Teacher Support Materials

Maths GCE

Paper Reference MM1B

Question 1A

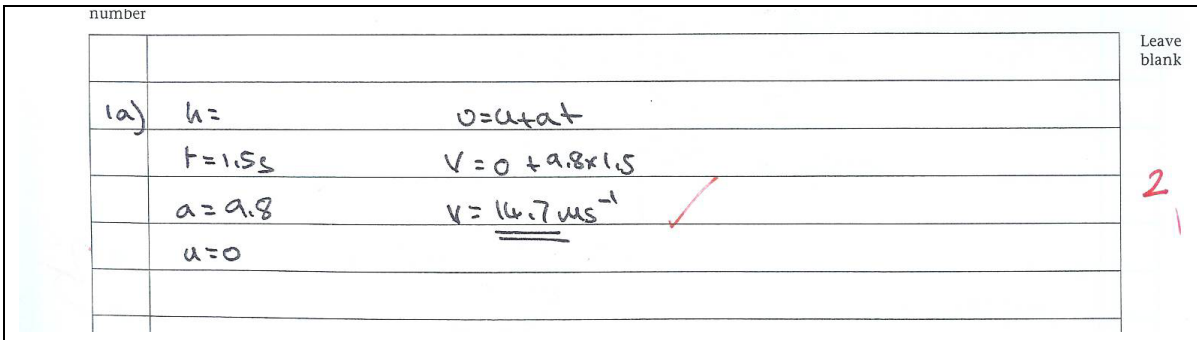
1 A ball is released from rest at a height h metres above ground level. The ball hits the ground 1.5 seconds after it is released. Assume that the ball is a particle that does not experience any air resistance.

(a) Show that the speed of the ball is $14.7\,\text{m s}^{-1}$ when it hits the ground. *(2 marks)*

(b) Find h. *(2 marks)*

(c) Find the distance that the ball has fallen when its speed is $5\,\text{m s}^{-1}$. *(3 marks)*

Student Response



Commentary

In this "Show that" question, the candidate clearly indicates the method to be used by stating the formula to be used and the values that are to be substituted into it. This clear piece of working shows a complete understanding and gains full marks, unlike responses that simply state $9.8\times1.5=14.7$.

Mark Scheme

Q	Solution	Marks	Total	Comments
1(a)	$v=0+1.5\times9.8$	M1		Use of constant acceleration equation to find v
	$=14.7\ \text{ms}^{-1}$	A1	2	AG Correct v from correct working $1.5\times9.8=14.7$ is not enough on its own

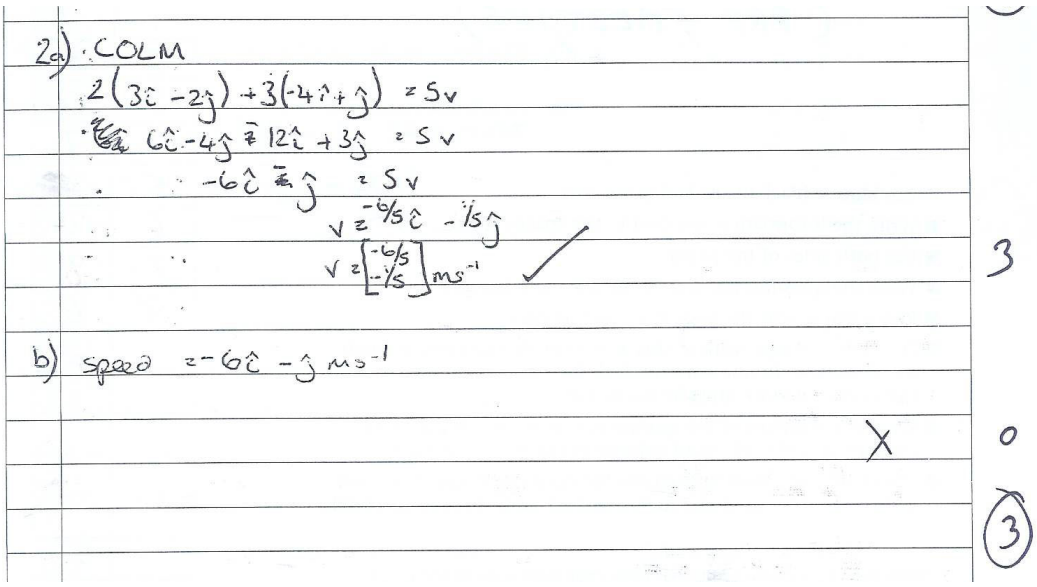
Question 2

2 Two particles, A and B, are moving on a smooth horizontal surface. Particle A has mass 2 kg and velocity $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ m s^{-1}. Particle B has mass 3 kg and velocity $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ m s^{-1}. The two particles collide, and they coalesce during the collision.

