓ ok! M1

M0

$[-x^{-1}]_1^x = [y + 4y^{1/2}]_4^y$ A0 A1

$(-1/x) - (-1) = (y + 4y^{1/2}) - (4+8)$ M1

$-1/x + 1 = y + 4y^{1/2} - 12$ A0 3

$4y^{1/2} + y = -1/x + 13$ M0

square everything $16y + y^2 = -1/x^2 + 169$

$y^2 = 16y - 1/x^2 + 169$ A0 0

$y = \sqrt{16y - 1/x^2 + 169}$.

3

## *Commentary*

In part (a) most candidates knew they were to separate the variables and then integrate on both sides of the equation. There were some impressive answers with many candidates doing this confidently in well presented and fully correct answers. Others indicated they weren't too sure what they were doing, and in some attempts dx and dy appeared as denominators. Lauren's response is like this; she is clearly trying to separate the variables but seems uncertain what to do with dx and dy. In her third line she looks to be really confused with an x on either side the integral sign but she has almost recovered to a correct integral in her fourth line; unfortunately the square root now only applies to y and not the whole expression in y. She now continues to integrate her expression correctly, cleverly using limits instead of finding a constant which is quite acceptable. In her eighth line of working Lauren has a solution to the differential equation but it was not the originally given equation.

She doesn't say she is now answering part (b) but she presumably is as she attempts to square both sides. She makes errors commonly seen by other candidates in this attempt. Her square of $-\frac{1}{x}$ is still negative and she has omitted the product term on both sides.

## *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $\int \frac{1}{\sqrt{1+2y}}\,dy = \int \frac{1}{x^2}\,dx$ | M1 | | attempt to separate and integrate |
| | $\int \frac{1}{\sqrt{1+2y}}\,dy = k\sqrt{1+2y}$ | m1 | | |
| | $\sqrt{1+2y} = -\frac{1}{x}(+c)$ | A1 | | OE A1 for $\sqrt{1+2y}$ depends on both Ms |
| | | A1 | | A1 for $-\frac{1}{x}$ depends on first M1 only |
| | $x = 1, y = 4 \Rightarrow c = 4$ | m1 | | $+c$ must be seen on previous line |
| | | A1F | 6 | ft on $k$ and $\pm\frac{1}{x}$ only |
| **(b)** | $1+2y = \left(4-\frac{1}{x}\right)^2$ | m1 | | need $k\sqrt{1+2y} =$ '$x$ expression with $+ c$' **and** attempt to square both sides |
| | $2y = 15 + \frac{1}{x^2} - \frac{8}{x}$ | A1 | 2 | terms on RHS in any order<br>AG – be convinced CSO |
| | **Total** | | **8** | |