OCR
Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics B (MEI)

Y413 Modelling with Algorithms

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **12** pages.

Answer **all** the questions

1 In Fig. 1 the weights on the arcs represent distances.

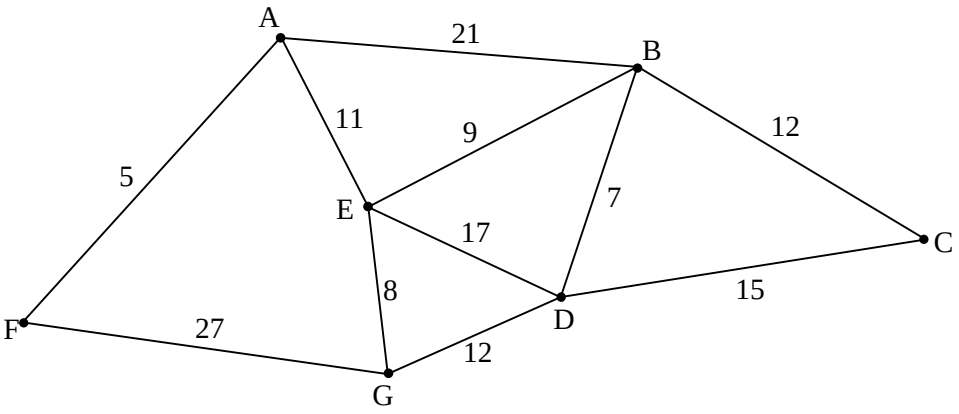


Fig. 1

Apply Dijkstra's algorithm to find the shortest path from A to D.

Give

- your shortest path

and

- its length. **[6]**

2 The instructions labelled Step 10 to Step 90 below describe a bubble sort algorithm to sort 4 numbers.

```
Step 10  Let i equal 1.
Step 20      Let j equal 1.
Step 30          If the jth number in the list is less than the (j+1)th, then swap them.
Step 40          Let the new value of j be j+1.
Step 50              If j is greater than 4–i, then go to Step 70.
Step 60              Go to Step 30.
Step 70      Let the new value of i be i+1.
Step 80  If i is equal to 4, then stop.
Step 90  Go to Step 20.
```

Four students take a test. Ali scores 57, Bill scores 67, Cleo scores 43 and Debbie scores 73.

(i) Use this bubble sort algorithm to rearrange the individuals from alphabetical order into descending order of their test scores. Record the names and scores in the order that they appear each time Step 70 is used. **[3]**

Ewan takes the test later, and his score of 60 is added to the list by comparing his score with the highest score, then with the second highest, and so on, until it can be put in the correct place.

(ii) Determine how many comparisons were made in using the bubble sort in part **(i)** and then inserting Ewan's score into the list. **[2]**

(iii) Describe how to amend the instructions so they give a bubble sort for 5 numbers. **[1]**

(iv) The five students are listed in alphabetical order. How many comparisons are made when your amended bubble sort is used to arrange their scores into descending order? You do not need to carry out the bubble sort. **[1]**

3 An industrial process is represented by the network shown in Fig. 3. The diagram also shows the precedences and durations, in minutes, for each activity, and the earliest event times for seven of the ten events.

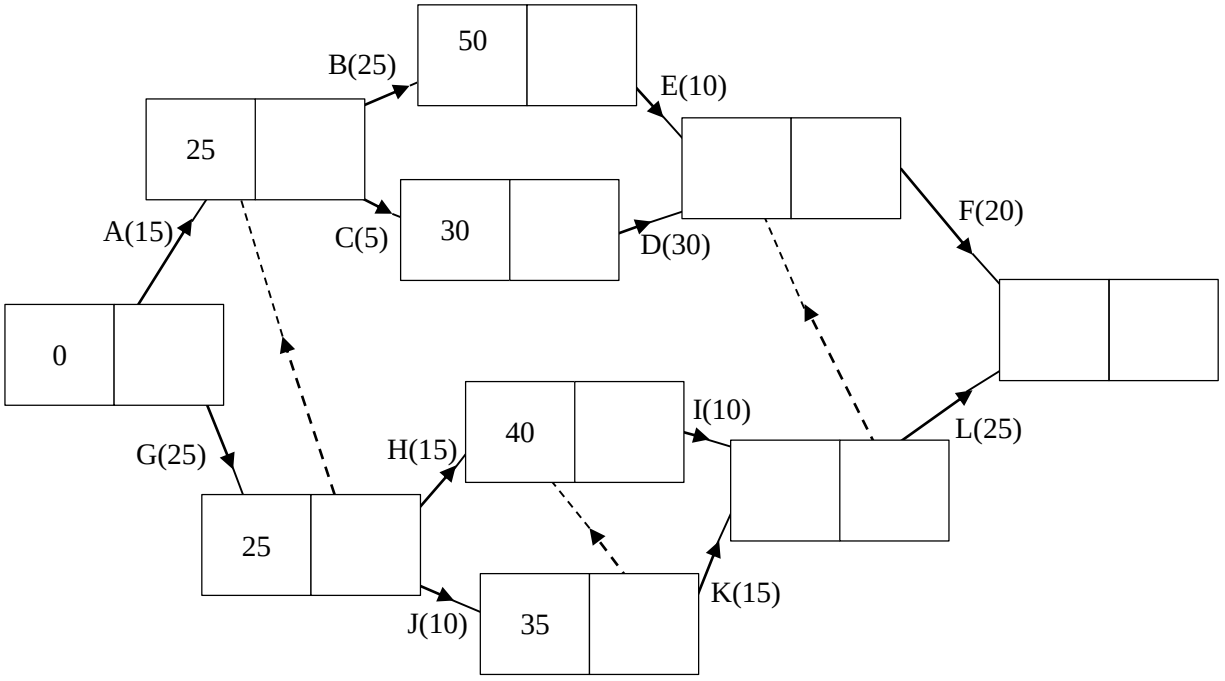


Fig. 3

(i) Find the minimum completion time for the process based on this information.