(a) Find the velocity of the combined particles after the collision. *(3 marks)*

(b) Find the speed of the combined particles after the collision. *(2 marks)*

Student response



Commentary

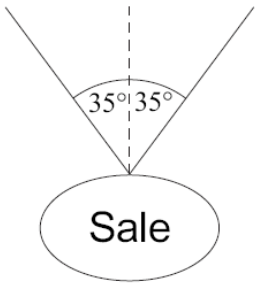
This response is from a candidate, who was able to work correctly with vectors and use the conservation of momentum to find the velocity after the collision. The solution shows a correct solution, although it appears that there were some problems with the negative signs which the candidate was able to sort out. A number of candidates did have problems with the manipulation of the negative signs, but did not correct them like this candidiate However, the candidate gives a vector as the answer to part (b), which asked for a speed. In this case the answer given is actually the momentum of the particle. A number of candidates did give their answers to part (b) as vectors.

Mark Scheme

2(a)	$2\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 3\begin{bmatrix} -4 \\ 1 \end{bmatrix} = 5\mathbf{v}$	M1 A1		Three term vector equation, with a '+' sign, for conservation of momentum Correct equation Deduct this first A mark for use of *mg*
	$\mathbf{v} = \frac{1}{5}\begin{bmatrix} -6 \\ -1 \end{bmatrix} = \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix}$	A1	3	Correct velocity
(b)	$v = \sqrt{1.2^2 + 0.2^2} = 1.22 \text{ ms}^{-1}$	M1 A1F	2	Finding speed from their velocity in part (a) (Must include addition of two terms) Correct speed from their velocity Accept 1.21
	Total		**5**	

Question 3a

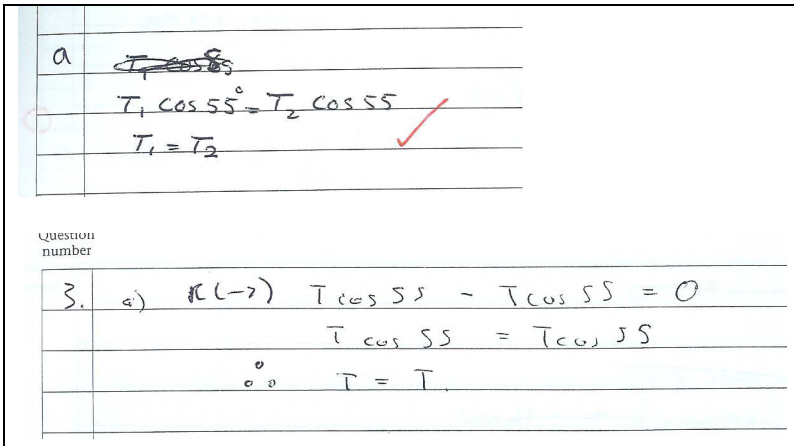
3 A sign, of mass 2 kg, is suspended from the ceiling of a supermarket by two light strings. It hangs in equilibrium with each string making an angle of 35° to the vertical, as shown in the diagram. Model the sign as a particle.



(a) By resolving forces horizontally, show that the tension is the same in each string.

(2 marks)

Student Response



Commentary

The two responses illustrate two common approaches to part (a) of this question. The first solution shows exactly what the examiners wanted to see for the award of full marks. (Note that cos55˚ could be replaced by sin35˚). The second solution, in contrast, gains no marks because the two tensions are treated as being the same throughout the solution.

