



General Certificate of Education

Mathematics 6360

MD02 Decision 2

Mark Scheme

2006 examination – January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

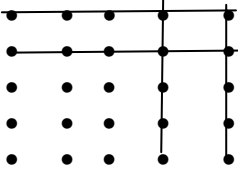
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MD02

Q	Solution	Marks	Total	Comments																														
1(a)	Add extra row with all values the same	B1	1	Usually all equal to 26 and below the other rows																														
(b)	Reduce columns first	M1	4	<table border="1" style="margin-left: 20px;"> <tr> <td>26</td><td>26</td><td>26</td><td>26</td><td>26</td> </tr> <tr> <td>16</td><td>19</td><td>18</td><td>25</td><td>24</td> </tr> <tr> <td>22</td><td>21</td><td>20</td><td>26</td><td>25</td> </tr> <tr> <td>21</td><td>22</td><td>23</td><td>21</td><td>24</td> </tr> <tr> <td>20</td><td>21</td><td>21</td><td>23</td><td>20</td> </tr> <tr> <td>26</td><td>26</td><td>26</td><td>26</td><td>26</td> </tr> </table>	26	26	26	26	26	16	19	18	25	24	22	21	20	26	25	21	22	23	21	24	20	21	21	23	20	26	26	26	26	26
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5	3	5	0	4																														
4	2	3	2	0																														
10	7	8	5	6																														
Reduce rows	M1	<p>These 2 marks available for those who reduce rows first</p> 																																
<table border="1" style="margin-left: 20px;"> <tr> <td>0</td><td>0</td><td>0</td><td>4</td><td>4</td> </tr> <tr> <td>4</td><td>0</td><td>0</td><td>3</td><td>3</td> </tr> <tr> <td>5</td><td>3</td><td>5</td><td>0</td><td>4</td> </tr> <tr> <td>4</td><td>2</td><td>3</td><td>2</td><td>0</td> </tr> <tr> <td>5</td><td>2</td><td>3</td><td>0</td><td>1</td> </tr> </table>	0		0	0	4	4	4	0	0	3	3	5	3	5	0	4	4	2	3	2	0	5	2	3	0	1	A1							
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Covering zeros requires 4 lines so adjust with least entry remaining being 2	M1																																	
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0	0	6	6																															
4	0	0	5	5																														
3	1	3	0	4																														
2	0	1	2	0																														
3	0	1	0	1																														
Match <i>A</i> – 1; <i>C</i> = 2; <i>D</i> –3; <i>E</i> –4	B1																																	
Expected minimum time $16 + 20 + 21 + 20 = 77$ min	B1	2																																
Total			9																															

MD02 (cont)

