ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2007 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

MPC4

Q	Solution	Marks	Total	Comments
1(a)(i)	$\frac{dx}{dt}=2$, $\frac{dy}{dt}=-8t$	B1, B1	2	CAO
(ii)	$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=\frac{-8t}{2}=-4t$	M1 A1F	2	Chain rule in correct form ft on sign coefficient errors (**not** power of t)
(b)	$m_T=-4$, $m_N=\frac{1}{4}$	B1F, B1F		ft on $\frac{dy}{dx}$ if f(t)
	$x=3$ $y=-3$			
	$\frac{y--3}{x-3}=\frac{1}{4}\Rightarrow\frac{y+3}{x-3}=\frac{1}{4}$	M1 A1	4	Use candidate's (x, y) and m_N Any correct form; ISW; CAO
(c)	$t=\frac{x-1}{2}$	M1		
	$y=1-4\left(\frac{x-1}{2}\right)^2$	M1A1	3	Substitute for t Simplification not required but CAO Or equivalent methods / forms: $y=2x-x^2$, $t^2=\frac{1-y}{4}$, $\left(\frac{x-1}{2}\right)^2=\frac{1-y}{4}$
	Total		**11**	
2(a)	$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-7\left(\frac{3}{2}\right)^2+13$	M1		Substitute $\pm\frac{3}{2}$ in f(x)
	$=4$	A1	2	
(b)	$g\left(\frac{3}{2}\right)=0\Rightarrow d+4=0\Rightarrow d=-4$	M1A1	2	AG (convincingly obtained) SC Written explanation with $g\left(\frac{3}{2}\right)=0$ not seen/clear E2,1,0
(c)	$a=-2$, $b=-3$	B1, B1	2	Inspection expected By division: M1 – complete method A1 CAO Multiply out and compare coefficients: M1 – evidence of use A1 – both a and b correct
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\cos 2x = 1 - 2\sin^2 x$	B1	1	
(b)(i)	$3\sin x - \cos 2x = 3\sin x - (1 - 2\sin^2 x)$	M1		Candidate's $\cos 2x$ or $\sin^2 x$
	$= 3\sin x - 1 + 2\sin^2 x$	A1	2	AG
(ii)	$2\sin^2 x + 3\sin x - 2 = 0$	M1		Soluble quadratic form
	$(2\sin x - 1)(\sin x + 2) = 0$	M1		Attempt to solve (allow one error in formula, allow sign errors)
	$\sin x = \frac{1}{2} \quad x = 30 \quad x = 150$	M1		$\sin^{-1}$ and two solutions ($0° < x < 360°$)
		A1	4	A0 if radians
	Allow misread for			
	$2\sin^2 x + 3\sin x - 1 = 0$	(M1)		Soluble quadratic form
	$\sin x = \frac{-3 \pm \sqrt{17}}{4}$	(M1)		Use of formula (allow one error)
	$x = 16.3°, 163.7°$	(A1)		Max 3/4
(c)	$\int \frac{1}{2}(1 - \cos 2x) = \frac{x}{2} - \frac{\sin 2x}{4} (+c)$	M1A1	2	M1 – solve integral, must have 2 terms for $\sin^2 x$ from (a)
			9	
4(a)(i)	$\frac{3x-5}{x-3} = 3 + \frac{4}{x-3}$	B1, B1	2	By division: B1 for 3, B1 for $\frac{4}{x-3}$ or $B = 4$ By partial fractions: M1 multiply by $x - 3$ and using 2 values of x, A1 both correct
(ii)	$\int 3 + \frac{4}{x-3}\,dx = 3x + 4\ln(x-3)(+c)$	M1A1F	2	M1 $\int 3 + \frac{4}{x-3}\,dx$ and attempt at integrals ft on A and B; condone omission of brackets around $x - 3$
	Alternative: By substitution $u = x - 3$			
	$\int \frac{3x-5}{x-3}\,dx = \int \frac{3u+4}{u}\,du$	(M1)		Integral in terms of u
	$= 3(x-3) + 4\ln(x-3)$	(A1)		Correct, in x
(b)(i)	$6x - 5 = P(2x-5) + Q(2x+5)$	M1		Clear evidence of use of cover-up rule M2
	$x = \frac{5}{2} \qquad x = -\frac{5}{2}$	m1		
	$10 = 10Q \qquad -20 = -10P$			
	$Q = 1 \qquad P = 2$	A1	3	
(ii)	$\int \frac{2}{2x+5} + \frac{1}{2x-5}\,dx$	M1		Attempt at ln integral $\left(a\ln(2x+5) + b\ln(2x-5)\right)$
	$\ln(2x+5) + \frac{1}{2}\ln(2x-5)(+c)$	M1 A1F	3	ft on P and Q; must have brackets
