

General Certificate of Education

Mathematics 6360

MS2A Statistics 2A

Mark Scheme

2006 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

М	mark is for method			
m or dM	mark is dependent on one or more M marks and is for method			
А	mark is dependent on M or m marks and is for accuracy			
В	mark is independent of M or m marks an	d is for method	l and accuracy	
E	mark is for explanation			
$\sqrt{100}$ or ft or F	follow through from previous			
	incorrect result	MC	mis-copy	
CAO	correct answer only	MR	mis-read	
CSO	correct solution only	RA	required accuracy	
AWFW	anything which falls within	FW	further work	
AWRT	anything which rounds to	ISW	ignore subsequent work	
ACF	any correct form	FIW	from incorrect work	
AG	answer given	BOD	given benefit of doubt	
SC	special case	WR	work replaced by candidate	
OE	or equivalent	FB	formulae book	
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme	
-x EE	deduct x marks for each error	G	graph	
NMS	no method shown	с	candidate	
PI	possibly implied	sf	significant figure(s)	
SCA	substantially correct approach	dp	decimal place(s)	

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2A				
Question	Solution	Marks	Total	Comments
1(a)(i)	$P(X \le 1) = 0.9098$	M1 A1	2	AWRT 0.91
(ii)	$P(Y=2) = \frac{e^{-2.5} (2.5)^2}{2!}$ = 0.257	M1 A1	2	AWFW 0.256 and 0.257
(b)(i)	$T = X + Y \sim P_o(3.0)$	B1		P(Total of 2 breakdowns) = P([x, y] = 2) = P([2,0]or[0,2]or[1,1])
	$P(T=2) = \frac{e^{-3.0} \times (3.0)^2}{2!}$ = 0.224	M1		$= [0.0758 \times 0.0821] + [0.6065 \times 0.25652] +$
	OR using tables: $= 0.4232 - 0.1991$ = 0.224	A1	3	$[0.20521 \times 0.3033] = 0.006223 + 0.155585 + 0.062233 = 0.224$
(ii)	$p = (0.224)^4 = 0.0025$	M1A1	2	
(c)(i)	$\frac{4X \sim P_o(2.0)}{4Y \sim P_o(10.0)} \Rightarrow \mu = 12.0$	B1	1	
(ii)	$P(T \ge 18) = 1 - P(T \le 17)$ = 1 - 0.9370	M1		
	= 0.0630	A1	2	
	Total		12	

