

General Certificate of Education  
June 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Decision 2**

**MD02**

Friday 22 June 2007 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

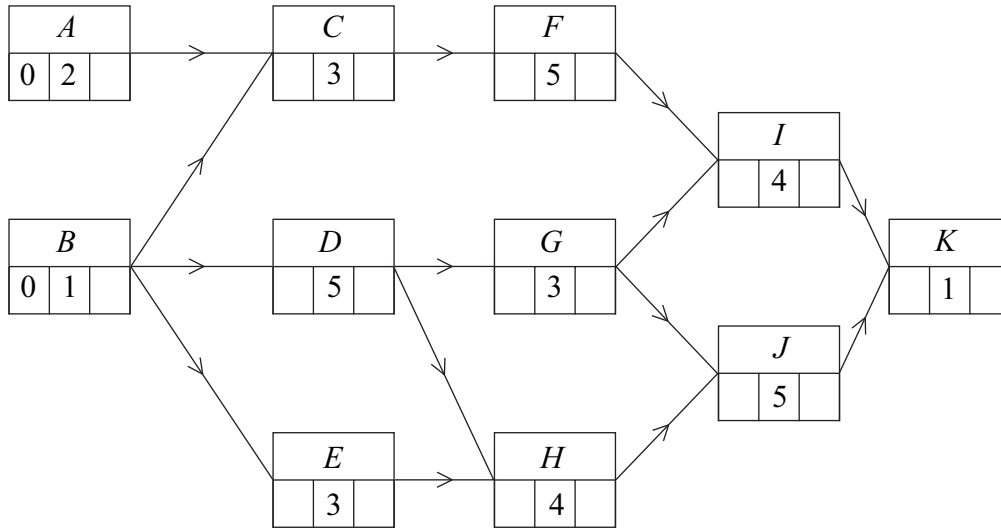
**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

**1** [Figures 1 and 2, printed on the insert, are provided for use in this question.]

The following diagram shows an activity diagram for a building project. The time needed for each activity is given in days.



- Complete the precedence table for the project on **Figure 1**. (2 marks)
- Find the earliest start times and latest finish times for each activity and insert their values on **Figure 2**. (4 marks)
- Find the critical path and state the minimum time for completion of the project. (2 marks)
- Find the activity with the greatest float time and state the value of its float time. (2 marks)

- 2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>Centre 1</b>	10	11	8	12	5
<b>Centre 2</b>	11	5	11	6	7
<b>Centre 3</b>	12	8	7	11	4
<b>Centre 4</b>	10	9	14	10	6
<b>Centre 5</b>	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

- (a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

- (b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros. (5 marks)
- (c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. (3 marks)
- (d) Find the value of this minimum daily total travel cost. (1 mark)

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- 3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

		Callum		
		$C_1$	$C_2$	$C_3$
Rose	$R_1$	5	2	-1
	$R_2$	-3	-1	5
	$R_3$	4	1	-2

- (a) (i) State the play-safe strategy for Rose and give a reason for your answer. *(2 marks)*
- (ii) Show that there is no stable solution for this game. *(2 marks)*
- (b) Explain why Rose should never play strategy  $R_3$ . *(1 mark)*
- (c) Rose adopts a mixed strategy, choosing  $R_1$  with probability  $p$  and  $R_2$  with probability  $1 - p$ .
- (i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. *(3 marks)*
- (ii) Illustrate graphically these expected gains for  $0 \leq p \leq 1$ . *(2 marks)*
- (iii) Hence determine the optimal mixed strategy for Rose. *(3 marks)*
- (iv) Find the value of the game. *(1 mark)*

- 4 A linear programming problem involving variables  $x$  and  $y$  is to be solved. The objective function to be maximised is  $P = 3x + 5y$ . The initial Simplex tableau is given below.

$P$	$x$	$y$	$s$	$t$	$u$	$value$
1	-3	-5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

- (a) In addition to  $x \geq 0$ ,  $y \geq 0$ , write down **three** inequalities involving  $x$  and  $y$  for this problem. *(2 marks)*
- (b) (i) By choosing the first pivot from the  **$y$ -column**, perform **one** iteration of the Simplex method. *(4 marks)*
- (ii) Explain how you know that the optimal value has not been reached. *(1 mark)*
- (c) (i) Perform one further iteration. *(4 marks)*
- (ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

**Turn over for the next question**

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5 [Figure 3, printed on the insert, is provided for use in this question.]