Identify

- the critical activities

and

- the total floats for the non-critical activities. **[5]**

Activities C and H need access to the same machine, so cannot happen simultaneously. Activities E and I need to be undertaken by the same person, so cannot happen simultaneously.

(ii) Explain why the process cannot be completed in the minimum completion time you found in part **(i).** **[3]**

(iii) Give a minimum completion time that takes these constraints into account. **[1]**

4 The table and the network in Fig. 4 represent the layout of cables joining nine junction boxes in a town; the weights on the arcs and the values in the table are the lengths of the cables, in kilometres.

	A	B	C	D	E	F	G	H	I
A		4	7		2				
B	4		4	5					
C	7	4		3	6				
D		5	3						
E	2		6			5			
F					5		3	1	
G						3		2	3
H						1	2		2
I							3	2	

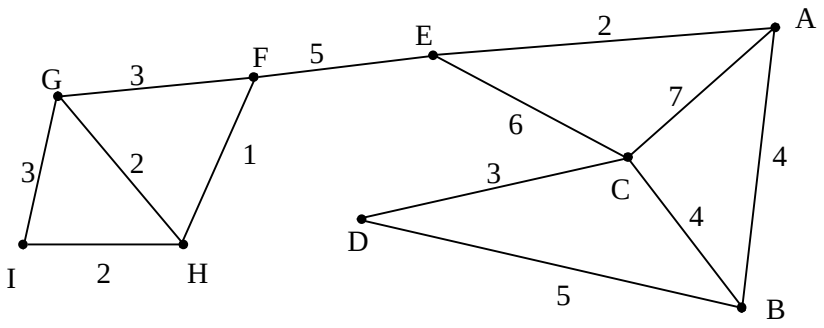


Fig. 4

Each month it costs £8 per kilometre to maintain the cables. Some of the cables are to be removed to save maintenance costs. The network must remain connected.

(i) (*A*) Which cables should be removed to save the maximum amount of money per month on maintenance? You should show sufficient working to make your reasoning clear. **[5]**

(*B*) What is the maximum amount of money which can be saved per month on maintenance? **[1]**

The cost of maintenance needs to be cut further. It is proposed to lay a new cable connecting the junction boxes at D and H. This cable costs £200 to lay, is 2 km long, and has the same maintenance costs as the other cables. When the new cable is laid, one or more of the old cables will be removed to obtain a further reduction in maintenance costs.

(ii) How many months will it take before the further reduction in maintenance costs is greater than the amount spent on laying the new cable? You should show sufficient working to support your reasoning. **[4]**

5 The following LP problem is to be solved.

$$\begin{aligned} \text{Maximise} \quad & P = \frac{1}{3}x + \frac{1}{2}y \\ \text{subject to} \quad & x + 2y \leq 9 \\ & 2x + 3y \leq 14 \\ & 2x + y \leq 10 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$

The graph in Fig. 5 shows the feasible region for the problem.

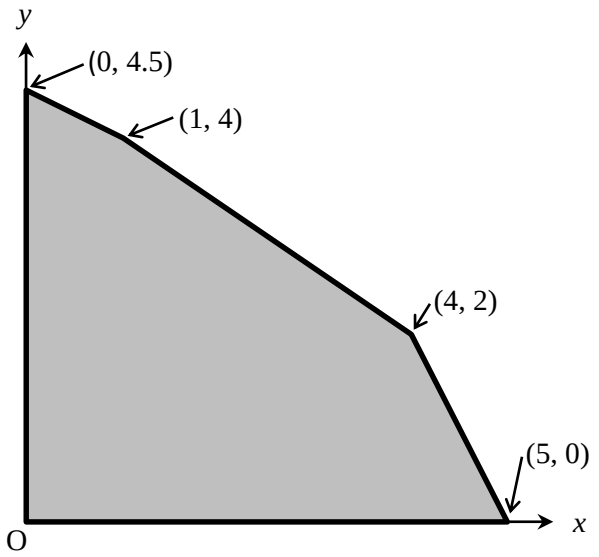


Fig. 5

(i) Use the graph to solve the LP problem. **[3]**

Chetan solves the problem using the simplex algorithm. His final tableau is shown below.

P	x	y	s_1	s_2	s_3	RHS
1	0	0	0	$\frac{1}{6}$	0	$\frac{7}{3}$
0	0	0	1	$-\frac{3}{4}$	$\frac{1}{4}$	1
0	1	0	0	$-\frac{1}{4}$	$\frac{3}{4}$	4
0	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	2

(ii) Interpret this tableau. **[3]**

(iii) (*A*) Perform another iteration using an entry in the s_3 column as the pivot element. **[2]**

(*B*) Comment on the result. **[1]**

6 Virginia is setting up an airline. She has a capital budget of \$500 million (\$500m) to buy aeroplanes. The capital budget cannot be used to pay running costs.

