OCR
Oxford Cambridge and RSA

Accredited

A Level Further Mathematics B (MEI)

Y433 Modelling with Algorithms

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Answer **all** the questions.

1 The following instructions generate a sequence. It uses Floor(x), which is the largest integer less than or equal to x. For example Floor(8.95) = 8.

Step 10 Let $n = 1$
Step 20 Let $r = \sqrt{2}$
Step 30 Let $\mathrm{u}(n) = \mathrm{Floor}(n \times r)$
Step 40 Print u(n)
Step 50 Let $n = n + 1$
Step 60 Go to Step 30

(i) Write down the first five terms of the sequence. **[1]**

(ii) Amend the instructions to produce an algorithm to give the first five terms of the sequence only. **[2]**

The original instructions are amended by replacing Step 20, Step 30 and Step 40. The new instructions are as follows.

Step 10 Let $n = 1$
Step 20 Let $s = 2 + \sqrt{2}$
Step 30 Let $\mathrm{v}(n) = \mathrm{Floor}(n \times s)$
Step 40 Print v(n)
Step 50 Let $n = n + 1$
Step 60 Go to Step 30

These new instructions generate a new sequence.

(iii) Find the first five terms of this new sequence. **[1]**

It is conjectured that every positive integer m appears in exactly one of the two sequences u(n) and v(n).

(iv) Show that the conjecture is true for $m = 100$. **[3]**

2 Activity X is part of a large project. The project has been modelled by an activity network. Fig. 2 shows that part of the activity network relating to X.

The events at the start and end of activity X are shown, together with all activities starting and ending at these events. Activity durations are shown. Some early and late event times are shown; these have been calculated from the full activity network.

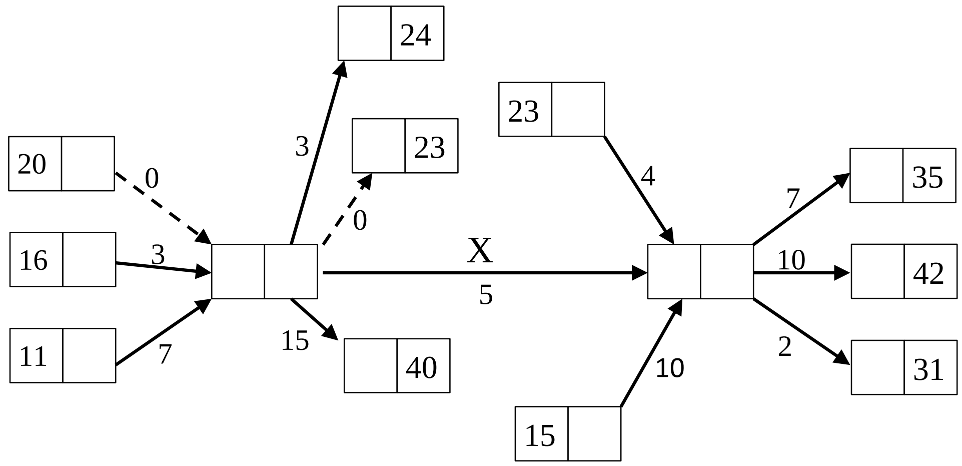


Fig. 2

Calculate the early and late times for the events at the start and end of activity X. **[4]**

3 Direct transport links between six cities A, B, C, D, E, F are shown in Fig. 3. The weights on the arcs are the times, in hours, for moving along those links.

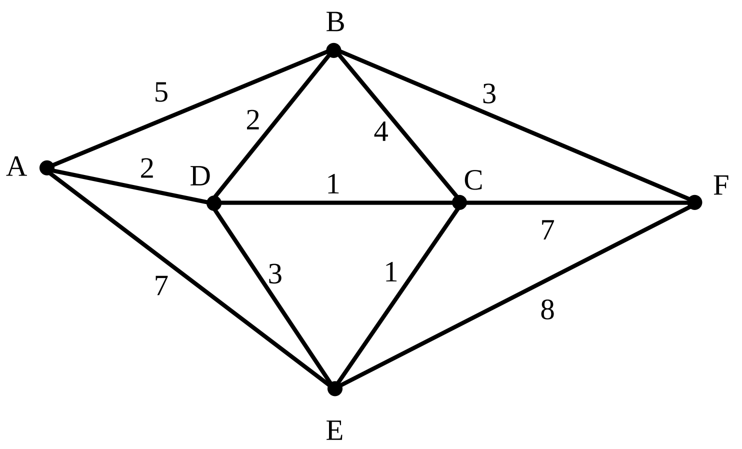


Fig. 3

The table below shows the complete set of shortest times between the cities. Some entries have been omitted.

	A	B	C	D	E	F
A	-	4	3	2	4	
B	4	-	3	2	4	
C	3	3	-	1	1	
D	2	2	1	-	2	
E	4	4	1	2	-	
F						-

(i) Use Dijkstra's algorithm to find the missing entries, and complete the table in the Printed Answer Booklet. **[3]**

A single application of Dijkstra's algorithm has complexity $O(n^2)$ where n is the number of vertices in the network. A computer program obtains a table of the complete set of shortest times by repeatedly applying Dijkstra's algorithm at each vertex. It takes the program t seconds to obtain a table with the complete set of shortest times for a network of p vertices.

(ii) Approximately how long will it take the program to carry out the task for a network of q vertices? Explain your reasoning. **[2]**

(iii) Explain why your answer to part **(ii)** is only an approximation. **[1]**

4 A list of n numbers is sorted by making passes through an algorithm as follows.

A pass consists of the following.

Compare the first and second number. If necessary, swap them so that the first number is less than or equal to the second number.
Compare the new second and third number. If necessary, swap them so that the second number is less than or equal to the third number.
Compare the third and fourth number. If necessary, swap them so that the third number is less than or equal to the fourth number.
…
Compare the $(n-1)$ th and n th number. If necessary, swap them so that the $(n-1)$ th number is less than or equal to the n th number.

Repeat this until a pass occurs with no swaps.

The algorithm is applied to the list of 10 numbers below.

30 29 28 27 26 25 24 23 22 21

(i) After one pass, in what position is
- the largest number,
- the smallest number? **[2]**

The algorithm is applied to a list of n numbers, with the largest number at the beginning of the list (in position 1) and the smallest at the end (in position n).

(ii) How many comparisons are made in sorting this list? **[4]**

The algorithm is applied to a list of n numbers. Nothing is known about the position of the numbers before the algorithm is applied.