Q	Solution	Marks	Total	Comments																																																																								
2(a)																																																																												
	Network diagram	M1 A1	2	SCA Correct																																																																								
(b)	Clear attempt to use Dynamic Programming Working backwards through network			Complete enumeration M0 Forwards through network																																																																								
	<table border="1"> <thead> <tr> <th>Month</th> <th>Already Built</th> <th>Machine Built</th> <th>Profit</th> <th>Total (Max*)</th> </tr> </thead> <tbody> <tr> <td rowspan="3">3</td> <td>A & B</td> <td>C</td> <td>64</td> <td>64*</td> </tr> <tr> <td>A & C</td> <td>B</td> <td>67</td> <td>67*</td> </tr> <tr> <td>B & C</td> <td>A</td> <td>69</td> <td>69*</td> </tr> <tr> <td rowspan="6">2</td> <td rowspan="2">A</td> <td>B</td> <td>58</td> <td>58+64 = 122*</td> </tr> <tr> <td>C</td> <td>54</td> <td>54+67 = 121</td> </tr> <tr> <td rowspan="2">B</td> <td>A</td> <td>70</td> <td>70+64 = 134*</td> </tr> <tr> <td>C</td> <td>54</td> <td>54+69 = 123</td> </tr> <tr> <td rowspan="2">C</td> <td>A</td> <td>68</td> <td>68+67 = 135*</td> </tr> <tr> <td>B</td> <td>63</td> <td>63+69 = 132</td> </tr> <tr> <td>1</td> <td>–</td> <td>A</td> <td>52</td> <td>52+122 = 174</td> </tr> <tr> <td rowspan="2">1</td> <td>–</td> <td>B</td> <td>47</td> <td>47+134 = 181</td> </tr> <tr> <td>–</td> <td>C</td> <td>48</td> <td>48+135 = 183*</td> </tr> </tbody> </table>	Month	Already Built	Machine Built	Profit	Total (Max*)	3	A & B	C	64	64*	A & C	B	67	67*	B & C	A	69	69*	2	A	B	58	58+64 = 122*	C	54	54+67 = 121	B	A	70	70+64 = 134*	C	54	54+69 = 123	C	A	68	68+67 = 135*	B	63	63+69 = 132	1	–	A	52	52+122 = 174	1	–	B	47	47+134 = 181	–	C	48	48+135 = 183*	M1 M1 A1 A1 A1		<table border="1"> <tbody> <tr> <td>A</td> <td>52</td> <td>52*</td> </tr> <tr> <td>B</td> <td>47</td> <td>47*</td> </tr> <tr> <td>C</td> <td>48</td> <td>48*</td> </tr> <tr> <td>AB</td> <td>110</td> <td>117</td> </tr> <tr> <td>AC</td> <td>106</td> <td>116</td> </tr> <tr> <td>BC</td> <td>101</td> <td>111</td> </tr> </tbody> </table> <p>six possibilities</p> <p>Correct max identified and rest correct BA 117*; CA 116*; CB 111*</p> <p>Exactly 3 totals considered</p> <p>Considering previous max to combine</p> <p>BAC 181; CAB 183; CBA 180 Everything correct and route clearly traceable</p>	A	52	52*	B	47	47*	C	48	48*	AB	110	117	AC	106	116	BC	101	111
Month	Already Built	Machine Built	Profit	Total (Max*)																																																																								
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BC	101	111																																																																										
	The machine should therefore be built in the order C then A then B	B1																																																																										
	Max profit = £183000	B1	2	condone 183																																																																								
	Total		9																																																																									

MD02 (cont)

Q	Solution	Marks	Total	Comments
3(a)	<p>Activity network SCA</p>	<p>M1 A1 A1</p>	<p>3</p>	<p>almost correct (up to 2 slips) all correct</p>
(b)	Forward pass for earliest times	M1 A1	2	
(c)	Backward pass	M1 A1	2	
(d)	Critical path is ACDHI Minimum completion 24 days	B1 B1	2	
(e)	Non-critical B E F G Float 2 5 4 2	M1 A1 [√]	2	At least 3 activities and float in one activity [√] correct [√] their earliest and latest times
(f)	<p>Resource histogram</p>	M1 A1 M1 A1	4	Histogram ≤ 11 Correct Rest as histogram – generally start activities ok All correct
(g)	Problems with D & E solved by E coming after D Problem at 16-18 days with F can be solved by moving F to 20-22 Must overrun by equivalent to duration of E (3 days)	M1 A1 B1	3	
	Total		18	

MD02 (cont)

Q	Solution	Marks	Total	Comments
4(a)		B1 B1 B1 B1	4	MN NT PQ NP
(b)(i)	<p>e.g. SMNT 2 SPQT 2</p>	M1 M1 A1 A1 M1 A1	6	initial flow indicated as surplus forward and backward flows use of flow augmentation one flow correctly identified all possible flows correct amending flows (dep on first M1) final situation with saturation at M and P
(ii)	Max flow = 14 	B1 B1	2	
(c)	Cut through 2 of their saturated arcs $\left\{ \begin{array}{l} \{S, M\} / \{P, N, Q, T\} \\ \text{or cuts through } MN, MP \text{ \& } SP \end{array} \right\}$	M1 A1	2	cut on original network described or drawn
Total			14	

MD02 (cont)

