

## **General Certificate of Education**

# Mathematics 6360 Statistics 6380

MS/SS1B Statistics 1B

# Mark Scheme 2006 examination – January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

### Key To Mark Scheme And Abbreviations Used In Marking

М	mark is for method							
m or dM	mark is dependent on one or more M marks and is for method							
А	mark is dependent on M or m marks and is for accuracy							
В	mark is independent of M or m marks and is for method and accuracy							
Е	mark is for explanation							
$\sqrt{100}$ or ft or F	follow through from previous							
	incorrect result	MC	mis-copy					
CAO	correct answer only	MR	mis-read					
CSO	correct solution only	RA	required accuracy					
AWFW	anything which falls within FW further work							
AWRT	anything which rounds to	ISW	ignore subsequent work					
ACF	any correct form	FIW	from incorrect work					
AG	answer given	BOD	given benefit of doubt					
SC	special case	WR	work replaced by candidate					
OE	or equivalent	FB	formulae book					
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme					
–x EE	deduct <i>x</i> marks for each error	G	graph					
NMS	no method shown	с	candidate					
PI	possibly implied	sf	significant figure(s)					
SCA	substantially correct approach	dp	decimal place(s)					

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

#### MS/SS1B

Q	Solution	Marks	Total	Comments
1(a)	Gradient, $b = 0.886$ to 0.887 b = 0.88 to 0.89	B2 (B1)		AWFW AWFW
	Intercept, $a = 2.31$ to 2.33 a = 2.3	B2 (B1)		AWFW AWRT
	Attempt at $\sum x \sum x^2 \sum y \sum xy$ or Attempt at $S_{xx} S_{xy}$ Attempt at a <b>correct</b> formula for <i>b</i> b = 0.886 to $0.887a = 2.31$ to $2.33$	(M1) (m1) (A1) (A1)		72, 624, 87, 720 105.6, 93.6 AWFW AWFW
	Accept <i>a</i> & <i>b</i> interchanged only if y = ax + b stated or subsequently used correctly in either (b) or (c)		4	
(b)	<i>a</i> : average <b>waiting time</b> of 2.32 minutes (139 seconds) when entering <b>empty restaurant</b>	B1		OE; accept minimum waiting time
	<i>b</i> : average <b>increase in waiting time</b> of 0.886 minutes (53 seconds) <b>for each customer in restaurant</b> on entry	B1	2	OE
(c)	Use of $y = a + 5b$ or $y = a + 25b$	M1		
(i)	For $x = 5$ $y = 6.6$ to $6.8$			
(ii)	For $x = 25$ $y = 24.3$ to 24.6	Al	2	Both; AWFW
(d)(i)	<b>Reliable as interpolation</b> and <b>small</b> residuals	B1 B1		Within range OE OE
	or Reliable as interpolation but large percentage residuals so inconclusive	(B1) (B1)		
	or Large percentage residuals so unreliable	(B1)		
(ii)	Unreliable as extrapolation	B1	3	Outside range OE
	Total		11	

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Q	Solution	Marks	Total	Comments
2(a)	P(X) = 0.3 $P(Y) = 0.4$ $P(Z) = 0.2$			
(i)	$P(X \cap Y \cap Z) = 0.3 \times 0.4 \times 0.2 = 0.024$	M1	1	
(ii)	$P(X' \cap Y' \cap Z') = 0.7 \times 0.6 \times 0.8$ = 0.336	M1 A1	2	At least 2 correct terms CAO
(iii)	$P(X' \cap Y' \cap Z) = 0.7 \times 0.6 \times 0.2$	M1		Correct numerical expression
	= 0.084	A1		CAO
(b)	P(W   Z) = 0.9 $P(W   Z') = 0.25$			
(i)	$P(Z \cap W) = 0.2 \times 0.9$ = 0.18	M1 A1	2	Correct numerical expression CAO
(ii)	$P((Z \cap W') \cup (Z' \cap W))$ or $1 - [P((Z \cap W) \cup (Z' \cap W'))]$			
	$= 0.2 \times (1 - 0.9)$	M1		$0.2 \times 0.9$ or (b)(i)
	$^+$ (1 – 0.2) × 0.25	M1		$(1-0.2) \times (1-0.25)$
				Cannot score an M1 in both methods
	= 0.02 + 0.20 = 0.22	A1	3	1 – (0.18 + 0.60) CAO
	Total		11	