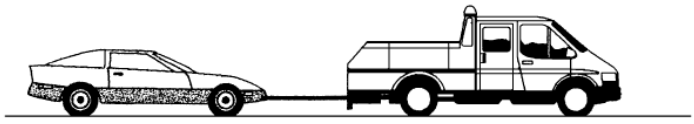
Mark Scheme

MM1B (cont)

Q	Solution	Marks	Total	Comments
3(a)	$T_1 \sin 35^\circ = T_2 \sin 35^\circ$	M1		Resolving two forces and forming an equation, with different tensions for each string
	$T_1 = T_2$	A1	2	Correct result from correct working
	OR			
	$T_1 \cos 55^\circ = T_2 \cos 55^\circ$			
	$T_1 = T_2$			

Question 4

4 A car, of mass 1200 kg, is connected by a tow rope to a truck, of mass 2800 kg. The truck tows the car in a straight line along a horizontal road. Assume that the tow rope is horizontal. A horizontal driving force of magnitude 3000 N acts on the truck. A horizontal resistance force of magnitude 800 N acts on the car. The car and truck accelerate at $0.4\,\text{m s}^{-2}$.



(a) Find the tension in the tow rope. *(3 marks)*

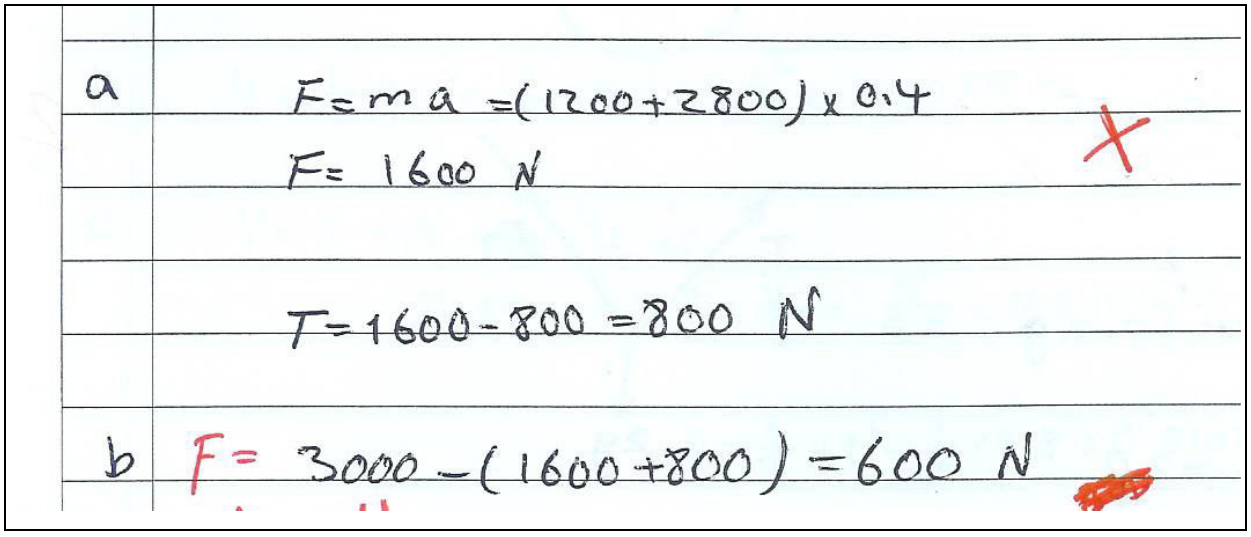
(b) Show that the magnitude of the horizontal resistance force acting on the truck is 600 N. *(4 marks)*

(c) In fact, the tow rope is **not** horizontal. Assume that the resistance forces and the driving force are unchanged.

Is the tension in the tow rope greater or less than in part (a)?

Explain why. *(2 marks)*

Student Response



Commentary

Many candidates found this question quite difficult. In part (a), a correct response requires a consideration of the forces acting on the car alone. This candidate has based their response on a consideration of the truck and the car together.

The solution to part (b) is shown here. This is a “Show that” question and this candidate has not shown enough working to gain any marks. The candidate appears to have simply selected numbers that can be manipulated to give the required answer of 600 N. There appears to be no evidence of the application of mechanics principles. Candidates should be encouraged to form what can be clearly identified as equations of motion, when answering questions of this type. (Note that the red “F =” was written by the examiner.)