Large aeroplanes cost \$18m each and have a capacity of 250 passengers. Large aeroplanes have fixed costs of \$3m each per year and variable costs of \$2.90 per mile.

Medium aeroplanes cost \$15m each and have a capacity of 200 passengers. Medium aeroplanes have fixed costs of \$3m each per year and variable costs of \$2 per mile.

Small aeroplanes cost \$12m each and have a capacity of 150 passengers. Small aeroplanes have fixed costs of \$1.5m each per year and variable costs of \$2 per mile.

Virginia's company will fly transatlantic routes and domestic routes. The average distances and demands for these routes are as shown in the table below.

	Distance (miles)	Demand (passengers per year)
Transatlantic	5000	1 000 000
Domestic	1000	2 250 000

Each aeroplane will fly for 300 days a year. Each aeroplane can make two transatlantic flights per day, or four domestic flights per day.

Let NL be the number of large aeroplanes, TL the number of transatlantic flights per year using large aeroplanes and DL the number of domestic flights per year using large aeroplanes. Use similar variables for medium and small aeroplanes.

(i) Explain why the annual running cost of using large aeroplanes is given by $3\,000\,000\,NL + 14\,500\,TL + 2900\,DL$. **[3]**

(ii) The inequality $0.5\,TL + 0.25\,DL \leq 300\,NL$ models the availability of large aeroplanes. What does each side of the inequality represent? **[2]**

(iii) Virginia wishes to minimise her annual running cost. Formulate an LP to find how many aeroplanes of each type Virginia should buy so that she can satisfy demand within her capital budget.

- For each type of aeroplane you will need an availability inequality.
- For each distance category you will need an inequality to ensure that there is sufficient capacity.
- You will need an inequality to ensure that the capital budget is not exceeded. **[6]**

The LP is run in a spreadsheet LP solver and the following output is obtained.

Result: Solver found an integer solution within tolerance. All Constraints are satisfied.

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$K$6	objective	0	123845400

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$B$5	NL	0	6	Integer
$C$5	TL	0	162	Integer
$D$5	DL	0	6876	Integer
$E$5	NM	0	8	Integer
$F$5	TM	0	4797	Integer
$G$5	DM	0	6	Integer
$H$5	NS	0	3	Integer
$I$5	TS	0	1	Integer
$J$5	DS	0	3532	Integer

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$K$10	availability 3	-16.5	$K$10<=$L$10	Not Binding	16.5
$K$11	capacity 1	1000050	$K$11>=$L$11	Not Binding	50
$K$12	capacity 2	2250000	$K$12>=$L$12	Binding	0
$K$13	capital	264	$K$13<=$L$13	Not Binding	236
$K$8	availability 1	0	$K$8<=$L$8	Binding	0
$K$9	availability 2	0	$K$9<=$L$9	Binding	0
$B$5:$J$5=Integer					

(iv) Interpret the output to advise Virginia how many aeroplanes of each type she should buy. **[2]**

(v) Give two criticisms of the LP model. **[2]**

Virginia uses the model and the output from the LP solver to set the price of tickets. She decides to set the cost per mile at a lower rate for transatlantic flights than for domestic flights. She wishes her income from tickets to be at least $25 million more than her annual running costs.

(vi) Showing your calculations, suggest what price she might charge for tickets on transatlantic and domestic flights. **[4]**

END OF QUESTION PAPER

BLANK PAGE

BLANK PAGE

OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

AS Level Further Mathematics B (MEI)

Y413 Modelling with algorithms

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 12 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
U1	Mark for correct units
G1	Mark for a correct feature on a graph
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

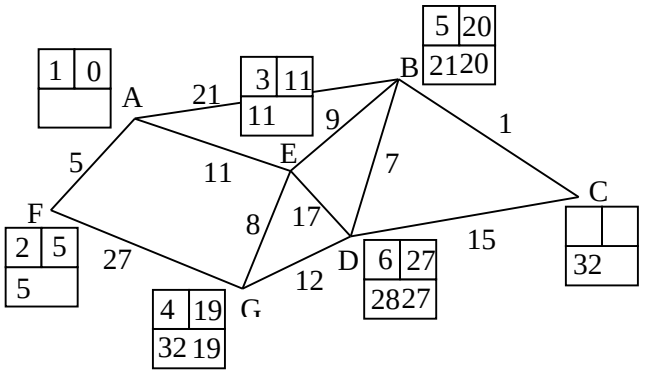
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

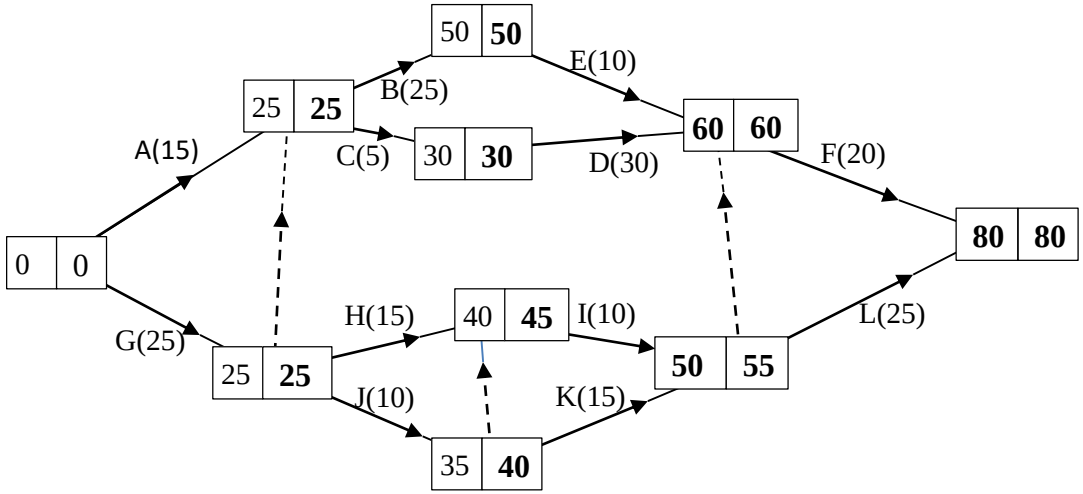
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