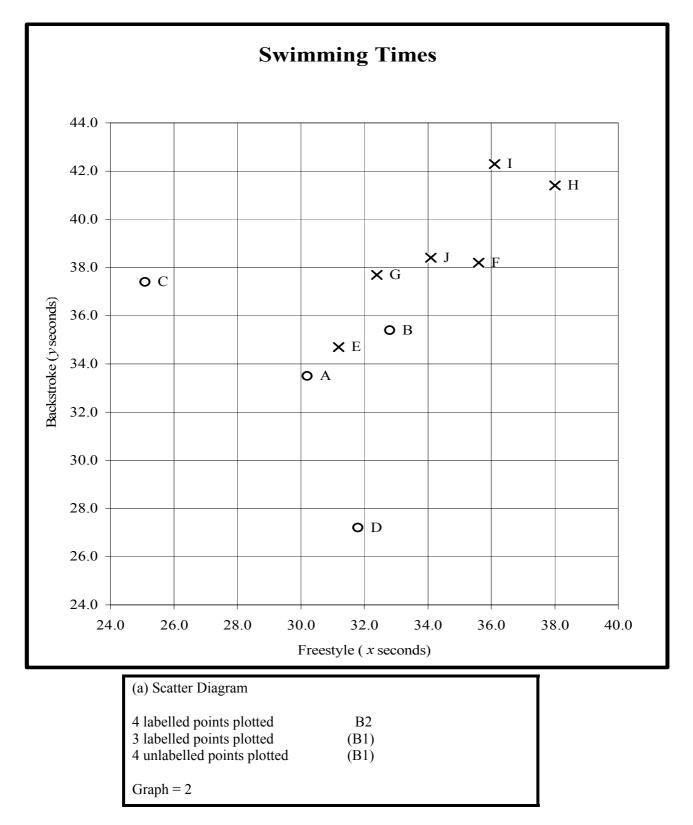
Q	Solution	Marks	Total	Comments
3(a)	Mean = $\frac{286.5}{50} = 5.73$	B1		CAO
	Standard deviation = $\sqrt{\frac{45.16}{49 \text{ or } 50}}$ =			
	0.95 to 0.961	B1	2	AWFW
(b)	$99\% \Rightarrow z = 2.57$ to $2.58$	B1		AWFW 2.5758
	CI for $\mu$ is $\overline{x} \pm z \times \frac{(\sigma \text{ or } s)}{\sqrt{n}}$	M1		Use of Must have $(\div \sqrt{n})$ with $n > 1$
	Thus $5.73 \pm 2.5758 \times \frac{(0.95 \text{ to } 0.961)}{\sqrt{50}}$	A1√		$$ on z and $s^2 > 0$ but not on $\overline{x}$ Accept only 50 or 49 for n
	$5.73 \pm (0.34 \text{ to } 0.36)$	$\uparrow$		Dependent
	5.37 to 5.39, 6.07 to 6.09)	A1	4	AWFW
(c)	CI excludes both values of 5 and 6 <sup>1</sup> / <sub>2</sub> so Neither claim appears valid	B1√ ↑ B1√		$\sqrt[]{}$ on (b); OE Dependent $\sqrt[]{}$ on (b); OE
	or			
	CI excludes 5 so claim not valid and	(B1√)		on (b); OE
	CI excludes $6\frac{1}{2}$ so claim not valid	(B1√)	2	on (b); OE
	Total		8	

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Q	Solution	Marks	Total	Comments
4(a)	$\sum fx = 8025$			
	$\sum fx^2 = 739975$			
	Mean $(\bar{x}) = 80.2$ to 80.3	B2		AWFW 80.25
	Standard Deviation $(s_n, s_{n-1}) = 30.9$ to 31.2	B2		AWFW 30.97882 or 31.13489
	MPs ( <i>x</i> ): 25, 35, 50, 70, 90, 110, 135, 165	(B1)		At least 4 correct
	Mean $(\overline{x}) = \frac{\Sigma fx}{100}$	(M1)	4	Use of
(b)(i)	Large $(n > 30)$ sample			
	or Central Limit Theorem	B1	1	OE
(ii)	Mean $(\overline{Y}) = 80.2$ to 80.3	B1√		on (a)
	Standard error $(\overline{Y}) = \frac{30.9 \text{ to } 31.2}{\sqrt{36}}$			
	= 5.1 to 5.25	M1	2	$\sqrt{s^2} > 0$ in (a) ÷ $\sqrt{36}$ or 6
(iii)	$P(\overline{Y} < 90) = P\left(Z < \frac{90 - (80.2 \text{ to } 80.3)}{(5.1 \text{ to } 5.25)}\right)$	M1 M1		Standardising 90 Using values from (b)(ii) with $\sqrt{s^2/36} > 0$ or $\sqrt{s^2/100} > 0$
	= P(Z < 1.84  to  1.93)			
	= 0.967 to 0.974	A1	3	AWFW
	Total		10	