Mark Scheme

4(a)	$T - 800 = 1200 \times 0.4$ $T = 800 + 480$ $= 1280$ N	M1 A1 A1	 3	Three term equation of motion for the car Correct equation Correct tension Treat calculation of two tensions as two methods unless one selected Treat sum or difference of two tensions as an incorrect method
(b)	$3000 - 800 - F = 4000 \times 0.4$ $F = 3000 - 800 - 1600$ $F = 600$ N **OR** $3000 - 1280 - F = 2800 \times 0.4$ $F = 3000 - 1280 - 1120$ $F = 600$ N	M1 A1 A1 A1	 4	Four term equation of motion (truck or both) Correct terms Correct signs AG Correct resistance force from correct working
(c)	Increase, because a greater tension would be needed so that the horizontal component would be the same as the tension above.	B1 B1	 2	Greater Reason Second B1 dependent on the first B1 mark
	Total		**9**	

Question 5

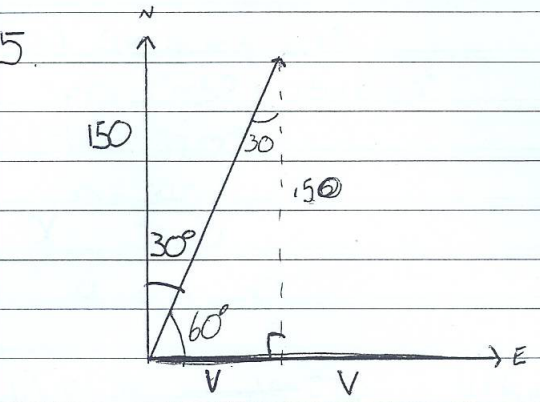
5 An aeroplane flies in air that is moving due east at a speed of $V\,\text{m s}^{-1}$. The velocity of the aeroplane relative to the air is $150\,\text{m s}^{-1}$ due north. The aeroplane actually travels on a bearing of 030°.

(a) Show that $V = 86.6\,\text{m s}^{-1}$, correct to three significant figures. *(2 marks)*

(b) Find the magnitude of the resultant velocity of the aeroplane. *(3 marks)*

Student Response

Question number 5

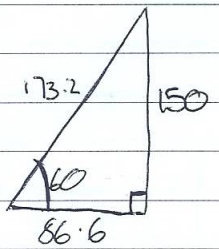


$$\frac{\sin 60}{150} = \frac{\sin 30}{V}$$

$$V = \frac{150 \sin 30}{\sin 60}$$

$$V = 86.6\ \text{ms}^{-1}$$

b)



$$r = \sqrt{86.6^2 + 150^2}$$

$$= 173.2\ \text{ms}^{-1}$$

Commentary

There were a lot of good responses to this question. Candidates were probably helped by the printed answer. The work below shows an approach that was used by a number of candidates to obtain a correct answer. This candidate's has used the sine rule to obtain the correct answer, rather than recognising that a simpler approach can be used based on using tan in the right angled triangle. Note the clear diagram that has been drawn, and which helps the candidate write down a correct equation.

The candidate produces a correct solution for part (b), although it does look as though an underlined letter r is used to denote the magnitude of the resultant velocity.

Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	$V = 150\tan 30°$	M1		Using trigonometry (usually tan or sine rule) to find V
	$= 86.6 \text{ ms}^{-1}$	A1	2	AG Correct answer from correct working (Division by 2 only acceptable if sin30° or cos60° seen)
	OR			
	$\frac{V}{\sin 30°} = \frac{150}{\sin 60°}$ AG			
	$V = 86.6 \text{ ms}^{-1}$			
(b)	$\frac{150}{v} = \cos 30°$	M1		Using trigonometry or Pythagoras to find v
		A1		Correct expression
	$v = \frac{150}{\cos 30°} = 173 \text{ ms}^{-1}$ (to 3sf)	A1	3	Correct answer
	Total		**5**	

Question 6

6 A box, of mass 3 kg, is placed on a slope inclined at an angle of 30° to the horizontal. The box slides down the slope. Assume that air resistance can be ignored.

(a) A simple model assumes that the slope is smooth.

(i) Draw a diagram to show the forces acting on the box. *(1 mark)*

(ii) Show that the acceleration of the box is $4.9\,\mathrm{m\,s^{-2}}$. *(2 marks)*

(b) A revised model assumes that the slope is rough. The box slides down the slope from rest, travelling 5 metres in 2 seconds.