Q	Solution	Marks	Total	Comments																												
5(a)	Introducing slack variables	M1																														
	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="text-align: right;"><i>P</i></td> <td style="text-align: center;"><i>x</i></td> <td style="text-align: center;"><i>y</i></td> <td style="text-align: center;"><i>z</i></td> <td style="text-align: center;"><i>r</i></td> <td style="text-align: center;"><i>s</i></td> <td style="text-align: right;">value</td> </tr> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;">-3</td> <td style="text-align: center;">-2</td> <td style="text-align: center;">-4</td> <td style="text-align: center;">0</td> <td style="text-align: center;">0</td> <td style="text-align: right;">0</td> </tr> <tr> <td style="text-align: right;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">4</td> <td style="text-align: center;">2</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> <td style="text-align: right;">8</td> </tr> <tr> <td style="text-align: right;">0</td> <td style="text-align: center;">2</td> <td style="text-align: center;">7</td> <td style="text-align: center;">3</td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: right;">21</td> </tr> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>r</i>	<i>s</i>	value	1	-3	-2	-4	0	0	0	0	1	4	2	1	0	8	0	2	7	3	0	1	21	A2	3	-1 EE
	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>r</i>	<i>s</i>	value																									
1	-3	-2	-4	0	0	0																										
0	1	4	2	1	0	8																										
0	2	7	3	0	1	21																										
(b)	Choosing correct pivot in <i>z</i> -column	M1		and perhaps dividing by 2																												
	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;">-1</td> <td style="text-align: center;">6</td> <td style="text-align: center;">0</td> <td style="text-align: center;">2</td> <td style="text-align: center;">0</td> <td style="text-align: right;">16</td> </tr> <tr> <td style="text-align: right;">0</td> <td style="text-align: center;">1/2</td> <td style="text-align: center;">2</td> <td style="text-align: center;">1</td> <td style="text-align: center;">1/2</td> <td style="text-align: center;">0</td> <td style="text-align: right;">4</td> </tr> <tr> <td style="text-align: right;">0</td> <td style="text-align: center;">1/2</td> <td style="text-align: center;">1</td> <td style="text-align: center;">0</td> <td style="text-align: center;">-3/2</td> <td style="text-align: center;">1</td> <td style="text-align: right;">9</td> </tr> </table>	1	-1	6	0	2	0	16	0	1/2	2	1	1/2	0	4	0	1/2	1	0	-3/2	1	9	M1		row operations							
1	-1	6	0	2	0	16																										
0	1/2	2	1	1/2	0	4																										
0	1/2	1	0	-3/2	1	9																										
		A1	3	correct																												
(c)(i)	Need to use <i>x</i> – column for pivot	M1																														
	Choosing correct pivot	A1																														
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0	0	-1	-1	-2	1	5																										
		A1		top row																												
		A1	5	third row																												
(ii)	Yes optimal	B1✓																														
	No negative values in top row	E1	2																													
Total			13																													

MD02 (cont)

Q	Solution	Marks	Total	Comments
6 (a)	$(-2, 2, 4) < (2, 4, 5)$ So S_1 dominated by S_2 $\begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} > \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$ So C_3 dominated by C_2	E1		note > sign
(b)	2×2 game now $\begin{matrix} & c_1 & c_2 \\ s_2 & \begin{bmatrix} 2 & 4 \end{bmatrix} \\ s_3 & \begin{bmatrix} 5 & 1 \end{bmatrix} \end{matrix}$ Minimum of rows $(2, 4) = 2$ Minimum of $(5, 1) = 1$ Choose maximum = $\textcircled{2}$ Max of column 1 = $\max(2, 5) = 5$ Max of column 2 = $\max(4, 1) = 4$ Choose minimum = 4 Since $2 \neq 4 \Rightarrow$ not stable solution	M1 A1 A1 E1	2 4	correct method for either S or C play safe for Sam is S_2 play safe for computer is C_2
(c)(i)	Computer picks C_1 Expected game = $2p + 5(1 - p)$ $= 5 - 3p$ Computer picks C_2 Expected gain = $4p + (1 - p)$ $= 1 + 3p$	M1 A1 A1	3	
(ii)	Best mixed strategy $5 - 3p = 1 + 3p$ $\Rightarrow p = \frac{2}{3}$	M1 A1	2	
(iii)	Expected points gain $= 5 - 3 \times \left(\frac{2}{3}\right)$ $= 3$	B1	1	Or $1 + 3 \left(\frac{2}{3}\right)$
	Total		12	
	Total		75	