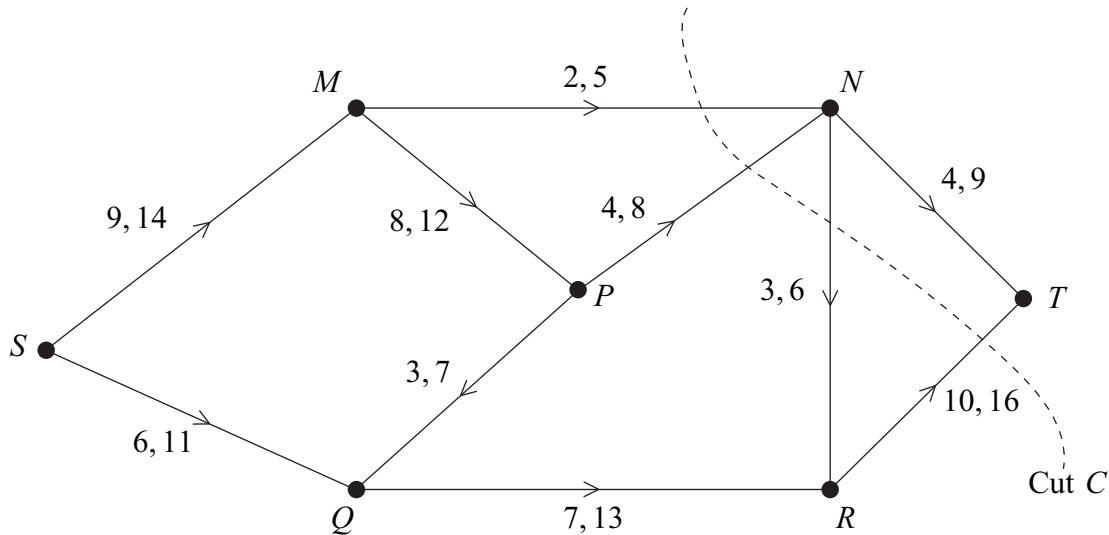
A maker of exclusive furniture is planning to build three cabinets *A*, *B* and *C* at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

Month	Already built	Cost		
		<i>A</i>	<i>B</i>	<i>C</i>
1	–	500	440	475
2	<i>A</i>	–	440	490
	<i>B</i>	510	–	500
	<i>C</i>	520	490	–
3	<i>A and B</i>	–	–	520
	<i>A and C</i>	–	500	–
	<i>B and C</i>	510	–	–

- (a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. (6 marks)
- (b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. (4 marks)

6 [Figures 4, 5 and 6, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



- (a) (i) Find the value of the cut  $C$ . (1 mark)
- (ii) State what can be deduced about the maximum flow from  $S$  to  $T$ . (1 mark)
- (b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from  $S$  to  $T$ . Indicate, on **Figure 4**, the flows along the edges  $MP$ ,  $PN$ ,  $QR$  and  $NR$ . (4 marks)
- (c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. (2 marks)
- (ii) Use flow augmentation on **Figure 5** to find the maximum flow from  $S$  to  $T$ . You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. (5 marks)
- (iii) Illustrate the maximum flow on **Figure 6**. (2 marks)

**END OF QUESTIONS**

**There are no questions printed on this page**



Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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# Insert

Insert for use in **Questions 1, 5 and 6.**

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

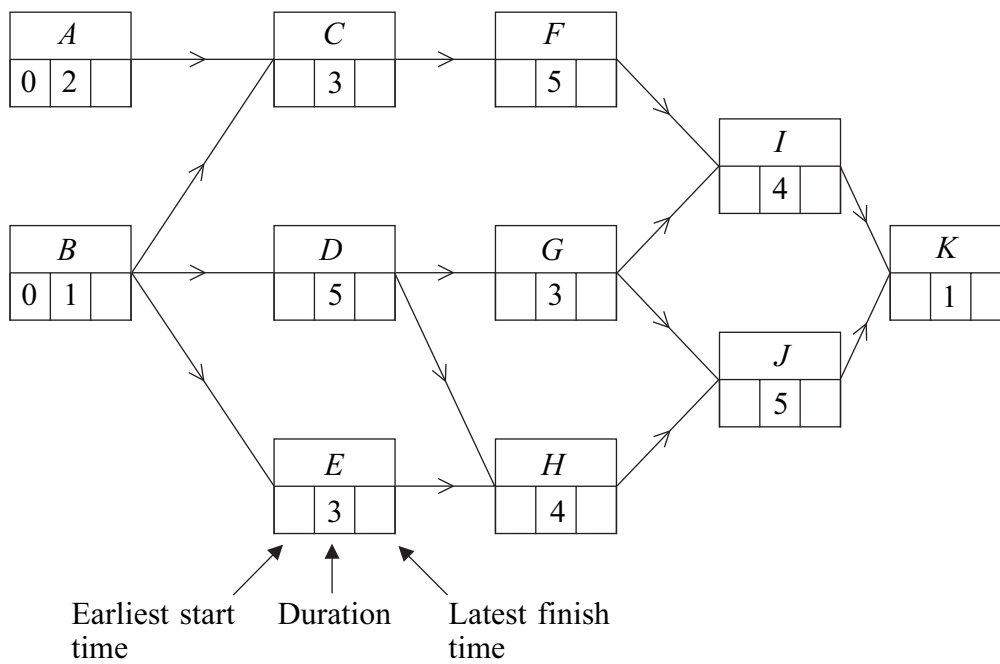
**Turn over for Figure 1**

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Figure 1 (for use in Question 1)