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}x^2$	M1		$1 + \frac{1}{3}x + kx^2$
		A1	2	
(b)(i)	$\sqrt[3]{8}\left(1+\frac{3}{8}x\right)^{\frac{1}{3}}$	B1		$8^{\frac{1}{3}}(1+kx)^{\frac{1}{3}}$
	$= 2\left(1 + \frac{1}{3}\left(\frac{3}{8}x\right) - \frac{1}{9}\left(\frac{3}{8}x\right)^2\right)$	M1		Replacing x with kx in answer to (a)
	$= 2 + \frac{1}{4}x - \frac{1}{32}x^2$	A1	3	For numerical expression which would evaluate to answer given
	Alternative: B1 – all powers of 8 correct: $8^{\frac{1}{3}}\ 8^{-\frac{2}{3}}\ 8^{-\frac{5}{3}}$ M1 – powers of $3x$ (condone $3x^2$) $2 + \frac{1}{8^{\frac{2}{3}}}x - \frac{1}{9}\frac{1}{8^{\frac{5}{3}}}9x^2$ A1 – see some arithmetic processing must see 9s in last term			
(ii)	$x = \frac{1}{3}$: $\sqrt[3]{8+1} = 2 + \frac{1}{4}\times\frac{1}{3} - \frac{1}{32}\times\left(\frac{1}{3}\right)^2$	M1		Using $x = \frac{1}{3}$ in given answer
	$\sqrt[3]{9} = \frac{576+24-1}{288} = \frac{599}{288}$	A1	2	Any correct numerical expression = $\frac{599}{288}$
	Total		**7**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\overrightarrow{BA}=\begin{bmatrix}3\\-2\\4\end{bmatrix}-\begin{bmatrix}5\\4\\0\end{bmatrix}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}$	M1A1	2	Attempt $\pm\overrightarrow{BA}$ $(OA-OB$ or $OB-OA)$
(ii)	$\overrightarrow{BC}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$	B1		Allow $\overrightarrow{CB}$; or $\begin{bmatrix}-6\\-2\\4\end{bmatrix}=\overrightarrow{BC}$ or $\overrightarrow{CB}=\begin{bmatrix}6\\2\\-4\end{bmatrix}$ May not see explicitly
	$\lvert\overrightarrow{BA}\rvert\left(=\sqrt{(-2)^2+(-6)^2+(4)^2}\right)=\sqrt{56}$	B1F		Calculate modulus of $\overrightarrow{BA}$ or $\overrightarrow{BC}$; for finding modulus of one of vectors they have used
	$\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\-6\\4\end{bmatrix}\bullet\begin{bmatrix}6\\2\\-4\end{bmatrix}=-12-12-16$	M1		Attempt at $\overrightarrow{BA}\bullet\overrightarrow{BC}$ with numerical answer; or $\overrightarrow{AB}\bullet\overrightarrow{CB}$
		A1		for –40, or correct if done with multiples of vectors
	$\cos ABC=\dfrac{-40}{\sqrt{56}\sqrt{56}}=-\dfrac{5}{7}$	A1	5	AG (convincingly obtained) Cosine rule: M1 attempt to find 3 sides A1 lengths of sides M1 cosine rule A1F correct A1 rearrange to get $\cos ABC=\dfrac{-5}{7}$ (ft on length of sides)

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6 (cont) (b)(i)	$\begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} \quad (\lambda = 3)$	M1A1	2	$\lambda = 3$ verified in three equations M1 for $\begin{cases} 11 = 8 + \lambda \\ 6 = -3 + 3\lambda \\ -4 = 2 - 2\lambda \end{cases}$ A1 for $\lambda = 3$ shown for all three equations $\lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} - \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} \therefore \lambda = 3$ M1A1 SC: $\lambda = 3$ written and nothing else: SC1
(ii)	$\begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ $\therefore$ same direction or same gradient or parallel	E1	1	
(c)	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$	B1		PI; $\overrightarrow{OD}$ = correct vector expression which may involve $\overrightarrow{AD}$