MS2A (cont)
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2(a)H_o : Choice independent of genderB1gender not associated with choiceSquash BadmintonArchery Hockey Male 53.5 16/14 3024.5 19/28 Female 4/5.5 20/22 33/38.5 53/44M1ECombine Squash and BadmintonM1EES&BArchery Hockey Male 21/17.5 10/24 5 19/28 Female 24/27.5 33/38.5 53/44M1E χ^2 values S & B Female 0.4455 0.7857 1.8409 $\chi^2_{nec} = 7.90$ M1M1 $\chi^2_{10}(2) = 5.991$ B1(7.8 to 7.9) $\nu = 2$ $\chi^2_{10}(2) = 5.991$ B1 ff(on their ν)Reject H_o sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and genderA1 ftreject H_o and H_o stated or statement in context(b)More females and fewer males chose to participate in hockey than expectedB1 223(a) $\bar{x} = 301.0$ B12 $y = 9$ $t = 3.250$ 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 A1 A1(their \bar{x}) $\pm tx \left(\frac{their \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 A15	Question	Solution	Marks	Total	Comments
Male 5/3.5 16/14 30/24.5 19/28 M1 Combine Squash and Badminton M1 E _i < 5 (Similar categories)	2(a)	H _o : Choice independent of gender	B1		gender not associated with choice
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		Male 5/3.5 16/14 30/24.5 19/28	M1		
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		Combine Squash and Badminton	M1		$E_i < 5$ (Similar categories)
$ \begin{array}{ c c c c c } \hline S & \& B & Archery & Hockey \\ \hline Male & 0.7000 & 1.2347 & 2.8928 \\ \hline Permale & 0.4455 & 0.7857 & 1.8409 \\ \hline \chi^2_{outc} = 7.90 & A1 \\ \hline V = 2 & B1 \\ \hline \chi^2_{5\%}(2) = 5.991 & B1ft & (on their \nu) \\ \hline Reject H_o \\ Sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and gender \\ \hline More females and fewer males chose to participate in hockey than expected B1 \\ B1 & 2 \\ \hline \hline & Total & 12 \\ \hline & Total & 12 \\ \hline & & & & & \\ \hline & & & & & \\ \hline & & & &$		Male 21/17.5 30/24.5 19/28			
$v = 2$ $\chi^2_{5\%}(2) = 5.991$ B1 B1ft(on their v)Reject Ho Sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and genderA1ftreject Ho and Ho stated or statement in context(b)More females and fewer males chose to participate in hockey than expectedB1 B12Total123(a) $\overline{x} = 301.0$ B1 $t = 3.250$ B1 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 A11(their \overline{x}) $\pm tx \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 B15		S & B Archery Hockey Male 0.7000 1.2347 2.8928 Female 0.4455 0.7857 1.8409	M1		
$\chi^2_{5\%}(2) = 5.991$ B1ft(on their v)Reject H_o Sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and genderA1ftreject H_o and H_o stated or statement in context(b)More females and fewer males chose to participate in hockey than expectedB1 B123(a) $\overline{x} = 301.0$ B112(b) $S^2 = (4.945)^2 = 24.5$ B1 $T = 3.250$ 2AWFW 24.4 to 24.5 $v = 9$ $t = 3.250$ B1B1 99% confidence interval for μ $= 301.0 \pm 5.08$ $= (295.9, 306.1)$ M1 A1 A1(their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 A15					(7.8 to 7.9)
Reject Ho Sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and genderA1ftreject Ho and Ho stated or statement in context(b)More females and fewer males chose to participate in hockey than expectedB1 B123(a) $\overline{x} = 301.0$ B112(b) $S^2 = (4.945)^2 = 24.5$ $v = 9$ $t = 3.250$ 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ $= 301.0 \pm 5.08$ $= (295.9, 306.1)$ M1 A1 A1(their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1ftI					
Sufficient evidence, at the 5% level of significance, to support an association between the choice of sport and gender (b) More females and fewer males chose to participate in hockey than expected $\overline{B1}$ 2 $\overline{B1}$ 3 $\overline{B1}$ 2 $\overline{B1}$ 3 $\overline{B1}$ 4 $\overline{B1}$		$\chi^{2}_{5\%}(2) = 5.991$	B1ft		(on their v)
significance, to support an association between the choice of sport and genderAlftor statement in context(b)More females and fewer males chose to participate in hockey than expectedB1 B123(a) $\overline{x} = 301.0$ B112(b) $S^2 = (4.945)^2 = 24.5$ B1 $t = 3.250$ 2 $v = 9$ $t = 3.250$ B1B1 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 A1(their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 A15		Reject H _o			
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participate in hockey than expectedB12Total12Total123(a) $\overline{x} = 301.0$ B1(b) $S^2 = (4.945)^2 = 24.5$ B12 $V = 9$ $t = 3.250$ B1B12 $V = 9$ $t = 3.250$ B1B14 $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 A1 A1(their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 A15				10	
3(a) $\overline{x} = 301.0$ B1B1(b) $S^2 = (4.945)^2 = 24.5$ B1B12AWFW 24.4 to 24.5 $v = 9$ $t = 3.250$ B1B1B1B1111 $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 $= 301.0 \pm 5.08$ $= (295.9, 306.1)$ M1 A1 A1(their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c)Since 290 is less than the lower limit of the confidence interval, it is probably safeA1 A15	(b)	participate in hockey than expected			
(b) $S^2 = (4.945)^2 = 24.5$ v = 9 t = 3.250 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ $= 301.0 \pm 5.08$ = (295.9, 306.1) (c) Since 290 is less than the lower limit of the confidence interval, it is probably safe				12	
(c) $3^{5} = (4.943)^{5} = 24.3^{5}$ v = 9 t = 3.250 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ $= 301.0 \pm 5.08$ = (295.9, 306.1) (c) Since 290 is less than the lower limit of the confidence interval, it is probably safe	3(a)	$\overline{x} = 301.0$	B1		
$t = 3.250$ 99% confidence interval for μ $= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right)$ M1 $= 301.0 \pm 5.08$ $= (295.9, 306.1)$ A1 $A1$ (their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (c) Since 290 is less than the lower limit of the confidence interval, it is probably safe	(b)	$S^2 = (4.945)^2 = 24.5$	B1	2	AWFW 24.4 to 24.5
$= 301.0 \pm 3.250 \times \left(\frac{4.945}{\sqrt{10}}\right) \qquad M1$ $= 301.0 \pm 5.08$ $= (295.9, 306.1) \qquad A1$ $A1$ $A1$ $A1$ 5 (their \overline{x}) $\pm t \times \left(\frac{\text{their } \sigma}{\sqrt{10}}\right)$ (their \overline{x}) $\pm t \times \left(\frac{1}{\sqrt{10}}\right)$		t = 3.250	B1B1		
$= 301.0 \pm 5.08$ $= (295.9, 306.1)$ (c) Since 290 is less than the lower limit of the confidence interval, it is probably safe $A1$ $A1$ 5		,	M1		(their \overline{x}) + $t \times \left(\frac{\text{their } \sigma}{\sigma}\right)$
= (295.9, 306.1) (c) Since 290 is less than the lower limit of the confidence interval, it is probably safe 5					$\sqrt{10}$
the confidence interval, it is probably safe				5	
	(c)	the confidence interval, it is probably safe to assume that the mean weight of packets	52	2	
of rice exceeds 290 grams E2 2 Total 9			E2		