(iii) What are the minimum and maximum number of comparisons which might be required to apply the algorithm? **[2]**

5 A network has 10 vertices, A to J. The table below shows the distance between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I	J
A			8	4						
B					10				7	
C	8			6			12			
D	4		6					6		7
E		10				8			9	
F					8				8	
G			12					6		9
H				6			6			7
I		7			9	8				
J				7			9	7		

(i) Apply the tabular form of Prim's algorithm to the network, starting at vertex A. **[3]**

(ii) Explain how the table shows that the algorithm terminates before connecting all the vertices. **[1]**

(iii) (*A*) Draw a minimum connector for the vertices which are connected to A. **[1]**

(*B*) Give the total length of the minimum connector in part **(iii)** (*A*). **[1]**

(iv) (*A*) Draw the network for the vertices not connected to A. **[1]**

(*B*) Draw a minimum connector for these vertices. **[1]**

(*C*) Give the total length of the minimum connector in part **(iv)** (*B*). **[1]**

A single arc connecting A and B is added to the original network.

(v) Explain why a minimum connector for the new network is given by the two minimum connectors from parts **(iii)** and **(iv)** together with the new arc. **[2]**

As well as the arc connecting A and B, another arc connecting C and E is added to the network. A tree is formed by the minimum connectors from parts **(iii)** and **(iv)**, together with the shorter of the two arcs AB and CE.

(vi) Determine whether it is always true, sometimes true or never true that this tree is a minimum connector for the new network. Give reasons for your answer. **[3]**

6 Three liquid medicines, X, Y and Z, are to be manufactured. All the medicines require ingredients A, B, C and D which are in limited supply. The table below shows how many grams of each ingredient are required for one litre of each medicine. It also shows how much of each ingredient is available.

	A	B	C	D
Each litre of X requires	2	0	2	4
Each litre of Y requires	5	2	4	3
Each litre of Z requires	3	1	2	2
Amount, in grams, of each ingredient available	20	10	70	30

When the medicines are sold, the profits are £5 per litre of X manufactured, £2 per litre of Y and £3 per litre of Z.

(i) Formulate an LP to maximise the total profit subject to the constraints imposed by the availability of the ingredients. Use x as the number of litres of X, y as the number of litres of Y and z as the number of litres of Z. **[3]**

The simplex algorithm is used to solve this LP. After the first iteration the tableau below is produced.

P	x	y	z	s_1	s_2	s_3	s_4	RHS
1	0	1.75	–0.5	0	0	0	1.25	37.5
0	0	3.5	2	1	0	0	–0.5	5
0	0	2	1	0	1	0	0	10
0	0	2.5	1	0	0	1	–0.5	55
0	1	0.75	0.5	0	0	0	0.25	7.5

(ii) (*A*) Perform a second iteration. **[2]**

(*B*) Give the maximum profit, and the number of litres of X, Y and Z which should be manufactured to achieve this profit. **[1]**

(iii) An extra constraint is imposed by a contract to supply at least 5 litres of Y. Produce an initial tableau which could be used to solve this new problem by using the two-stage simplex method. **[3]**

7 A series of drainage pipes allows rainwater to flow downhill under gravity from a field on a hillside to a river in a valley below. The directed network in Fig. 7 models this system, with S representing the field and T the river. The weights represent the capacities of the pipes, in litres per minute.

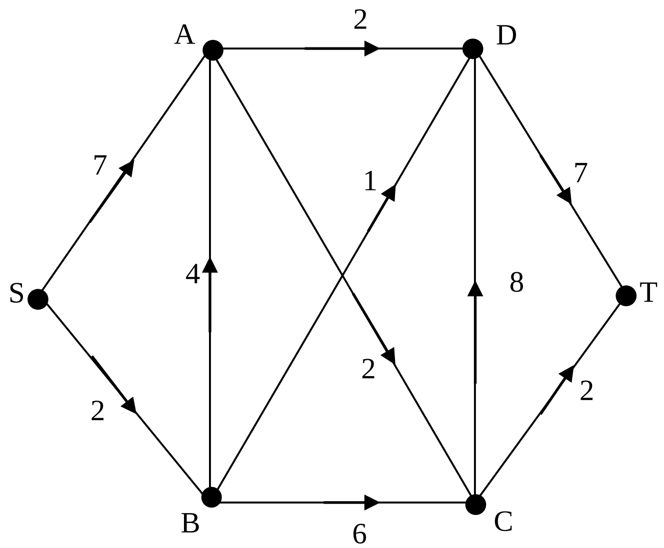


Fig. 7

(i) What aspect of the network models the fact that the point represented by C is higher up the hillside than the point represented by D? **[1]**

The following LP formulation finds the maximum flow through the network. The variable SA represents the flow along the arc from vertex S to vertex A, and similarly for other arcs.

Maximise $\mathrm{SA}+\mathrm{SB}$

subject to

$$\mathrm{SA}+\mathrm{BA}-\mathrm{AD}-\mathrm{AC}=0$$
$$\mathrm{SB}-\mathrm{BA}-\mathrm{BC}-\mathrm{BD}=0$$
$$\mathrm{BC}+\mathrm{AC}-\mathrm{CD}-\mathrm{CT}=0$$
$$\mathrm{AD}+\mathrm{BD}-\mathrm{DT}+\mathrm{CD}=0$$
$$\mathrm{SA}\le 7$$
$$\mathrm{AD}\le 2$$
$$\mathrm{BC}\le 6$$
$$\mathrm{CT}\le 2$$
$$\mathrm{SB}\le 2$$
$$\mathrm{AC}\le 2$$
$$\mathrm{CD}\le 8$$
$$\mathrm{BA}\le 4$$
$$\mathrm{BD}\le 1$$
$$\mathrm{DT}\le 7$$

(ii) Explain the purpose of each of the following lines from the LP formulation.

(*A*) Maximise $\mathrm{SA}+\mathrm{SB}$ **[2]**

(*B*) $\mathrm{SA}+\mathrm{BA}-\mathrm{AD}-\mathrm{AC}=0$ **[2]**

(*C*) $\mathrm{CT}\leq 2$ **[1]**

The LP is run in an Online LP Solver and in a Spreadsheet LP Solver, and the following outputs are obtained.

Online LP Solver

Objective	6
Variable	Value
SA	4
SB	2
BA	0
AD	2
AC	2
BD	1
BC	1
CD	3
DT	6
CT	0

This question is continued on the next page

Spreadsheet LP Solver

Result: Solver found a solution. All Constraints and optimality conditions are satisfied.