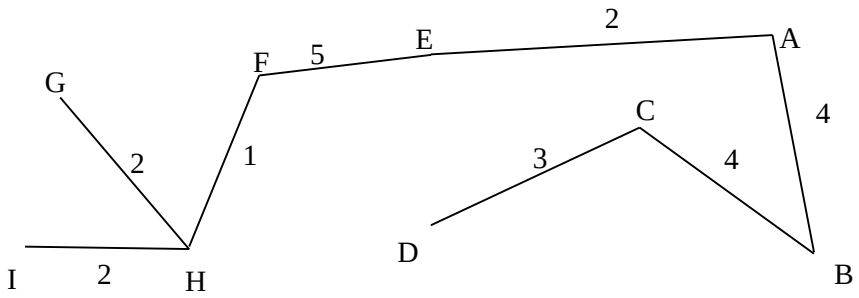
i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance
1			 AEBD 27	M1 A1 A1 A1 B1 B1 [6]	1.2 1.1a 1.1a 1.1 1.1 1.1	correct at G working values labels order of labelling
2	(i)		e.g. starting from left with smaller moving right: B67 A57 D73 C43 B67 D73 A57 C43 D73 B67 A57 C43	 B1 B1 B1 [3]	 1.1 1.1 1.1	
2	(ii)		6 comparisons + 3 comparisons to insert Ewan 9	M1 A1 [2]	1.1 1.1	either one as part of a sum
2	(iii)		Replace 4 by 5 in Steps 50 and Step 80.	B1 [1]	2.1	OR new Steps written out
2	(iv)		10	B1 [1]	2.2a	

Question			Answer	Marks	AOs	Guidance
3	(i)			**M1** **A1**	**1.1** **1.1**	Forward pass and attempt at backward pass All correct
			Minimum completion time = 80 minutes Critical activities: G, B, C, D, E, F Total floats: A 10 mins; H 5 mins; I 5 mins; J 5 mins; K 5 mins; L 5 mins	**B1** **A1** **A1** **[5]**	**1.1** **1.2** **1.2**	
3	(ii)		Constraint on C and H means C runs from 25 to 30 because C critical, H runs from 30 to 45 So I runs from 45 to 55 But E runs 50 – 60, so E and I overlap, which is not possible.	**E1** **E1** **E1** **[3]**	**1.1** **2.2a** **3.5a**	Or similar argument starting with E and I
3	(iii)		Minimum completion time 85 minutes	**B1** **[1]**	**1.1**	

Question			Answer	Marks	AOs	Guidance
4	(i)	(*A*)	Save most money when remaining network is a minimum spanning tree	E1	2.4	might be seen later
			E.g. Attempt at Kruskal, Prim graphical or Prim tabular or equivalent	M1	3.1b	May split into two networks and include bridge EF
			Detail shown: Kruskal, order in which choose cables. FH, {GH, HI, AE}, CD, {AB, BC}, EF Prim graphical, order in which choose cables. E.g. AE, AB, BC, CD, EF, FH, {HG, HI} Prim tabular, starting vertex and order in which connect vertices. E.g. A, E, B, C, D, F, H, {G, I}	A1	1.1	
			Minimum spanning tree shown  	A1	1.1	soi
			Remove GI, FG, EC, AC, DB	A1 [5]	1.1	
4	(i)	(*B*)	Removed GI (3), FG (3), EC (6), AC(7), DB (5)			
			Monthly saving (3+3+6+7+5) × 8 = £192	A1 [1]	3.2a	
4	(ii)		Only one cable can be removed to remain connected, and EF (the longest) can be removed	E1	3.1b	accept new minimum connector drawn
			saving in total length 3km	M1	1.1	
			so further reduction in maintenance costs of 3 × £8 = £24.	M1	3.2a	
			200 ÷ 24 = 8.333, during the 9th month	A1 [4]	1.1	Accept 8.3[333]

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>5</td><td>(i)</td><td></td><td>$(0, 4.5) \rightarrow 2.25$
$(1, 4) \rightarrow \frac{7}{3}$
$(4, 2) \rightarrow \frac{7}{3}$
$(5, 0) \rightarrow \frac{5}{3}$
So $\frac{7}{3}$ at either (1, 4) or (4, 2), or
anywhere on the line segment joining them.</td><td>M1

A1

B1

[3]</td><td>1.1

1.1

1.1</td><td></td></tr>
<tr><td>5</td><td>(ii)</td><td></td><td>$P = \frac{7}{3}$
…. at (4, 2)
$s_1 = 1 \quad s_2 = s_3 = 0$</td><td>B1