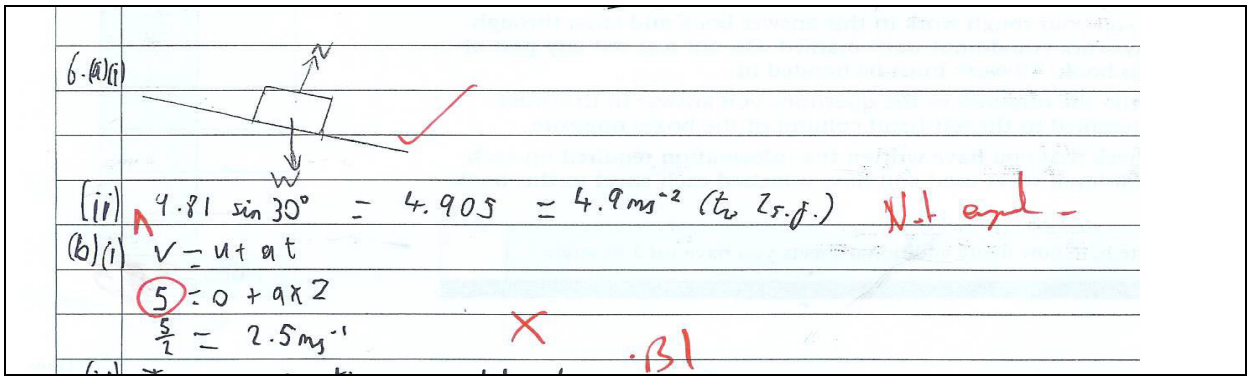
(i) Show that the acceleration of the box is $2.5\,\mathrm{m\,s^{-2}}$. *(2 marks)*

(ii) Find the magnitude of the friction force acting on the box. *(3 marks)*

(iii) Find the coefficient of friction between the box and the slope. *(5 marks)*

(iv) In reality, air resistance affects the motion of the box. Explain how its acceleration would change if you took this into account. *(2 marks)*

Student Response



Commentary

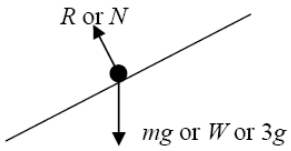
This extract shows a good force diagram, which uses two arrows to show the direction of the two forces and that is clearly labelled.

In (a) part (i), the candidate does draw a correct force diagram and gains the mark that is available.

In (a) part (ii), the candidate simply calculates $g\sin 30°$. Although this is the correct value, no credit is given because there is no working to support this answer. It is also worth noting here that the candidate uses 9.81 as the value of *g* instead of 9.8. The policy is that the first time this is done the candidate is penalised by one mark. In subsequent questions the candidate would not be penalised again, except in the case of a given answer that the candidate could not obtain when using 9.81 instead of 9.8. In spite of the policy teachers are advised to discourage candidates from using a value of 9.8.

In (b) part (i), the candidates assumes that the velocity of the box is 5 $\mathrm{ms^{-1}}$ after it has been moving for 2 seconds. Although this is correct, no credit can be given here as the candidate has not justified using this value.

Mark Scheme

6(a)(i)		B1	1	Correct diagram with arrows and labels
(ii)	$3a = 3g\sin 30°$	M1		Two term equation of motion
	$a = g\sin 30° = 4.9\ \text{ms}^{-2}$	A1	2	AG Correct acceleration from correct working (Allow $a = g\sin 30°$)
(b)(i)	$5 = \frac{1}{2}a \times 2^2$	M1		Constant acceleration equation with $u = 0$
	$a = 2.5\ \text{ms}^{-2}$	A1	2	AG Correct answer from correct working. (Use of $v = 5$ must be justified)
(ii)	$3 \times 2.5 = 3g\sin 30° - F$	M1 A1		Three term equation of motion Correct equation
	$F = 3g\sin 30° - 7.5$ $= 7.20$ N (to 3 sf)	A1	3	Correct F Accept 7.2 N
(iii)	$R = 3g\cos 30°$ (= 25.46)	M1		Resolving perpendicular to the slope to find R
		A1		Correct R
	$7.2 = \mu \times 3g\cos 30°$	M1		Use of $F = \mu R$
		A1F		Correct expression
	$\mu = \frac{7.2}{3g\cos 30°} = 0.283$	A1F	5	Correct μ Accept 0.282 (Follow through from incorrect F from above, but not an incorrect R)
(iv)	Reduce a, as the air resistance would reduce the magnitude of the resultant force or because the air resistance increases as the velocity increases towards its terminal value	B1 B1	2	Reduces Explanation Second B1 dependent on the first B1 mark
	Total		**15**	

Question 7

7 An arrow is fired from a point A with a velocity of $25\,\text{m s}^{-1}$, at an angle of $40°$ above the horizontal. The arrow hits a target at the point B which is at the same level as the point A, as shown in the diagram.