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$M$5	capacity	0	6

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$4	SA	0	4	Contin
$D$4	SB	0	2	Contin
$E$4	BA	0	0	Contin
$F$4	AD	0	2	Contin
$G$4	AC	0	2	Contin
$H$4	BD	0	1	Contin
$I$4	BC	0	1	Contin
$J$4	CD	0	1	Contin
$K$4	DT	0	4	Contin
$L$4	CT	0	2	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$M$10	SA<=7	4	$M$10<=$N$10	Not Binding	3
$M$11	AD <=2	2	$M$11<=$N$11	Binding	0
$M$12	BC<=6	1	$M$12<=$N$12	Not Binding	5
$M$13	CT<=2	2	$M$13<=$N$13	Binding	0
$M$14	SB <=2	2	$M$14<=$N$14	Binding	0
$M$15	AC<=2	2	$M$15<=$N$15	Binding	0
$M$16	CD<=8	1	$M$16<=$N$16	Not Binding	7
$M$17	BA<=4	0	$M$17<=$N$17	Not Binding	4
$M$18	BD<=1	1	$M$18<=$N$18	Binding	0
$M$19	DT<=7	4	$M$19<=$N$19	Not Binding	3
$M$6	SA+BA-AD-AC=0	0	$M$6=$N$6	Binding	0
$M$7	SB-BA-BC-BD=0	0	$M$7=$N$7	Binding	0
$M$8	BC+AC-CD-CT=0	0	$M$8=$N$8	Binding	0
$M$9	AD+BD-DT+CD=0	0	$M$9=$N$9	Binding	0

(iii) (*A*) For each solver, interpret the output to give a maximum flow on the diagram in the Printed Answer Booklet. **[2]**

(*B*) State the maximum capacity of the network. **[1]**

(iv) Give a cut with capacity equal to the maximum flow. **[1]**

(v) In order to save money it is proposed to shut down as many drainage pipes as possible, while still allowing the maximum flow through the system. Show how this can be done. **[2]**

END OF QUESTION PAPER

Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics B (MEI)

Y433 Modelling with Algorithms

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

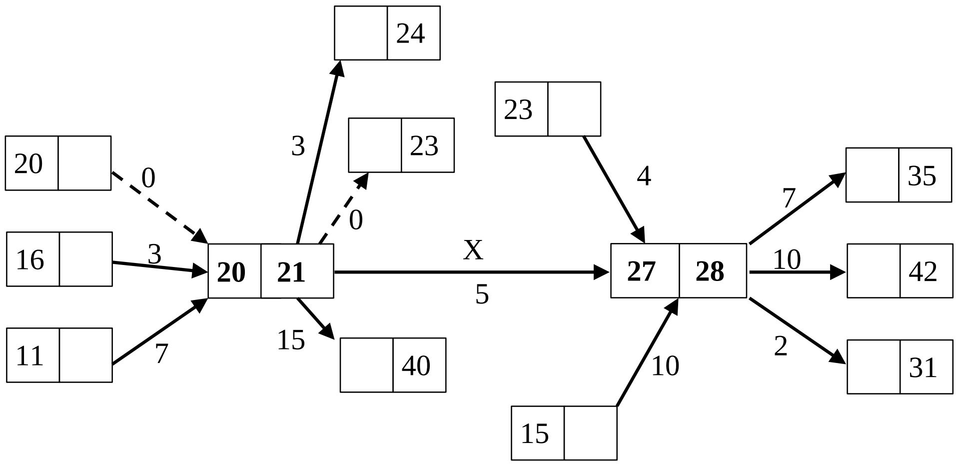
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

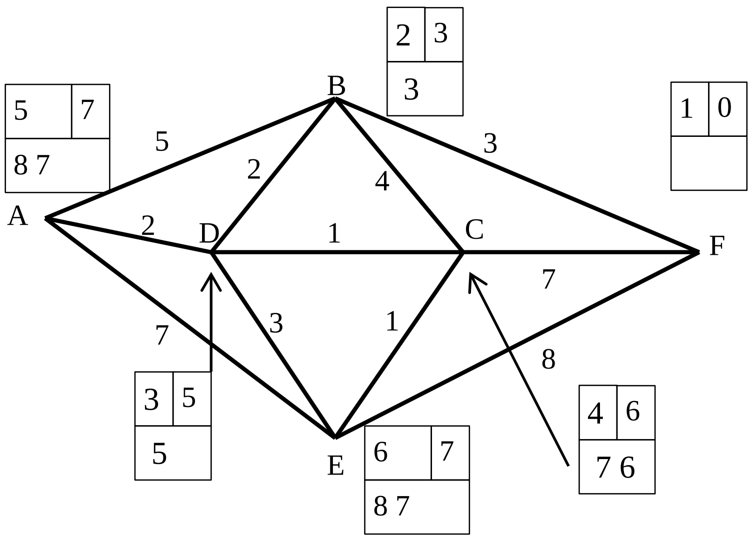
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like

Question			Answer	Marks	AOs	Guidance	
1	(i)		1,2,4,5,7	B1 [1]	1.1		
1	(ii)		E.g. Step 60 If $n < 6$ then go to Step 30 Step 70 Stop	M1 A1 [2]	1.1 1.2	Stopping condition… …in the right place and gives 5 terms	
1	(iii)		3,6,10,13,17	B1 [1]	1.1		
1	(iv)		u(71) = 100 v(29)=99, v(30) = 102 100 is in u(n) but not v(n), so true for m = 100.	B1 B1 E1 [3]	3.2a 1.1 2.1		
2				M1 A1 M1 A1 [4]	1.1a 1.1 1.1a 1.1	Forward pass Backward pass	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th colspan="2">Guidance</th></tr>
<tr><td>3</td><td>(i)</td><td></td><td>


<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr>
<tr><td>A</td><td>-</td><td>4</td><td>3</td><td>2</td><td>4</td><td>7</td></tr>
<tr><td>B</td><td>4</td><td>-</td><td>3</td><td>2</td><td>4</td><td>3</td></tr>
<tr><td>C</td><td>3</td><td>3</td><td>-</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td>D</td><td>2</td><td>2</td><td>1</td><td>-</td><td>2</td><td>5</td></tr>
<tr><td>E</td><td>4</td><td>4</td><td>1</td><td>2</td><td>-</td><td>7</td></tr>
<tr><td>F</td><td>7</td><td>3</td><td>6</td><td>5</td><td>7</td><td>-</td></tr>
</table>
</td><td>M1