B1
B1
[3]</td><td>1.1

1.1
1.1</td><td></td></tr>
<tr><td>5</td><td>(iii)</td><td>(A)
<table><tr><th>P</th><th>x</th><th>y</th><th>s_1</th><th>s_2</th><th>s_3</th><th>RHS</th></tr><tr><td>1</td><td>0</td><td>0</td><td>0</td><td>$\frac{1}{6}$</td><td>0</td><td>$\frac{7}{3}$</td></tr><tr><td>0</td><td>0</td><td>0</td><td>4</td><td>−3</td><td>1</td><td>4</td></tr><tr><td>0</td><td>0</td><td>1</td><td>2</td><td>−1</td><td>0</td><td>4</td></tr><tr><td>0</td><td>1</td><td>0</td><td>−3</td><td>2</td><td>0</td><td>1</td></tr></table></td><td>M1

A1
[2]</td><td>1.1

1.1</td><td>pivot and pivot row

rest</td></tr>
</table>

Question			Answer	Marks	AOs	Guidance
5	(iii)	(*B*)	This represents the other optimal vertex, (1, 4)	B1 [1]	2.2a	
6	(i)		3 000 000 is fixed cost For variable cost $5000 \times 2.9 = 14\,500$. $1000 \times 2.9 = 2900$	B1 B1 B1 [3]	2.4 1.1 1.1	
6	(ii)		RHS = large plane days available. LHS = large plane days used	B1 B1 [2]	2.4 2.2a	
6	(iii)		Minimise $C = 3000000NL + 14500TL + 2900DL + 3000000NM + 10000TM + 2000DM + 1500000NS + 10000TS + 2000DS$ subject to $0.5TL + 0.25DL - 300NL \leq 0$ $0.5TM + 0.25DM - 300NM \leq 0$ $0.5TS + 0.25DS - 300NS \leq 0$ $250TL + 200TM + 150TS \geq 1000000$ $250DL + 200DM + 150DS \geq 2250000$ $18NL + 15NM + 12NS \leq 500$ All the variables are integers	B1 B1 B1 B1 B1 [6]	3.3 1.1 3.3 1.1 3.3 2.5	objective plane availability passenger demand purchase cost integer programming
6	(iv)		Virginia should buy 6 large planes, 8 medium planes and 3 small planes. Her annual running costs will be \$123 845 400	B1 B1 [2]	3.4 3.4	

Question			Answer	Marks	AOs	Guidance
6	(v)		Modelling assumptions may be unlikely e.g. • Every flight full to capacity • Scheduling issues • All transatlantic flights (for instance) 5000 miles • replacement costs	**B1** **B1** **[2]**	**3.5b** **3.5b**	two relevant correct statements
6	(vi)		Sensible attempt Solution e.g. domestic \$24, transatlantic \$96 showing … transatlantic < 5 × domestic because 5 ×24 = 120 income = 1 000 000 × \$96 + 2 250 000 × \$24 = \$150m and \$150m – \$123 845 400 > \$25m	**M1** **E1** **E1** **E1** **[4]**	**3.1b** **2.4** **2.2b** **3.2a**	

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	6	0	0	0	6
2i	3	0	0	0	3
2ii	2	0	0	0	2
2iii	0	1	0	0	1
2iv	0	1	0	0	1
3i	5	0	0	0	5
3ii	1	1	0	1	3
3iii	1	0	0	0	1
4iA	3	1	1	0	5
4iB	0	0	1	0	1
4ii	2	0	2	0	4
5i	3	0	0	0	3
5ii	3	0	0	0	3
5iiiA	2	0	0	0	2
5iiiB	0	1	0	0	1
6i	2	1	0	0	3
6ii	0	2	0	0	2
6iii	2	1	0	3	6
6iv	0	0	0	2	2
6v	0	0	0	2	2
6vi	0	2	2	0	4
Totals	35	11	6	8	60

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

BLANK PAGE

OCR
Oxford Cambridge and RSA

Accredited

AS Level Further Mathematics B (MEI)

Y413 Modelling with Algorithms

Printed Answer Booklet Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Question Paper Y413 (inserted)
- Formulae Further Mathematics B (MEI)