Question	Solution	Marks	Total	Comments
4(a)(i)	Area = $k(b-a) = 1$			
	$\Rightarrow \qquad k = \frac{1}{b-a}$	E1	1	AG
(ii)	а	M1		
	$=\left(\frac{kx^2}{2}\right)\Big _a^b$	A1		
	$=\frac{1}{2}k(b^2-a^2)$			
	$=\frac{1}{2}\times\frac{1}{(b-a)}\times(b-a)(a+b)$	M1		(factors shown)
	$=\frac{1}{2}(a+b)$	A1	4	AG
(b)(i)	$\mu = 1$	B1	1	
(ii)	$\sigma^{2} = \operatorname{Var}(X) = \frac{1}{12}(b-a)^{2}$			
	$=\frac{1}{12}\times 6^2$	M1		
	$\therefore \qquad \sigma = \sqrt{3}$	A1	2	AWRT 1.73
(iii)	$rac{12}{= 3}$ $\therefore \qquad \sigma = \sqrt{3}$ $P\left(X < \frac{2-\mu}{\sigma}\right) = P\left(X < \frac{1}{\sqrt{3}}\right)$	M1ft		(on their μ and σ)
	$=\frac{1}{6} \times 2.577$	M1ft		
	= 0.430	A1	3	
	Total		11	

Question	Solution	Marks	Total	Comments
5(a)	$E(X) = \sum_{\text{all } x} x P(X = x)$			
	all x = 50	B1		CAO
		DI		CAO
	$\mathbf{E}(X^{2}) = \sum_{\text{all } x} x^{2} \mathbf{P}(X = x)$	M1		
	=2602.6(0)			
	$\operatorname{Var}(X) = \operatorname{E}(X^{2}) - \left[\operatorname{E}(X)\right]^{2}$			
	$= 2602.6 - 50^2$			
	=102.6(0)	M1		
	\Rightarrow standard deviation (X) = 10.13	A1	4	(to nearest 1p)
(b)	$\mathrm{E}(Y) = \mu = \mathrm{E}(10X + 250)$			
	$=10\times \mathrm{E}(X)+250$			
	= 750	B1ft		On their $E(X)$
	$s.d(Y) = 10 \times 10.1$			
	=101	B1ft	2	On their $sd(X)$
	Total		6	
6	$H_{o}: \mu = 1000$	DI		
	$H_1: \mu \neq 1000$	B1		2-tailed test
	$\overline{x} = \frac{12036}{12} = 1003$	D1		
	12	B1		
	<i>S</i> = 5.444	B1		$S^2 = 29.6$
	v = 12 - 1 = 11	B1		
	$t = \frac{\overline{x} - \mu}{1003 - 1000} = 1.91$	M1		
	$t = \frac{\overline{x} - \mu}{S / \sqrt{n}} = \frac{1003 - 1000}{5.444 / \sqrt{12}} = 1.91$	A1ft A1		
		AI B1ft		(on their v)
	$t_{crit} = \pm 2.201$	A1ft		(on their t – values)
	Accept H _o Insufficient evidence to indicate a change			(on then $i = values)$
	in the mean content of sherry in a bottle	E1ft	10	
	Total		10	
	TOTAL		60	