A1

B1

[3]</td><td>1.1a

1.1

1.1</td><td>Dijkstra, starting from F
All correct

All correct</td><td>Order of labelling on A and E could be reversed</td></tr>
<tr><td>3</td><td>(ii)</td><td></td><td>Repeatedly applying Dijkstra will be $O(n^3)$
$\left(\frac{q}{p}\right)^3 t$</td><td>E1
B1

[2]</td><td>2.4
3.3</td><td></td><td></td></tr>
<tr><td>3</td><td>(iii)</td><td></td><td>$O(n^3)$ does not mean proportional to n^3, which is the assumption behind answer to (ii).</td><td>E1

[1]</td><td>3.5b</td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
4	(i)		Largest (30) in 10th/last place Smallest (21) in 9th/penultimate place	**B1** **B1** **[2]**	**1.1** **1.1**		
4	(ii)		Each pass moves the smallest number 1 to left, so need $n - 1$ passes to get smallest number to left One checking pass with no swaps, so n passes altogether $n - 1$ comparisons in one pass $n(n - 1)$ comparisons altogether	**E1** **E1** **M1** **A1** **[4]**	**2.1** **2.4** **1.1** **2.2a**		
4	(iii)		$n - 1$ $n(n - 1)$	**B1** **B1** **[2]**	**1.1** **2.2a**		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th colspan="2">Guidance</th></tr>
<tr><td>5</td><td>(i)</td><td></td><td>
<table>
<tr><td></td><td>1</td><td></td><td>3</td><td>2</td><td></td><td></td><td>5</td><td>4</td><td></td><td>6</td></tr>
<tr><td></td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td></tr>
<tr><td>A</td><td></td><td></td><td>~~8~~</td><td>~~4~~</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>B</td><td></td><td></td><td></td><td></td><td>10</td><td></td><td></td><td></td><td>7</td><td></td></tr>
<tr><td>C</td><td>~~8~~</td><td></td><td></td><td>6</td><td></td><td></td><td>~~12~~</td><td></td><td></td><td></td></tr>
<tr><td>D</td><td>4</td><td></td><td>~~6~~</td><td></td><td></td><td></td><td></td><td>~~6~~</td><td></td><td>~~7~~</td></tr>
<tr><td>E</td><td></td><td>10</td><td></td><td></td><td></td><td>8</td><td></td><td></td><td>9</td><td></td></tr>
<tr><td>F</td><td></td><td></td><td></td><td></td><td>8</td><td></td><td></td><td></td><td>8</td><td></td></tr>
<tr><td>G</td><td></td><td></td><td>~~12~~</td><td></td><td></td><td></td><td></td><td>6</td><td></td><td>~~9~~</td></tr>
<tr><td>H</td><td></td><td></td><td></td><td>6</td><td></td><td></td><td>~~6~~</td><td></td><td></td><td>~~7~~</td></tr>
<tr><td>I</td><td></td><td>7</td><td></td><td></td><td>9</td><td>8</td><td></td><td></td><td></td><td></td></tr>
<tr><td>J</td><td></td><td></td><td></td><td>~~7~~</td><td></td><td></td><td>~~9~~</td><td>7</td><td></td><td></td></tr>
</table>
</td><td>M1

M1

A1

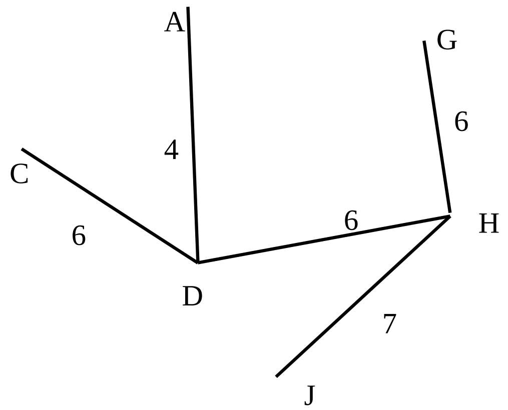
[3]</td><td>1.1

1.1

1.1</td><td>Choosing 4 in column A
Selecting at least two entries in column D
All correct

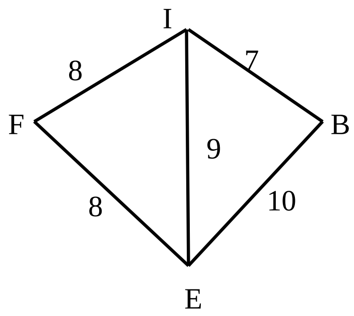
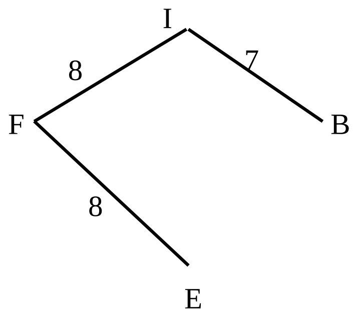
Order could be ADHCGJ or ADHGCJ

JD replacing JH is an alternative solution</td><td></td></tr>
<tr><td>5</td><td>(ii)</td><td></td><td>Algorithm terminates as no remaining values in cols
A C D G H J</td><td>E1

[1]</td><td>2.2a</td><td></td><td></td></tr>
<tr><td>5</td><td>(iii)</td><td>(*A*)</td><td></td><td>B1

[1]</td><td>1.1</td><td>Connector

JD replacing JH is an alternative solution</td><td></td></tr>
<tr><td>5</td><td>(iii)</td><td>(*B*)</td><td>Length of minimum connector = 29</td><td>B1
[1]</td><td>1.1</td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance	
5	**(iv)**	(*A*)		**B1** **[1]**	**1.1**	Network	
5	**(iv)**	(*B*)		**B1** **[1]**	**1.2**	Minimum connector	
5	**(iv)**	(*C*)	Length of minimum connector = 23	**B1** **[1]**	**1.1**		
5	**(v)**		Arc AB must be in minimum connector otherwise not connected, and AB cannot be part of a cycle. If a shorter spanning tree than the one suggested is possible then there will be a shorter minimum connector for one or both of ACGHJ, BEFI	**E1** **E1** **[2]**	**2.1** **2.3**		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th colspan="2">Guidance</th></tr>
<tr><td>5</td><td>(vi)</td><td></td><td>E.g. If arc length AB = 1 and arc length CE = 100 [then applying Kruskal] will produce the suggested minimum connector.
E.g. If arc length AB = arc length CE = 1 [then applying Kruskal] will produce a minimum connector with both of AB and CE in it.
So sometimes true</td><td>E1