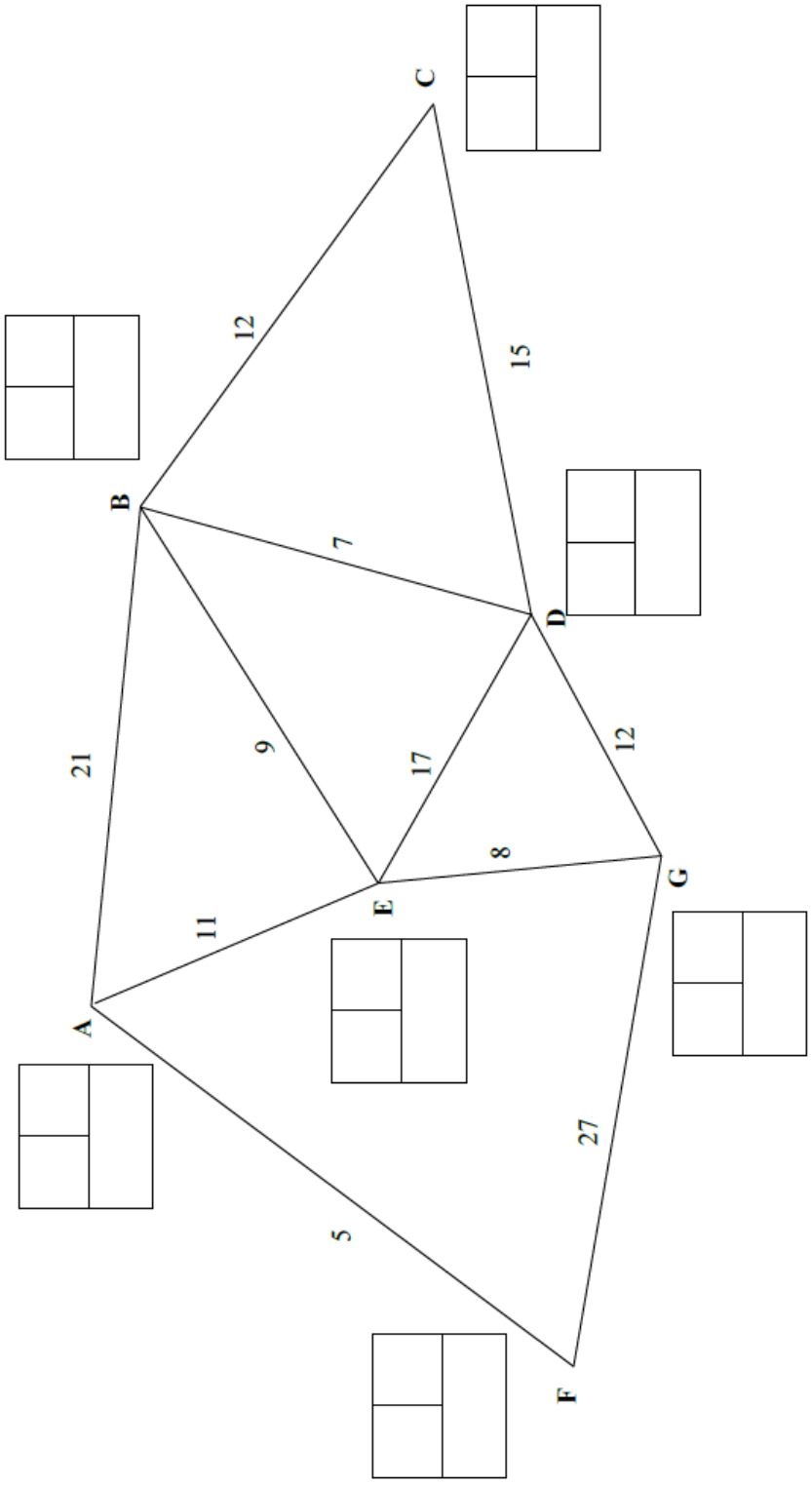
You may use:
- a scientific or graphical calculator

First name									
Last name									
Centre number						**Candidate number**				

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

1



A spare copy of the diagram for question 1 can be found on page 13

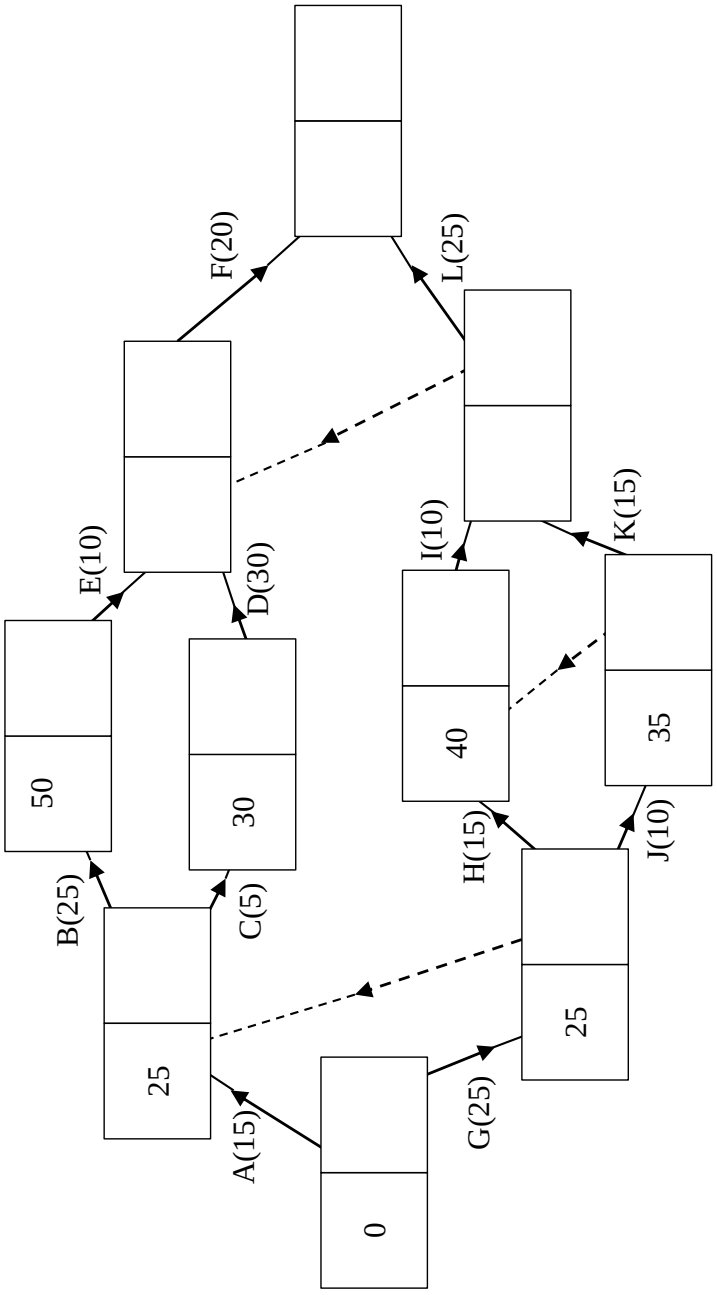
2 (i)