	$= \begin{bmatrix} 11 \\ 6 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} \quad D$ is $(9, 0, 0)$	M1A1	3	M1 for substituting into vector expression for $\overrightarrow{OD}$ NMS 3/3
	Total		**13**	
7(a)	$\tan(x + x) = \dfrac{\tan x + \tan x}{1 - \tan x \tan x} \left(= \dfrac{2\tan x}{1 - \tan^2 x} \right)$	M1 A1	2	$A = B = x$ used
(b)	$2 - 2\tan x - \dfrac{2\tan x(1 - \tan^2 x)}{2\tan x}$	M1		Substitute from (a)
	$2 - 2\tan x - (1 - \tan x)(1 + \tan x)$	M1		Simplification $2 - 2\tan x - (1 - \tan^2 x)$
	$(1 - \tan x)(2 - (1 + \tan x))$	M1		$2 - 2\tan x - 1 + \tan^2 x$
	$(1 - \tan x)^2$	A1	4	AG (convincingly obtained) $= (\tan x - 1)^2 = (1 - \tan x)^2$ Any equivalent method
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\int \frac{\mathrm{d}y}{y} = \int \sin t \,\mathrm{d}t$	M1		Attempt to separate and integrate
	$\ln y = -\cos t + C$	A1,A1		A1 for ln *y*; A1 for –cos*t*; condone missing *C*
	$y = A\mathrm{e}^{-\cos t}$	A1	4	*A* present; or $y = \mathrm{e}^{-\cos t + C}$
(ii)	$y = 50,\ t = \pi:\quad 50 = A\mathrm{e}^{-\cos\pi} = A\mathrm{e}$	M1 A1		Substitute *y* = 50, $t = \pi$ to find constant Can have $50 = \mathrm{e}^{1+C}$ if substituted in above $\mathrm{e}^{C} = \frac{50}{\mathrm{e}}$
	$y = 50\mathrm{e}^{-1}\mathrm{e}^{-\cos t}$	A1	3	AG (convincingly obtained)
	Alternative: Must have a constant in answer to (a)(i) $y = A\mathrm{e}^{-\cos t}$ or $y = \mathrm{e}^{-\cos t + c}$ or $\ln y = -\cos t + c$			**Alternative:** Substitute *y* = 50, $t = \pi$ into $\ln y = -\cos t + c$ M1 $\ln y = -\cos t + \ln 50 - 1$ A1
	$50 = A\mathrm{e}^{-\cos\pi}$ $\quad 50 = \mathrm{e}^{-\cos\pi + c}$ $\quad \ln 50 = -\cos\pi + c$	(M1)		$\ln\frac{y}{50} = -1 - \cos t$ (AG) A1
	$50 = A\mathrm{e}$ $\quad 50 = \mathrm{e}^{1+c}$ $\quad \ln y = -\cos t + \ln 50 - 1$	(A1)		
	$y = 50\mathrm{e}^{-1-\cos t}$ $\quad y = \mathrm{e}^{-\cos t}\frac{50}{\mathrm{e}}$ $\quad \ln\left(\frac{y}{50}\right) = -1 - \cos t$	(A1)		
(b)(i)	$t = 6:\ \ y = 50\mathrm{e}^{-1}\mathrm{e}^{-\cos 6} = 7.0417... \approx 7\text{cm}$	M1A1	2	Degrees 6.8 SC1 7 or 7.0 for A1
(ii)	$t = \pi \Rightarrow (\sin t = 0 \Rightarrow)\ \frac{\mathrm{d}y}{\mathrm{d}t} = 0$	B1		Condone *x* for *t*
	$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = y\cos t + \frac{\mathrm{d}y}{\mathrm{d}t}\sin t$	M1		For attempt at product rule including $\frac{\mathrm{d}y}{\mathrm{d}t}$ term; must have $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} =$
	$t = \pi$	A1		
	$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = y\cos\pi + \frac{\mathrm{d}y}{\mathrm{d}t}\sin\pi$ $= -50 \Rightarrow$ max	A1	4	Accept = – *y*, with explanation that *y* is never negative

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii) (cont)	**Alternative:** $y = 50\mathrm{e}^{-(1+\cos t)} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \sin t = 0$ at $t = \pi$ $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \cos t + \frac{50}{\mathrm{e}}\mathrm{e}^{-\cos t} \times \sin^2 t$ Substitute $t = \pi \rightarrow -50 \Rightarrow$ max	 (B1) (M1) (A1) (A1)		 Attempt at product rule Correct
	Total		**13**	
	TOTAL		**75**	