E1

E1 dep
[3]</td><td>3.1a

2.4

2.2a</td><td>Example of true

Example when false</td><td></td></tr>
<tr><td>6</td><td>(i)</td><td></td><td>Maximise $5x + 2y + 3z$
subject to $2x+5y+3z \le 20$
$2y+ z \le 10$
$2x+4y+2z \le 70$
$4x+3y+2z \le 30$
$x, y, z \ge 0$</td><td>M1
A2

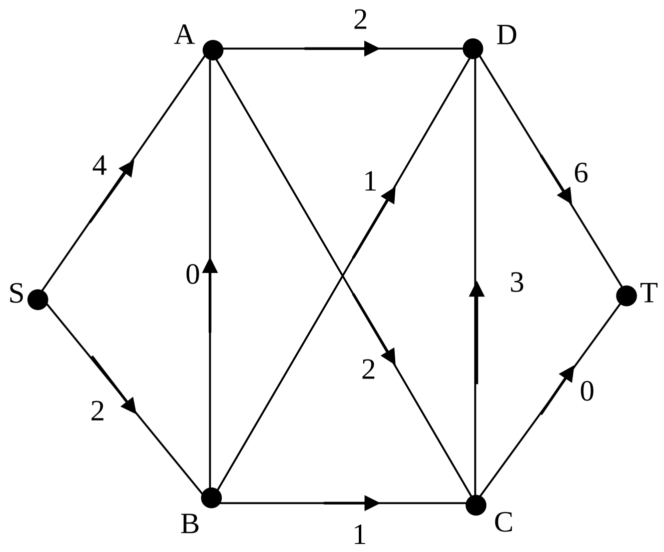
[3]</td><td>3.3
3.3
1.1</td><td>objective
constraints
-1 each error</td><td></td></tr>
<tr><td>6</td><td>(ii)</td><td>(A)</td><td><table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>RHS</th></tr><tr><td>1</td><td>0</td><td>2.625</td><td>0</td><td>0.25</td><td>0</td><td>0</td><td>1.125</td><td>38.75</td></tr><tr><td>0</td><td>0</td><td>1.75</td><td>1</td><td>0.5</td><td>0</td><td>0</td><td>–0.25</td><td>2.5</td></tr><tr><td>0</td><td>0</td><td>0.25</td><td>0</td><td>–0.5</td><td>1</td><td>0</td><td>0.25</td><td>7.5</td></tr><tr><td>0</td><td>0</td><td>0.75</td><td>0</td><td>–0.5</td><td>0</td><td>1</td><td>–0.25</td><td>52.5</td></tr><tr><td>0</td><td>1</td><td>– 0.125</td><td>0</td><td>–0.25</td><td>0</td><td>0</td><td>0.375</td><td>6.25</td></tr></table></td><td>M1
A1

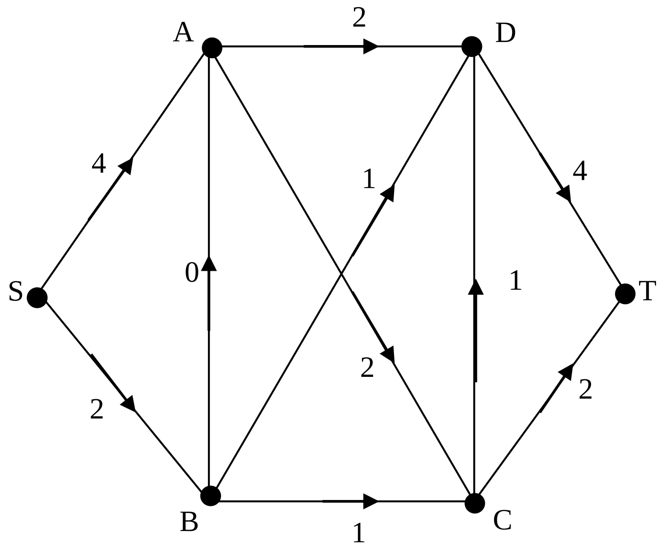
[2]</td><td>3.4
1.1</td><td>Pivot
all correct</td><td></td></tr>
<tr><td>6</td><td>(ii)</td><td>(B)</td><td>Maximum profit of £38.75 manufacturing 6.25 litres of X, none of Y and 2.5 litres of Z.</td><td>B1

[1]</td><td>3.2a</td><td></td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th>Guidance</th></tr>
<tr><td>6</td><td>(iii)</td><td></td><td>
<table>
<tr><th>Q</th><th>P</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>s_5</th><th>a_5</th><th>RHS</th></tr>
<tr><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>–1</td><td>0</td><td>5</td></tr>
<tr><td>0</td><td>1</td><td>–5</td><td>–2</td><td>–3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>–1</td><td>1</td><td>5</td></tr>
<tr><td>0</td><td>0</td><td>2</td><td>5</td><td>3</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>20</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>2</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>10</td></tr>
<tr><td>0</td><td>0</td><td>2</td><td>4</td><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>70</td></tr>
<tr><td>0</td><td>0</td><td>4</td><td>3</td><td>2</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>30</td></tr>
</table>
</td><td>B1
B1
B1

[3]</td><td>3.3
3.5c
1.1</td><td>Surplus
Additional variable
New objective</td></tr>
<tr><td>7</td><td>(i)</td><td></td><td>Flow is in direction C to D.</td><td>E1
[1]</td><td>3.3</td><td></td></tr>
<tr><td>7</td><td>(ii)</td><td>(A)</td><td>SA+ SB is the total flow leaving S
This line maximises the total flow in the network, so finds the capacity.</td><td>E1
E1
[2]</td><td>1.2
3.3</td><td></td></tr>
<tr><td></td><td></td><td>(B)</td><td>SA + BA is the flow into vertex A. AD + AC is the flow out of vertex A
This line achieves: flow into A = flow out of A</td><td>E1
E1
[2]</td><td>1.1
3.3</td><td></td></tr>
<tr><td></td><td></td><td>(C)</td><td>This constrains the flow from C to T to be not greater than its capacity of 2.</td><td>E1
[1]</td><td>2.5</td><td></td></tr>
</table>

Question			Answer	Marks	AOs	Guidance
7	(iii)	(*A*)	 Online LP Solver	B1	3.4	

Question			Answer	Marks	AOs	Guidance	
			 Spreadsheet LP Solver	B1 [2]	3.4		
7	**(iii)**	(*B*)	The maximum capacity of the network is 6.	**B1** **[1]**	**3.2a**		
7	**(iv)**		{S, A} {B, C, D, T}	**B1** **[1]**	**3.4**		
7	**(v)**		AB and CT can be shut with flow as online solver Also BD can be shut – increase BC, CD, DT by 1	**B1** **B1** **[2]**	**3.1b** **3.2a**	Any 2 shut All three shut	