2 (ii)

2 (iii)

2 (iv)

3 (i)



A spare copy of the diagram for question 3 (i) can be found on page 14

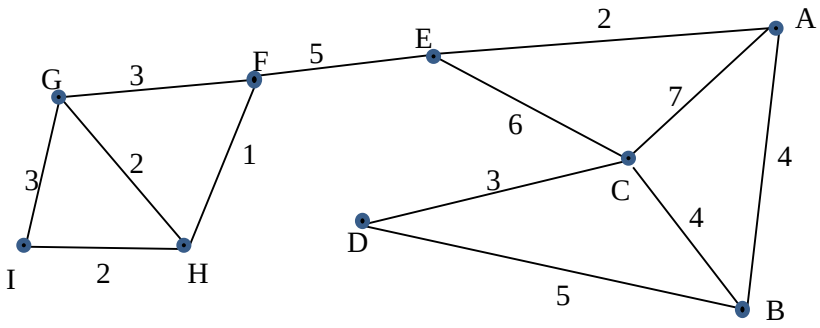
(answer space continued on next page)

3 (i)	**(continued)**
3 (ii)	
3(iii)	

DO NOT WRITE IN THIS SPACE

4 (i) (*A*)

	A	B	C	D	E	F	G	H	I
A		4	7		2				
B	4		4	5					
C	7	4		3	6				
D		5	3						
E	2		6			5			
F					5		3	1	
G						3		2	3
H						1	2		2
I							3	2	



(answer space continued on next page)

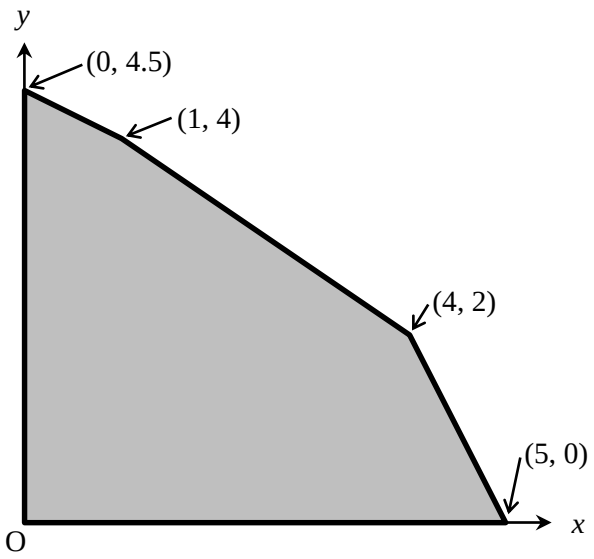
4 (i) (*A*) **(continued)**

4 (i) (*B*)

4 (ii)	

DO NOT WRITE IN THIS SPACE

5 (i)



5 (ii)

5 (iii) (*A*)

P	x	y	s_1	s_2	s_3	RHS
1	0	0	0	$\frac{1}{6}$	0	$\frac{7}{3}$
0	0	0	1	$-\frac{3}{4}$	$\frac{1}{4}$	1
0	1	0	0	$-\frac{1}{4}$	$\frac{3}{4}$	4
0	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	2

P	x	y	s_1	s_2	s_3	RHS

Spare copy of tableau

P	x	y	s_1	s_2	s_3	RHS

5 (iii) (*B*)

