

General Certificate of Education  
June 2006  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 3**

**MPC3**

Thursday 15 June 2006 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The curve  $y = x^3 - x - 7$  intersects the  $x$ -axis at the point where  $x = \alpha$ .
- (a) Show that  $\alpha$  lies between 2.0 and 2.1. *(2 marks)*
- (b) Show that the equation  $x^3 - x - 7 = 0$  can be rearranged in the form  $x = \sqrt[3]{x+7}$ .  
*(1 mark)*
- (c) Use the iteration  $x_{n+1} = \sqrt[3]{x_n+7}$  with  $x_1 = 2$  to find the values of  $x_2$ ,  $x_3$  and  $x_4$ , giving your answers to three significant figures. *(3 marks)*
- 2 (a) Find  $\frac{dy}{dx}$  when  $y = (3x - 1)^{10}$ . *(2 marks)*
- (b) Use the substitution  $u = 2x + 1$  to find  $\int x(2x + 1)^8 dx$ , giving your answer in terms of  $x$ . *(4 marks)*
- 3 (a) Solve the equation  $\sec x = 5$ , giving all the values of  $x$  in the interval  $0 \leq x \leq 2\pi$  in radians to two decimal places. *(3 marks)*
- (b) Show that the equation  $\tan^2 x = 3 \sec x + 9$  can be written as
- $$\sec^2 x - 3 \sec x - 10 = 0 \quad \text{span style="float: right;">*(2 marks)*$$
- (c) Solve the equation  $\tan^2 x = 3 \sec x + 9$ , giving all the values of  $x$  in the interval  $0 \leq x \leq 2\pi$  in radians to two decimal places. *(4 marks)*
- 4 (a) Sketch and label on the same set of axes the graphs of:
- (i)  $y = |x|$ ; *(1 mark)*
- (ii)  $y = |2x - 4|$ . *(2 marks)*
- (b) (i) Solve the equation  $|x| = |2x - 4|$ . *(3 marks)*
- (ii) Hence, or otherwise, solve the inequality  $|x| > |2x - 4|$ . *(2 marks)*

5 (a) A curve has equation  $y = e^{2x} - 10e^x + 12x$ .

(i) Find  $\frac{dy}{dx}$ . (2 marks)

(ii) Find  $\frac{d^2y}{dx^2}$ . (1 mark)

(b) The points  $P$  and  $Q$  are the stationary points of the curve.

(i) Show that the  $x$ -coordinates of  $P$  and  $Q$  are given by the solutions of the equation

$$e^{2x} - 5e^x + 6 = 0 \quad (1 \text{ mark})$$

(ii) By using the substitution  $z = e^x$ , or otherwise, show that the  $x$ -coordinates of  $P$  and  $Q$  are  $\ln 2$  and  $\ln 3$ . (3 marks)

(iii) Find the  $y$ -coordinates of  $P$  and  $Q$ , giving each of your answers in the form  $m + 12 \ln n$ , where  $m$  and  $n$  are integers. (3 marks)

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. (3 marks)

6 (a) Use the mid-ordinate rule with four strips to find an estimate for  $\int_1^5 \ln x \, dx$ , giving your answer to three significant figures. (3 marks)

(b) (i) Given that  $y = x \ln x$ , find  $\frac{dy}{dx}$ . (2 marks)

(ii) Hence, or otherwise, find  $\int \ln x \, dx$ . (2 marks)

(iii) Find the exact value of  $\int_1^5 \ln x \, dx$ . (2 marks)

7 (a) Given that  $z = \frac{\sin x}{\cos x}$ , use the quotient rule to show that  $\frac{dz}{dx} = \sec^2 x$ . (3 marks)

(b) Sketch the curve with equation  $y = \sec x$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ . (2 marks)

(c) The region  $R$  is bounded by the curve  $y = \sec x$ , the  $x$ -axis and the lines  $x = 0$  and  $x = 1$ .

Find the volume of the solid formed when  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis, giving your answer to three significant figures. (3 marks)

**Turn over ►**

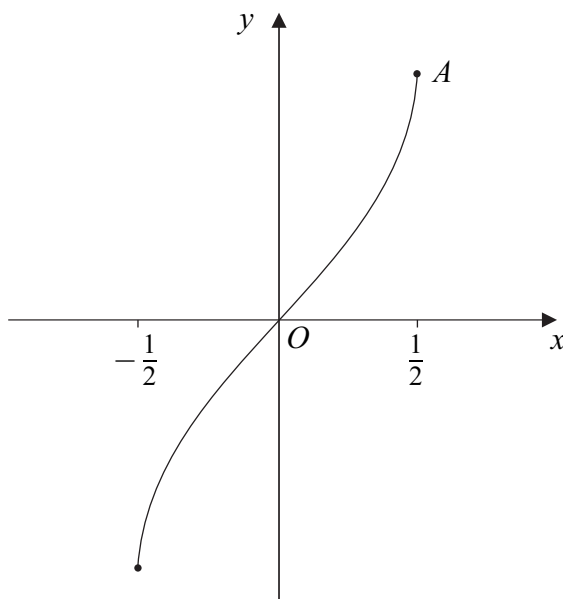
8 A function  $f$  is defined by  $f(x) = 2e^{3x} - 1$  for all real values of  $x$ .

(a) Find the range of  $f$ . (2 marks)

(b) Show that  $f^{-1}(x) = \frac{1}{3} \ln \left( \frac{x+1}{2} \right)$ . (3 marks)

(c) Find the gradient of the curve  $y = f^{-1}(x)$  when  $x = 0$ . (4 marks)

9 The diagram shows the curve with equation  $y = \sin^{-1} 2x$ , where  $-\frac{1}{2} \leq x \leq \frac{1}{2}$ .



(a) Find the  $y$ -coordinate of the point  $A$ , where  $x = \frac{1}{2}$ . (1 mark)

(b) (i) Given that  $y = \sin^{-1} 2x$ , show that  $x = \frac{1}{2} \sin y$ . (1 mark)

(ii) Given that  $x = \frac{1}{2} \sin y$ , find  $\frac{dx}{dy}$  in terms of  $y$ . (1 mark)

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} \quad (4 \text{ marks})$$

**END OF QUESTIONS**