Question	AO1	AO2	AO3(PS)	AO3(M)	Totals
1i	1	0	0	0	1
1ii	2	0	0	0	2
1iii	1	0	0	0	1
1iv	1	1	1	0	3
2	4	0	0	0	4
3i	3	0	0	0	3
3ii	0	1	0	1	2
3iii	0	0	0	1	1
4i	2	0	0	0	2
4ii	1	3	0	0	4
4iii	1	1	0	0	2
5i	3	0	0	0	3
5ii	0	1	0	0	1
5iiiA	1	0	0	0	1
5iiiB	1	0	0	0	1
5ivA	1	0	0	0	1
5ivB	1	0	0	0	1
5ivC	1	0	0	0	1
5v	0	2	0	0	2
5vi	0	2	1	0	3
6i	1	0	0	2	3
6iiA	1	0	0	1	2
6iiB	0	0	1	0	1
6iii	1	0	0	2	3
7i	0	0	0	1	1
7ii	2	1	0	2	5
7iiiA	0	0	0	2	2
7iiiB	0	0	1	0	1
7iv	0	0	0	1	1
7v	0	0	2	0	2
Totals	29	12	6	13	60

Summary of Updates

Date	Version	Change
October 2019	2	Amendments to the front cover rubric instructions to candidates

BLANK PAGE

Accredited

A Level Further Mathematics B (MEI)

Y433 Modelling with Algorithms

Printed Answer Booklet — Version 2

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

You must have:
- Question Paper Y433 (inserted)
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

First name	
Last name	

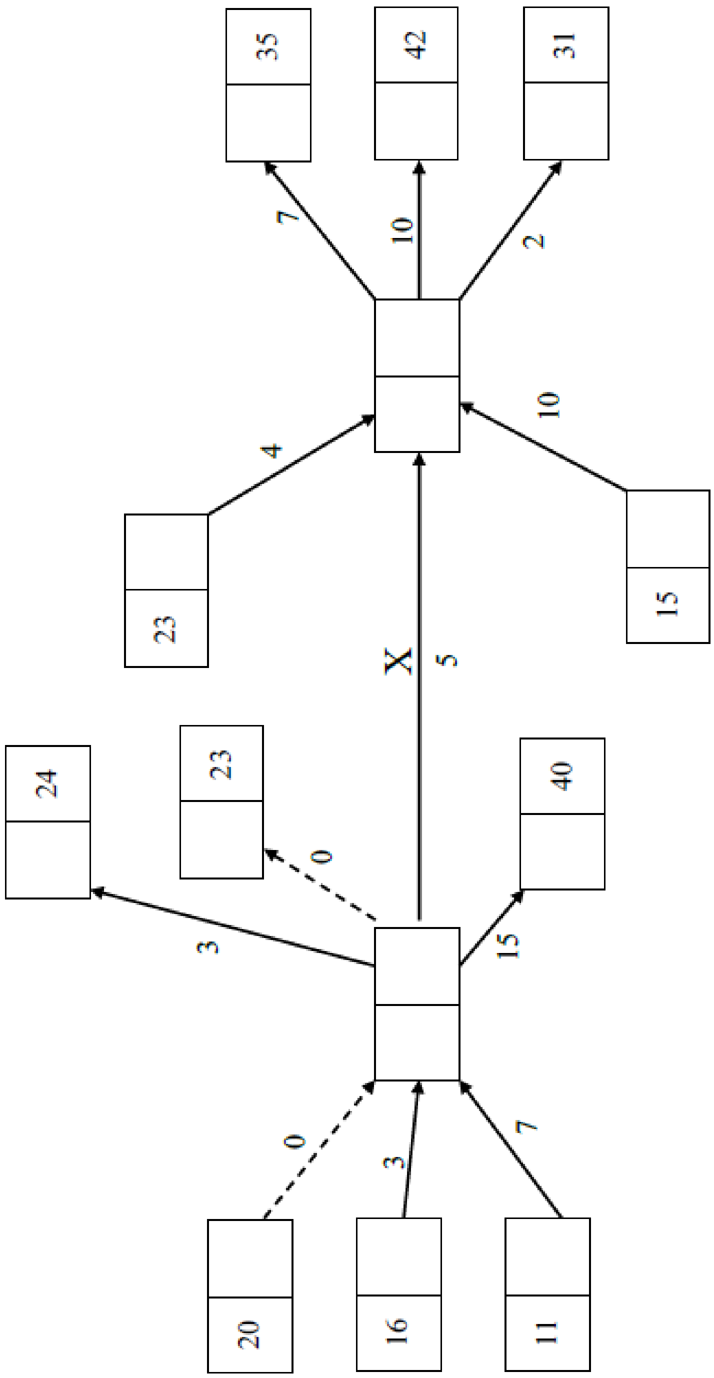
Centre number						Candidate number				

INSTRUCTIONS
- The Question Paper will be found inside the Printed Answer Booklet.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

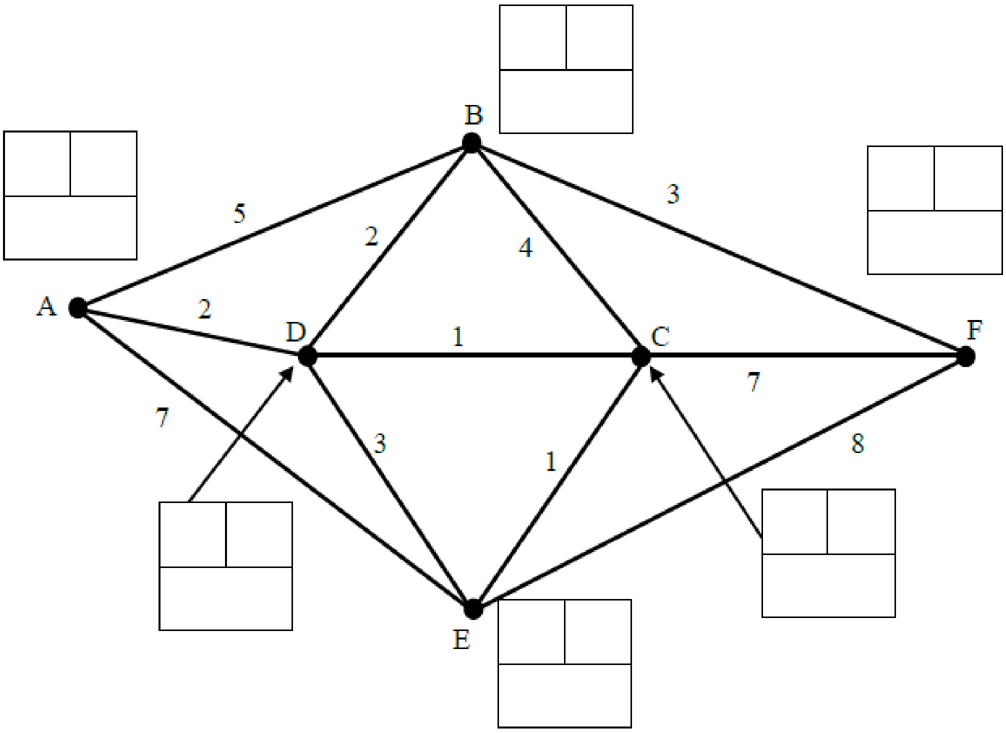
1 (i)	
1 (ii)	
1 (iii)	
1 (iv)	

