General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require: • an 8-page answer book; • the **blue** AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The point A has coordinates $(11, 2)$ and the point B has coordinates $(-1, -1)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation

$$x - 4y = 3$$ *(2 marks)*

(b) The line with equation $3x + 5y = 26$ intersects the line AB at the point C. Find the coordinates of C. *(3 marks)*

2 A curve has equation $y = x^5 - 6x^3 - 3x + 25$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) The point P on the curve has coordinates $(2, 3)$.

(i) Show that the gradient of the curve at P is 5. *(2 marks)*

(ii) Hence find an equation of the normal to the curve at P, expressing your answer in the form $ax + by = c$, where a, b and c are integers. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = 1$. *(2 marks)*

3 A circle has equation $x^2 + y^2 - 12x - 6y + 20 = 0$.

(a) By completing the square, express the equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of the centre of the circle; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The line with equation $y = x + 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 - 5x + 6 = 0$$ *(2 marks)*

(ii) Find the coordinates of P and Q. *(4 marks)*

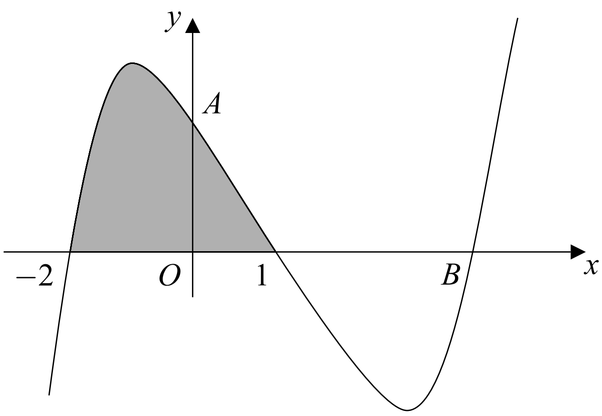
4 (a) The function f is defined for all values of x by $f(x) = x^3 - 3x^2 - 6x + 8$.

(i) Find the remainder when $f(x)$ is divided by $x + 1$. *(2 marks)*

(ii) Given that $f(1) = 0$ and $f(-2) = 0$, write down two linear factors of $f(x)$. *(2 marks)*

(iii) Hence express $x^3 - 3x^2 - 6x + 8$ as the product of three linear factors. *(2 marks)*

(b) The curve with equation $y = x^3 - 3x^2 - 6x + 8$ is sketched below.



(i) The curve intersects the y-axis at the point A. Find the y-coordinate of A. *(1 mark)*

(ii) The curve crosses the x-axis when $x = -2$, when $x = 1$ and also at the point B. Use the results from part (a) to find the x-coordinate of B. *(1 mark)*

(c) (i) Find $\int (x^3 - 3x^2 - 6x + 8)\,dx$. *(4 marks)*

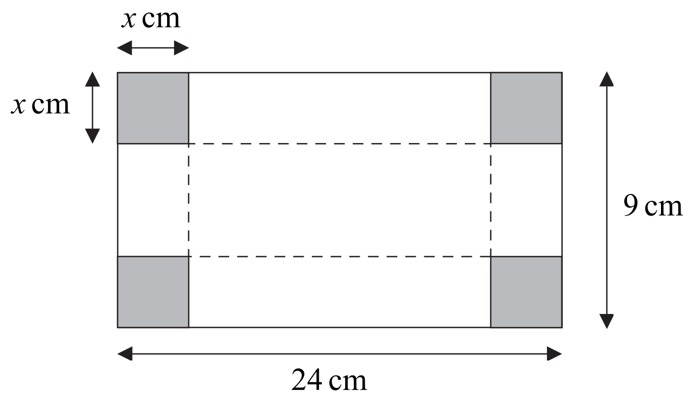
(ii) Hence find the area of the shaded region bounded by the curve and the x-axis. *(3 marks)*

5 (a) Simplify $(\sqrt{12} + 2)(\sqrt{12} - 2)$. *(2 marks)*

(b) Express $\sqrt{12}$ in the form $m\sqrt{3}$, where m is an integer. *(1 mark)*

(c) Express $\dfrac{\sqrt{12} + 2}{\sqrt{12} - 2}$ in the form $a + b\sqrt{3}$, where a and b are integers. *(4 marks)*

6 The diagram below shows a rectangular sheet of metal 24 cm by 9 cm.



A square of side x cm is cut from each corner and the metal is then folded along the broken lines to make an open box with a rectangular base and height x cm.

(a) Show that the volume, V cm^3, of liquid the box can hold is given by

$$V = 4x^3 - 66x^2 + 216x$$ *(3 marks)*

(b) (i) Find $\frac{dV}{dx}$. *(3 marks)*

(ii) Show that any stationary values of V must occur when $x^2 - 11x + 18 = 0$. *(2 marks)*

(iii) Solve the equation $x^2 - 11x + 18 = 0$. *(2 marks)*

(iv) Explain why there is only one value of x for which V is stationary. *(1 mark)*

(c) (i) Find $\frac{d^2V}{dx^2}$. *(2 marks)*

(ii) Hence determine whether the stationary value is a maximum or minimum. *(2 marks)*

7 (a) Simplify $(k+5)^2 - 12k(k+2)$. *(2 marks)*

(b) The quadratic equation $3(k+2)x^2 + (k+5)x + k = 0$ has real roots.

(i) Show that $(k-1)(11k+25) \leqslant 0$. *(5 marks)*

(ii) Hence find the possible values of k. *(3 marks)*

END OF QUESTIONS