(a) State **two** assumptions that you should make in order to model the motion of the arrow. *(2 marks)*

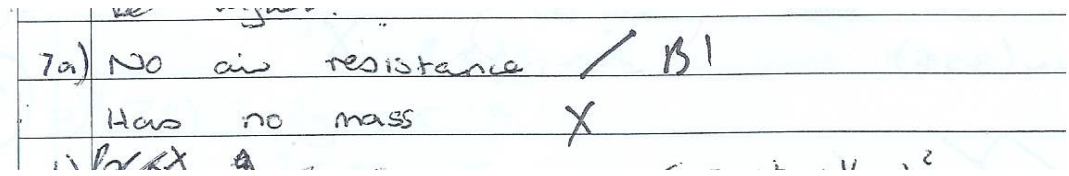
(b) Show that the time that it takes for the arrow to travel from A to B is 3.28 seconds, correct to three significant figures. *(4 marks)*

(c) Find the distance between the points A and B. *(2 marks)*

(d) State the magnitude and direction of the velocity of the arrow when it hits the target. *(2 marks)*

(e) Find the minimum speed of the arrow during its flight. *(2 marks)*

Student Response (contd on next page)



number

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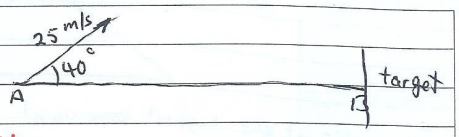
7



a - No air resistance ✓ B1 — 1

b $x = V\cos\theta\, t = 25\cos 40 \times t$

$y = V\sin\theta\, t - \frac{1}{2}gt^2 = 25\sin 40 \times t - \frac{1}{2}gt^2$

at the target $y = 0 \Rightarrow$

(also at the beginning)

$25\sin 40 \times t - \frac{1}{2}gt^2 = 0$

$t = 0$

$\frac{1}{2}gt = 25\sin 40$

$t = \frac{50\sin 40}{g}$

$t = 3.28$ s ✓ — 4

c $x = 25\cos 40 \times 3.28 = 62.8$

So the distance between A and B is 62.8 m ✓ — 2

d $V = V\cos\theta\, i + (V\sin\theta - gt)\, j$

$V = 25\cos 40\, i + (25\sin 40 - 9.8 \times 3.28)\, j$

$V = 19.15\, i - 16.07\, j$

$|V| = \sqrt{19.15^2 + 16.07^2} = 25$ m/s

$V = 25$ ms^{-1} ✓ B1 — 1

No angle.

Commentary

These two samples show the difficulties that some candidates had stating assumptions. The first candidate states one correct assumption and then that the particle has no mass. This response was seen on a fair number of scripts, and in some cases seemed to be regarded as a consequence of modelling the arrow as a particle.

The further work of the second candidate shows good solutions to parts (b) and (c), but a long approach to part (d). The intention of the question was that candidates would realise that the arrow would be travelling at 25 ms^{-1} and at angle of 40° below the horizontal. This candidate did a lot of work to obtain the speed and gained only one mark. In this case the candidate did not specify an angle, but some candidates found the 40° angle but did not specify that it was below the horizontal.

Mark Scheme

MM1B (cont)

Q	Solution	Marks	Total	Comments
7(a)	A particle or no spin No air resistance or no wind or only gravity acting	B1 B1	2	First assumption Second assumption If more than 2 assumptions given, subtract one mark for each incorrect additional assumption
(b)	$0 = 25\sin 40° t - 4.9t^2$ $0 = t(25\sin 40° - 4.9t)$ $t = 0 \quad \text{or} \quad t = \dfrac{25\sin 40°}{4.9}$ Time of flight = 3.28 s	M1 A1 dM1 A1	4	Equation for time of flight Correct equation Solving for t AG Correct final answer from correct working (Verification method M1A1M1A0)
(c)	$s = 3.28 \times 25\cos 40° = 62.8$ m	M1 A1	2	Finding range Correct range
(d)	25 ms^{-1} at 40° **below** the horizontal	B1 B1	2	Speed Direction
(e)	$v_{min} = 25\cos 40° = 19.2$ ms^{-1} **OR** $v_{min} = \dfrac{62.807}{3.2795} = 19.2$ ms^{-1}	M1 A1	2	Horizontal component of velocity Correct speed Accept 19.1 ms^{-1}
	Total		**12**	