2



A spare copy of the diagram for question 2 can be found on page 14

3 (i)



	A	B	C	D	E	F
A	-	4	3	2	4	
B	4	-	3	2	4	
C	3	3	-	1	1	
D	2	2	1	-	2	
E	4	4	1	2	-	
F						-

A spare copy of the diagram and table for question 3 (i) can be found on page 15

3 (ii)	
3 (iii)	

DO NOT WRITE IN THIS SPACE

4 (i)

4 (ii)

4 (iii)

5 (i)

	A	B	C	D	E	F	G	H	I	J
A			8	4						
B					10				7	
C	8			6			12			
D	4		6					6		7
E		10				8			9	
F					8				8	
G			12					6		9
H				6			6			7
I		7			9	8				
J				7			9	7		

A spare copy of the table for question 5 (i) can be found on page 16

5 (ii)

5 (iii) (*A*)

5 (iii) (*B*)

5 (iv) (*A*)	
5 (iv) (*B*)	
5 (iv) (*C*)	
5 (v)	

5 (vi)	
6 (i)	

6 (ii) (*A*)

[You may not need to use all of these tableaux]

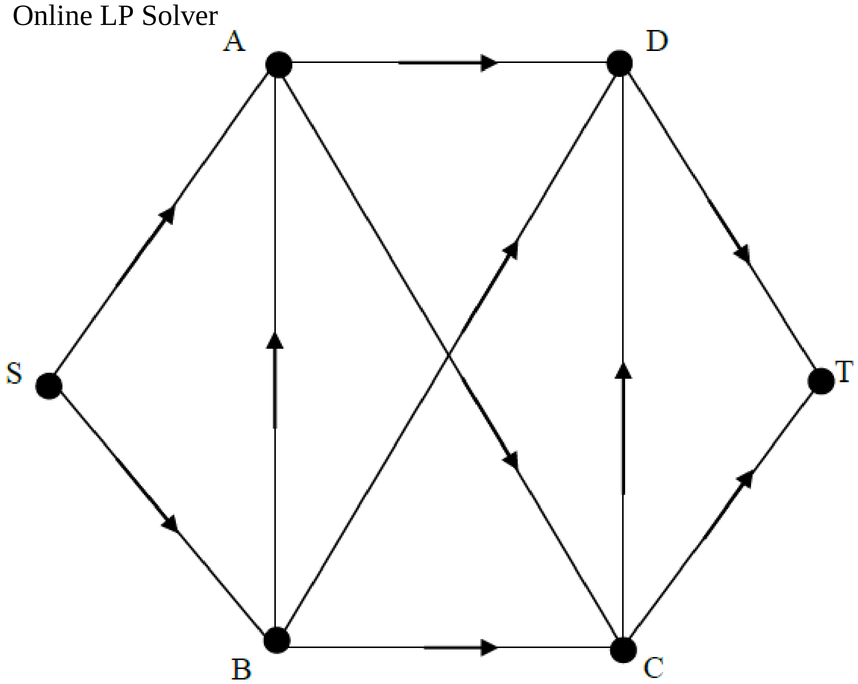
P	x	y	z	s_1	s_2	s_3	s_4	RHS
1	0	1.75	–0.5	0	0	0	1.25	37.5
0	0	3.5	2	1	0	0	–0.5	5
0	0	2	1	0	1	0	0	10
0	0	2.5	1	0	0	1	–0.5	55
0	1	0.75	0.5	0	0	0	0.25	7.5