Q	Solution	Marks	Total	Comments
5(a)	Scatter Diagram or or	B2 (B1) (B1)	2	<ul><li>4 labelled points plotted</li><li>3 labelled points plotted</li><li>4 unlabelled points plotted</li></ul>
(b)(i)	Positive/linear correlation/relationship except for	B1		OE
	two unusual values/results	B1	2	OE
(ii)	0.462	B1	1	CAO; accept 3 <sup>rd</sup> /final/last value
(c)	C and D	B1		CAO
	C is likely <b>freestyle</b> champion <b>D</b> is likely <b>backstroke</b> champion	B1		Style identified
	or C is likely freestyle champion D is likely backstroke champion	(B1) (B1)	2	
(d)(i)	r = 0.912 to 0.913	В3		AWFW
	or $r = 0.91$ to 0.92 or 0.46 to 0.47	B2		AWFW
	or $r = 0.9$	B1		AWRT
	Attempt at $\Sigma x \Sigma x^2$ $\Sigma y \Sigma y^2$ $\Sigma xy$			270.4, 9188.46 301.6, 11437.84 10246.53
	orAttempt at $S_{xx}$ $S_{yy}$ $S_{xy}$	(M1)		48.94, 67.52, 52.45
	Attempt at a <b>correct</b> formula for <i>r</i>	(m1)		
	r = 0.912 to 0.913	A1	3	AWFW
(ii)	Boys are <b>faster/slower</b> at <b>both strokes</b> or Boys are <b>equally good</b> at <b>both strokes</b>	B1	1	OE;do not accept freestyle times are proportional to backstroke times
	Total		11	

Question 5(a)



Q	Solution	Marks	Total	Comments
6(a)(i)	B(50, 0.2) P( $R \le 15$ ) = 0.969 to 0.97	M1 A1	2	Use of in (a) AWFW 0.9692
(ii)	$P(R = 10) = P(R \le 10) - P(R \le 9)$			Stated or implied
	or P(R = 10) = $\binom{50}{10} (0.2)^{10} (0.8)^{40}$	M1		Stated or implied
	= 0.5836 - 0.4437 = 0.139 to 0.141	Al	2	AWFW 0.1399
(iii)	P(5 < R < 15) = P(R \le 14 or 15) = 0.9393 or 0.9692	M1		Accept values to 3 dp
	minus $P(R \le 5 \text{ or } 4) = 0.0480 \text{ or}$ 0.0185	M1		Accept values to 3 dp
	= 0.89 to 0.893 or	A1		AWFW 0.8913
	B(50, 0.2) expressions stated for at least 3 of $5 \le R \le 15$	(M1)		Or implied by a correct answer
	Answer	(A2)	3	
(b)	Mean, $\mu = np = 50 \times 0.2 = 10$	B1		Either; CAO
	or			
	Estimate of $p$ , $\hat{p} = 0.21$			
	Variance, $\sigma^2 = np(1-p) = 10 \times 0.8 = 8$	B1		CAO
	Mean or Estimate of <i>p</i> is similar to that expected			10.5 and 10 or 0.21 and 0.2
	but Variance (standard deviation) is different from that expected	B1		Either point 20.41 and 8 or 4.5 and 2.8
	Reason to <b>doubt validity</b> of Sly's claim	B1	4	Must be based on both 10 or 0.2 and 8 or on both 10 or 0.2 and 2.8 correctly
	Total		11	

Q	Solution	Marks	Total	Comments
7 (a) (i)	Weight, $X \sim N(406, 4.2^2)$ P(X < 400) = P $\left(Z < \frac{400 - 406}{4.2}\right)$	M1		Standardising (399.5, 400 or 400.5) with 406 and ( $\sqrt{4.2}$ , 4.2 or 4.2 <sup>2</sup> ) and/or (406 – <i>x</i> )
	= P(Z < -1.428  to  -1.43) = 1 - P(Z < 1.428 to 1.43)	ml		$\Phi(-z) = 1 - \Phi(z)$
	= 0.076 to 0.077	A1	3	AWRT 0.07636
(ii)	P(402.5 < X < 407.5) = P(X < 407.5) - P(X < 402.5) =	M1		Difference OE
	P(Z < 0.36) - P(Z < -0.83)	B2,1		AWRT; ignoring signs
	= 0.64058 - (1 - 0.79673) = 0.433 to 0.44	Al	4	AWFW 0.43731
(b)(i)	$0.975 \implies z = 1.96$	M1		Accept explanation in words
	$P(Y < 310) = P\left(Z < \frac{310 - \mu}{\sigma}\right)$ or	M1		Standardising 310 using $\mu$ and $\sigma$
	$x = \mu + \pm z\sigma$			Accept in words
	Thus $\frac{310 - \mu}{\sigma} = 1.96 \implies \text{result}$	m1		Equating
	$310 = \mu + 1.96\sigma \implies \text{result}$			Substitution
	NB: Working backwards from given equation $\Rightarrow$ at most M1 M0 mo		3	
(ii)	$0.86 \implies z = 1.08$	B1		AWRT 1.0803
	$310 - \mu = 1.96\sigma$ $307.5 - \mu = 1.08\sigma$			
	$2.5 = 0.88 \sigma$	M1		Attempt at solving 2 equations each of form $x - \mu = z\sigma$
	$\sigma$ = 2.84 to 2.842	A1		AWFW 2.841
	$\mu = 304.4$ to 304.5	A1	4	AWFW 304.43
	Total		14	
	TOTAL		75	