Question 8

8 A boat is initially at the origin, heading due east at $5\,\text{m s}^{-1}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\text{m s}^{-2}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

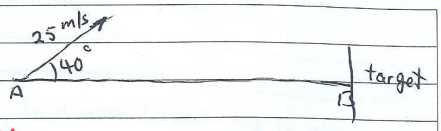
Student Response (next page)

7a) No air resistance ✓ B1

Has no mass ✗

number

7



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a - No air resistance ✓ B1 — 1

b $x = V\cos\theta\, t = 25\cos 40 \times t$

$y = V\sin\theta\, t - \frac{1}{2}gt^2 = 25\sin 40 \times t - \frac{1}{2}gt^2$

at the target $y = 0 \Rightarrow$

(also at the beginning)

$25\sin 40 \times t - \frac{1}{2}gt^2 = 0$

$t = 0$

$\frac{1}{2}gt = 25\sin 40$

$t = \frac{50\sin 40}{g}$

$t = 3.28$ s ✓ — 4

c $x = 25\cos 40 \times 3.28 = 62.8$

So the distance between A and B is 62.8 m ✓ — 2

d $\underline{V} = V\cos\theta\, \underline{i} + (V\sin\theta - gt)\underline{j}$

$\underline{V} = 25\cos 40\, \underline{i} + (25\sin 40 - 9.8 \times 3.28)\underline{j}$

$\underline{V} = 19.15\, \underline{i} - 16.07\, \underline{j}$

$V = \sqrt{19.15^2 + 16.07^2} = 25$ m/s

$\underline{V} = 25$ ms^{-1} ✓ B1 — 1

No cwtc.

Commentary

This candidate's solution shows a good answer to part (a). In part (b) the candidates gains no marks because the answer given is incorrect and there is no supporting working to justify a method mark. A number of candidates did give answers to part (b) with poor notation. This often involved writing the 5$\underline{i}$ as a scalar, typically just putting the number 5 as in this case. Some solutions were written with a "+" sign between the 5 and the bracket.

Interestingly, many candidates that gave imprecise answers to part (b) were able to gain marks on part (c). Here the candidate obtains the required result of $t = 25$, by dividing 5 by 0.2. Ideally more working would have been shown.

Mark Scheme

MM1B (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\mathbf{u} = 5\mathbf{i}$ or $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$	B1	1	Correct velocity
(b)	$\mathbf{v} = 5\mathbf{i} + (-0.2\mathbf{i} + 0.25\mathbf{j})t$	M1		Use of constant acceleration equatic with **u** and **a** not zero
		A1	2	Correct velocity M1A0 for using 5**j** or just 5
	OR			
	$\mathbf{v} = \begin{bmatrix} 5 - 0.2t \\ 0.25t \end{bmatrix}$			
(c)	$5 - 0.2t = 0$	M1		Easterly component zero
		A1		Correct equation
	$t = \dfrac{5}{0.2} = 25$ seconds	A1	3	Correct t
(d)	$\mathbf{r} = 5\mathbf{i} \times 25 + \dfrac{1}{2}(-0.2\mathbf{i} + 0.25\mathbf{j}) \times 25^2$	M1		Use of constant acceleration equatic t from part (c)
		A1F		Correct expression based on t from
	$= 62.5\mathbf{i} + 78.125\mathbf{j}$	A1		Correct simplification CAO
	$\theta = \tan^{-1}\left(\dfrac{62.5}{78.125}\right)$	dM1		Using tan to find the angle
		A1F		Correct expression based on t from with correct two values(either way)
	$= 038.7°$	A1	6	Correct angle Accept 38.6° or 039°
	OR			
	$\mathbf{r} = \dfrac{1}{2}(5\mathbf{i} + 6.25\mathbf{j}) \times 25$	(M1) (A1F) (A1)		
	$\theta = \tan^{-1}\left(\dfrac{5}{6.25}\right) = 038.7°$	(dM1) (A1F) (A1)		
	Total		**12**	
	TOTAL		**75**	