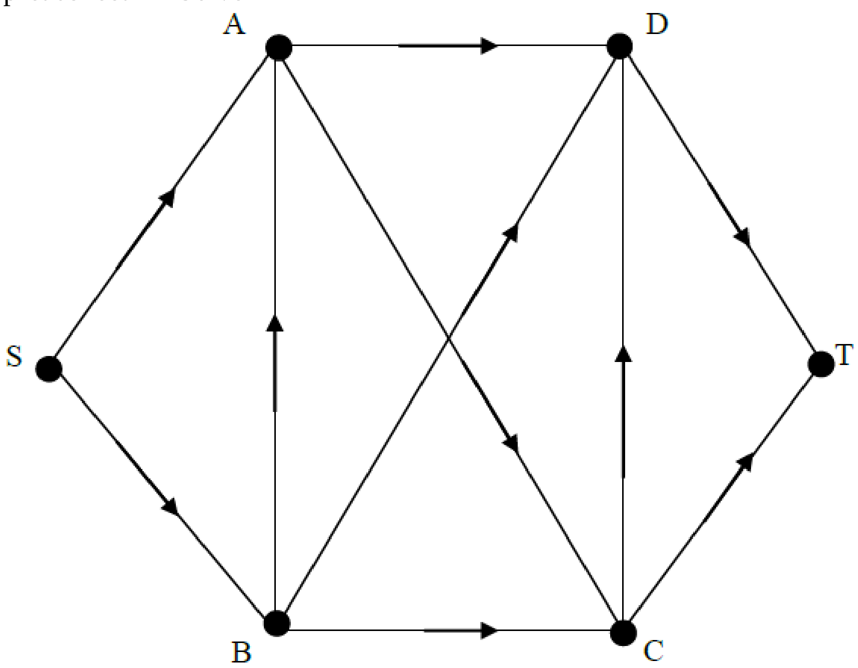
P	x	y	z	s_1	s_2	s_3	s_4	RHS

P	x	y	z	s_1	s_2	s_3	s_4	RHS

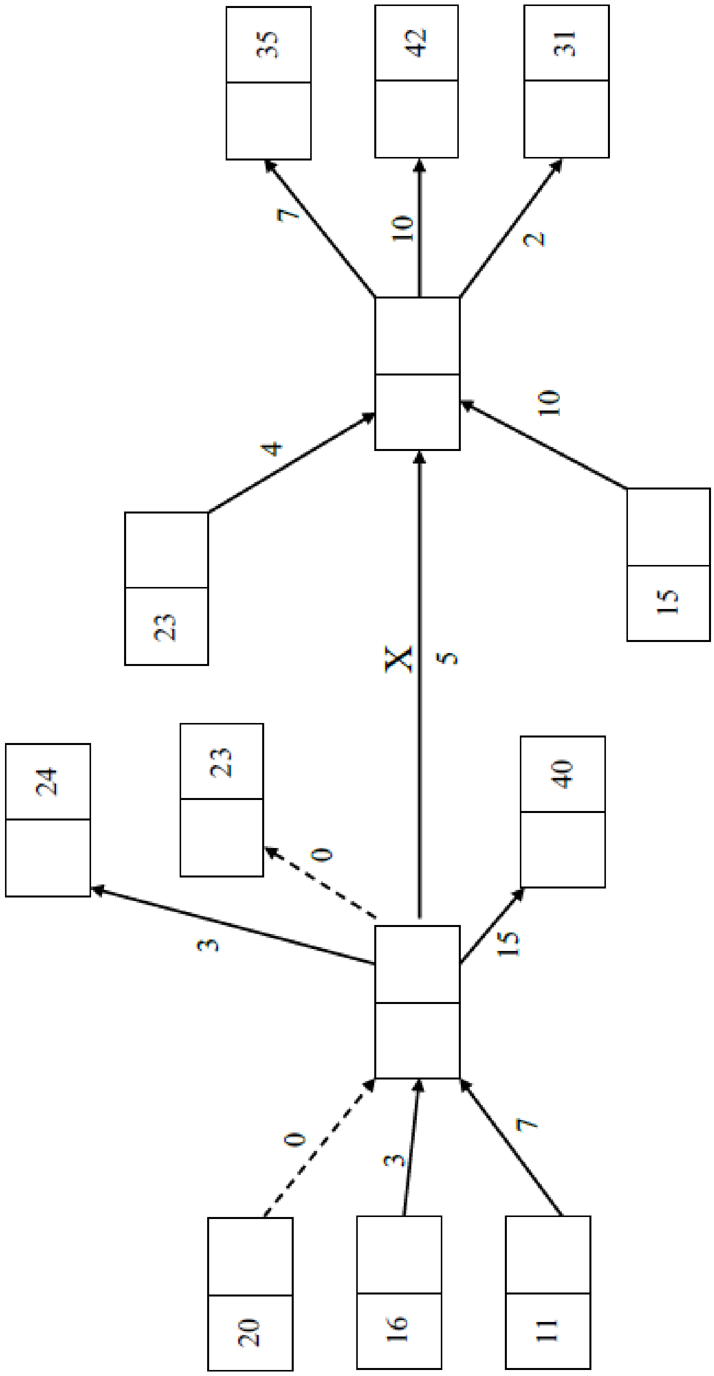
6 (ii) (*B*)

6 (iii)

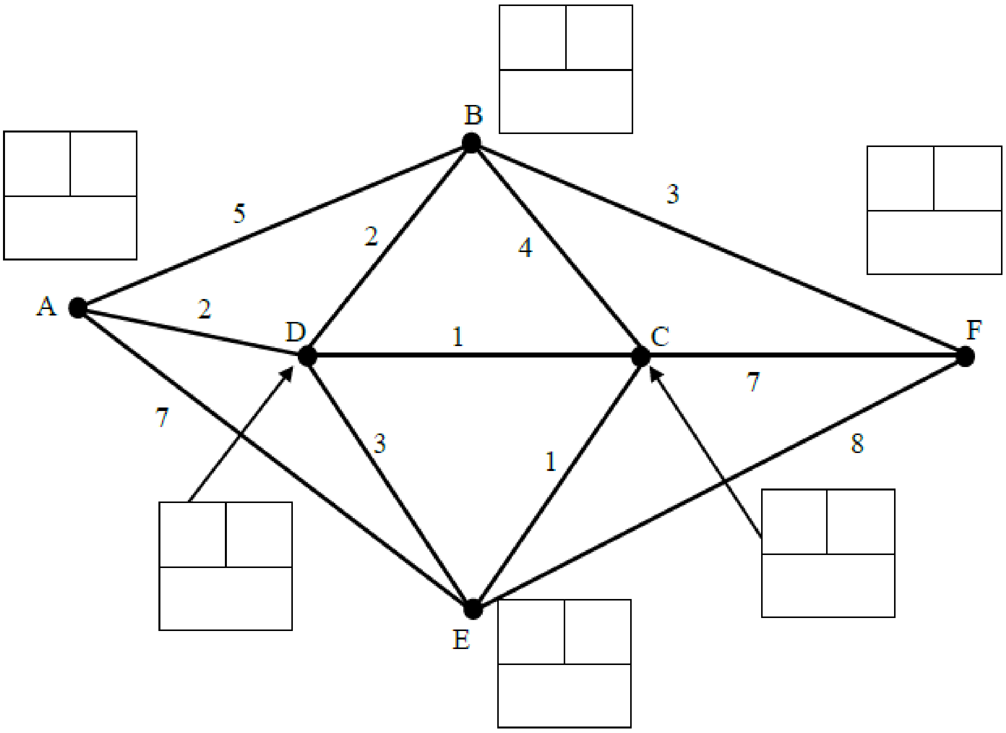
7 (i)	
7 (ii) (*A*)	
7 (ii) (*B*)	
7 (ii) (*C*)	
7 (iii) (*A*)	Online LP Solver  **(answer space continues on next page)**

7 (iii) (*A*)	**(continued)**
	Spreadsheet LP Solver
	
7 (iii) (*B*)	
7 (iv)	
7 (v)	

Spare copy of diagram for question 2



Spare copy of diagram and table for question 3 (i)



	A	B	C	D	E	F
A	-	4	3	2	4	
B	4	-	3	2	4	
C	3	3	-	1	1	
D	2	2	1	-	2	
E	4	4	1	2	-	
F						-

Spare copy of table for question 5 (i)

	A	B	C	D	E	F	G	H	I	J
A			8	4						
B					10				7	
C	8			6			12			
D	4		6					6		7
E		10				8			9	
F					8				8	
G			12					6		9
H				6			6			7
I		7			9	8				
J				7			9	7		

DO NOT WRITE IN THIS SPACE