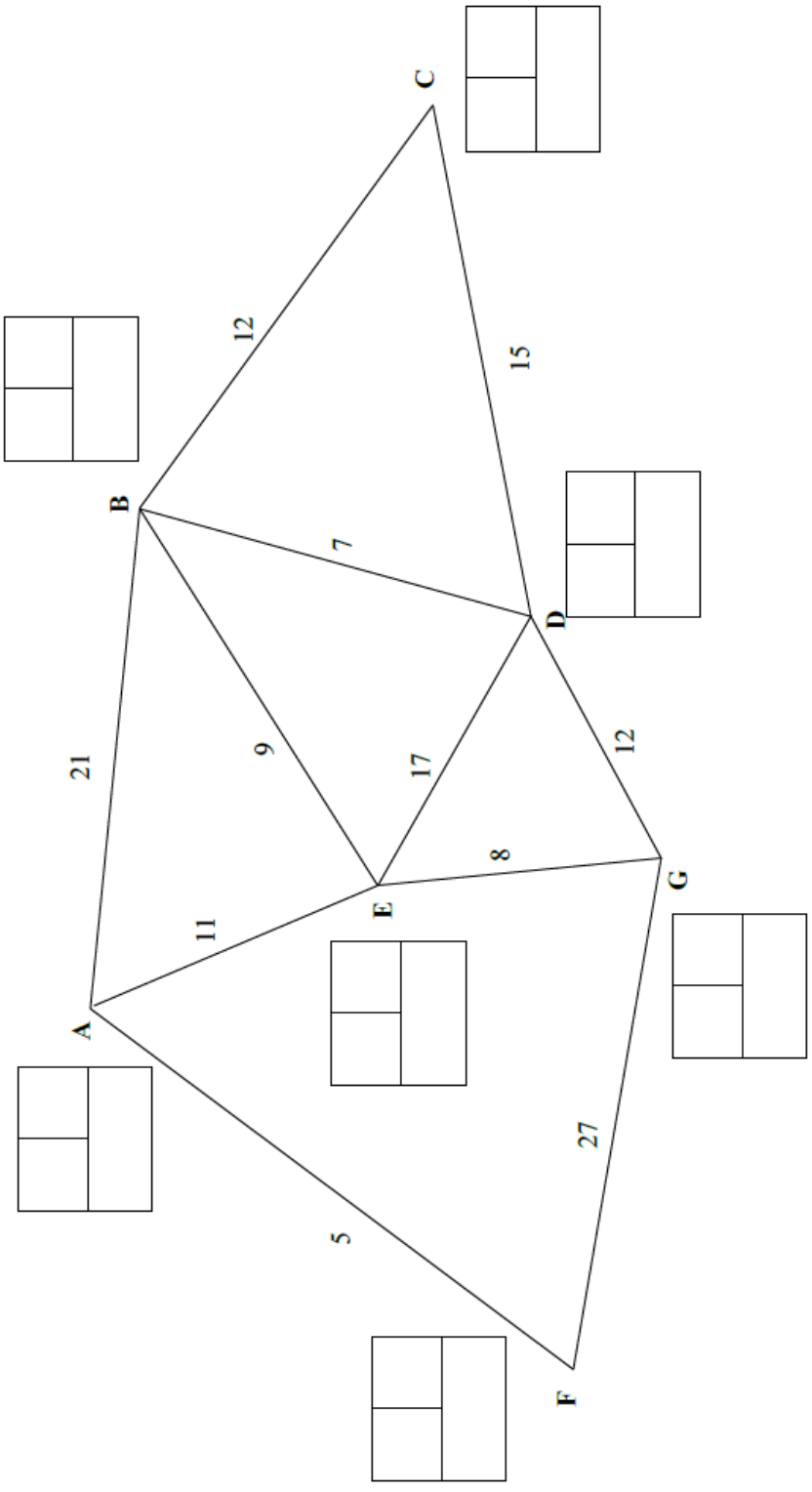
6 (i)

6 (ii)

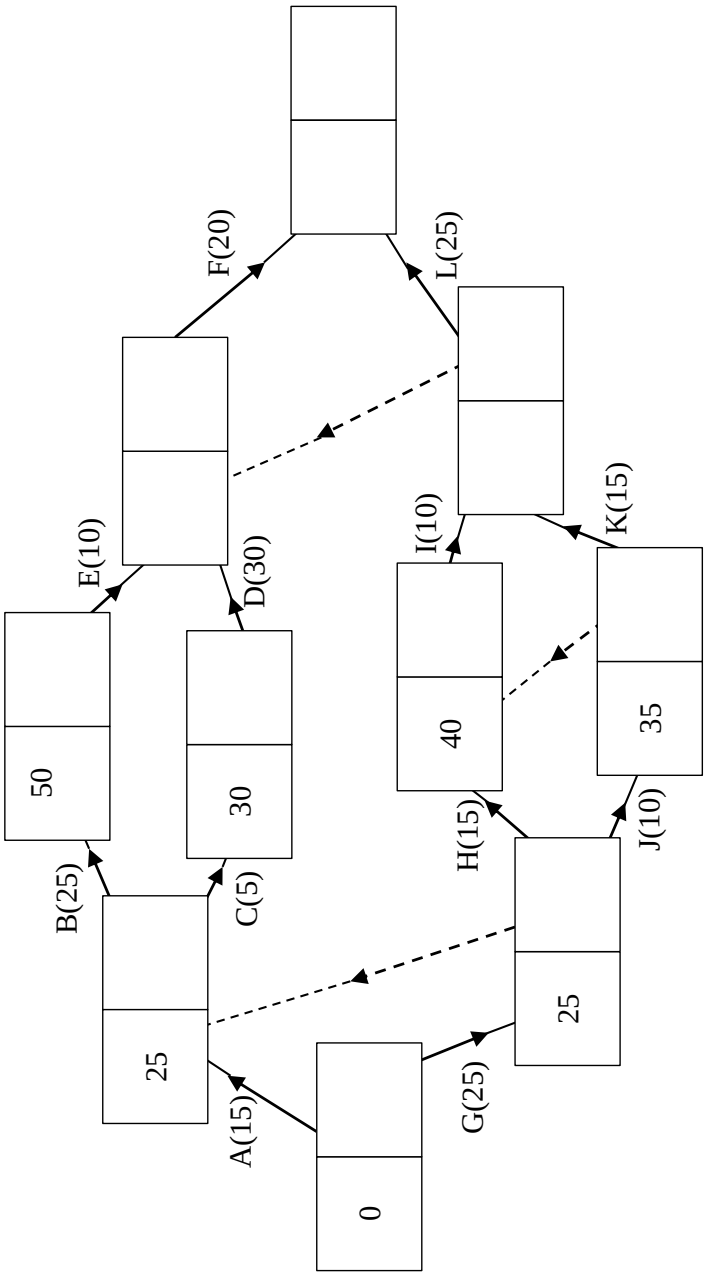
6 (iii)

6 (iv)	
6 (v)	
6 (vi)	

Spare copy of diagram for question 1



Spare copy of diagram for question 3 (i)



DO NOT WRITE ON THIS PAGE

DO NOT WRITE ON THIS PAGE