Activity	Immediate Predecessors
<i>A</i>	–
<i>B</i>	–
<i>C</i>	
<i>D</i>	
<i>E</i>	
<i>F</i>	
<i>G</i>	
<i>H</i>	
<i>I</i>	
<i>J</i>	
<i>K</i>	

Figure 2 (for use in Question 1)

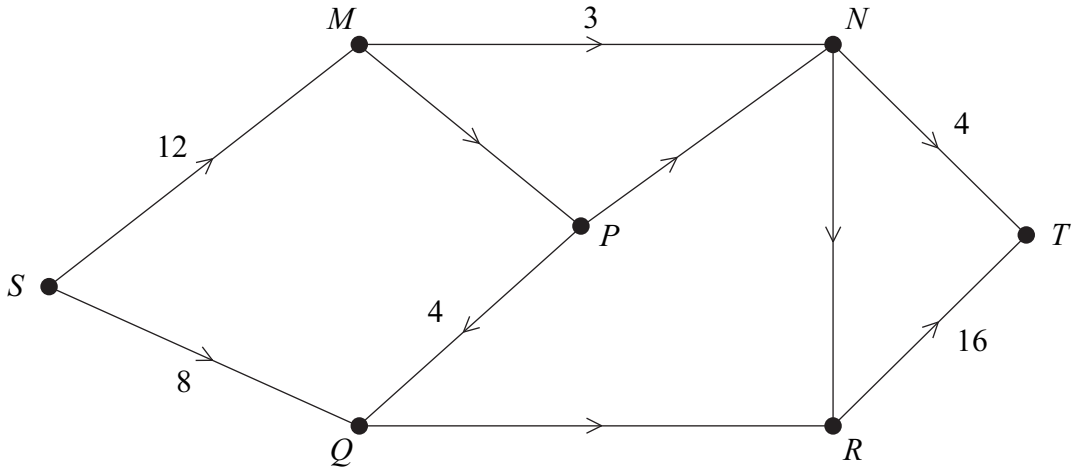


**Figure 3 (for use in Question 5)**

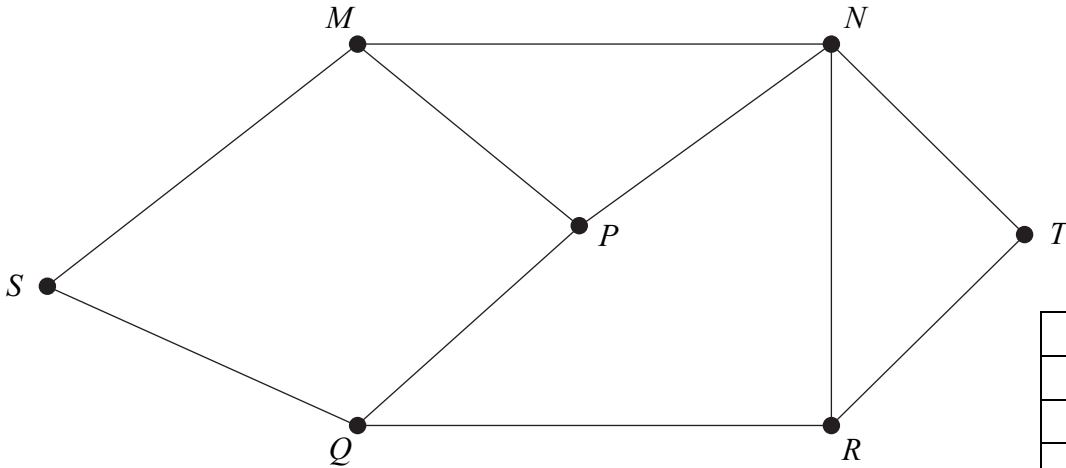
<b>Month</b>	<b>Already built</b>	<b>Machine built</b>	For use in part (a)	For use in part (b)
<b>3</b>	<i>A</i> and <i>B</i>	<i>C</i>		
	<i>A</i> and <i>C</i>	<i>B</i>		
	<i>B</i> and <i>C</i>	<i>A</i>		
<b>2</b>	<i>A</i>	<i>B</i>		
		<i>C</i>		
	<i>B</i>	<i>A</i>		
		<i>C</i>		
	<i>C</i>	<i>A</i>		
		<i>B</i>		

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**Figure 4 (for use in Question 6)**



**Figure 5 (for use in Question 6)**



Path	Flow

**Figure 6 (for use in Question 6)**

